Convex Optimization Theory
Homework 2

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Homework Requirement:

a. Can be written in either English or Chinese.

b. Must be submitted on time, due on Nov.30.

c. It is recommended to use Latex to complete the homework.

Problem 1. Consider the optimization problem

$$\begin{aligned} \min_{x_1,x_2} \quad & f(x_1,x_2) \\ s.t. \quad & 2x_1 + x_2 \geq 1 \\ & x_1 + 3x_2 \geq 1 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned} \tag{1}$$

(1). **Sketch the feasible set**. (ps: you can plot it by Matlab or Python.)
(2). For the objective function in (a),(b), first find the set of optimal solutions, and then solve them by Matlab or Python. For the objective function in (c) and (d), solve them by Programming and show the output.

a. $f(x_1,x_2) = x_1 + x_2$

b. $f(x_1,x_2) = -x_1 - x_2$

c. $f(x_1,x_2) = \max\{x_1,x_2\}$

d. $f(x_1,x_2) = x_1^2 + 9x_2^2$

Problem 2. Consider the QCQP

$$\begin{aligned} \min_{x} \quad & x_1^2 + x_2^2 \\ s.t. \quad & (x_1-1)^2 + (x_2-1)^2 \leq 1 \\ & (x_1-1)^2 + (x_2+1)^2 \leq 1 \end{aligned} \tag{2}$$

with variable $x \in \mathbf{R^2}$.

a. Sketch the feasible set and level sets of the objective. Find the optimal point x^* and optimal value p^*.

b. Give the KKT conditions. Do there exist Lagrange multipliers λ_1^* and λ_2^* that prove that x^* is optimal?

c. Derive and solve the Lagrange dual problem. Does strong duality hold?

Problem 3. Let $w \in \mathbf{R}^2$ and

$$X = \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad y = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$$

Solve the following problems using CVX with Matlab or CVXPY, and show the outputs.

a. Linear least square regression

$$\min_{w} \quad \|Xw - y\|_2^2$$

b. Lasso

$$\begin{aligned} \min_{w} \quad & \|Xw - y\|_2^2 \\ s.t. \quad & \|w\|_1 \leq t. \end{aligned}$$

for $t = 1$ and $t = 10$. In each case, is the solution the same as that of (a)?

c. Ridge regression

$$\begin{aligned} \min_{w} \quad & \|Xw - y\|_2^2 \\ s.t. \quad & \|w\|_2^2 \leq t. \end{aligned}$$

for $t = 1$ and $t = 10$. In each case, is the solution the same as that of (a)?