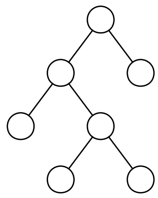
Written Homework 2

Due: 10월 26일 오후 9시

In class, we discussed the *height* of a binary search tree is an important measure of its efficiency of searching. This homework deals with a few related questions. **No late submission is accepted for this homework. Make sure to submit it correctly!**

1. (a) Fill in the nodes of the following binary tree with seven key values, $1, 2, 3, 4, 5, 6$, and 7, so that the tree becomes a binary search tree.



   (b) What is the number of binary search trees with the keys, $1, 2, 3, 4, 5, 6$, and 7, whose root node contains 4?

2. Consider the following two functions:

```
f(t)
{
  if(t=null) return 0
  l=f(LLINK(t))
  r=f(RLINK(t))
  if(l<r) return r+1
  else return l+1
}
```

```
g(t)
{
  if(t=null) return 0
  l=g(LLINK(t))
  r=g(RLINK(t))
  return l+r+1
}
```

   Given a binary tree represented as linked nodes and the pointer `t` to its root, what do `f(t)` and `g(t)` return?

3. Suppose that 1023 keys $\{4k + 1 \mid k = 1, 2, \ldots, 1023\} = \{5, 9, 13, \ldots, 4093\}$ are inserted into a binary search tree. Each of the 1023 keys is inserted once and only once. But we do not know the order of the insertions.

   (a) What is the maximum possible height of the resulting binary search tree?

   (b) Consider a sequence of the insertions of the keys that results in a binary search tree of the maximum height. List the first five keys. (There can be many answers.)

   (c) What is the minimum possible height of the resulting binary search tree?

   (d) 10 Suppose that a sequence of the insertions of the keys resulted in a binary search tree of the minimum height. List the first five keys that were inserted. (There is only one answer.)

   (e) What are the minimum and maximum possible total numbers of key comparisons it takes to insert all the 1023 keys? Hint: Note that $1023 = 2^{10} - 1$. Use the formula $\sum_{k=1}^{n} k \cdot 2^k = (n-1)2^{n+1} + 2$.

4. Suppose that n keys $\{1, 2, \ldots, n\}$ are inserted into a binary search tree. Each of the 1023 keys is inserted once and only once, and, therefore, there are $n!$ possible ways to insert the n keys. Recall that there are C_n different binary trees. (Recall WHW1.)

   (a) Consider the first a few entries of C_n and $n!$ as follows:

n	C_n	$n!$
0	1	1
1	1	1
2	2	2
3	5	6
4	14	24
5	42	120
6	132	720
7	429	5040
8	1430	40320
9	4862	362880
10	16796	3628800

   Prove that $C_n < n!$, for all $n \geq 3$. (Use mathematical induction.)

   (b) Prove that each of all possible C_n binary trees arises from an insertion order, among $n!$ possible ways, of the above n keys. (Again, use mathematical induction!)

5. Use the code for HW5, together with properly completed `add()`, and write a program to perform the following experiment that estimates the average height of BST, for $N = 50, 100, 200$ and 500:

   (a) Generate N random data using the standard C library function `rand()`.

   (b) Insert the generated data into a BST, and obtain the height of the resulting BST, whose height must be between $\lg N$ and $N - 1$.

   (c) Repeat Step (b) 100 times. And calculate the average of 100 results.

   Present your program and discuss your experiment result.