Simplicity of the Automorphism Groups of Some Homogeneous Structures

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BPGMT 2018, University of Oxford

Outline

- Amalgamation classes and semi-free amalgamation.
- Stationary independence relation on the structures.
- Automorphism groups of structures with the stationary independence relation

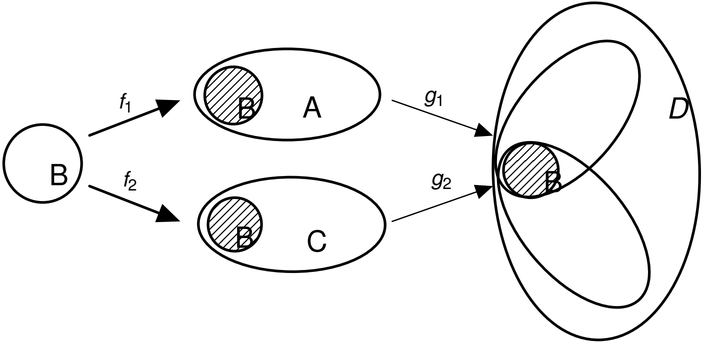
Amalgamation Class

Definition

Let $\mathcal{L}$ be a relational language, an amalgamation class is a class $\mathcal{C}$ of finite $\mathcal{L}$-structures satisfying the following three conditions:

1. Hereditary property: *For every $A \in \mathcal{C}$ and any substructure $B \subseteq A$ we have $B \in \mathcal{C}$;*
2. Joint embedding property: *For every $A, B \in \mathcal{C}$ there exists $C \in \mathcal{C}$ such that C contains both A and B as substructures;*
3. Amalgamation property: *For A, B, C and $f_1 : B \to A$, $f_2 : B \to C$ are embeddings, there is $D \in \mathcal{C}$ and embeddings $g_1 : A \to D$, $g_2 : C \to D$ such that $g_1 \circ f_1 = g_2 \circ f_2$.*

Amalgamation Property



Free Amalgamation

- the class of all finite graphs with free amalgamation

Definition

We say A, C are freely amalgamated over B, if for any $a \in g_1(A) \setminus g_1 f_1(B), c \in g_2(C) \setminus g_2 f_2(B)$, a, c are not in any relation.

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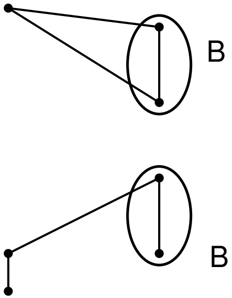

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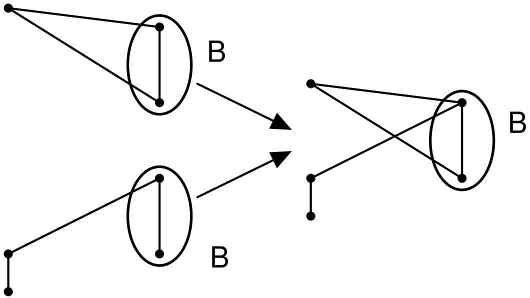

Fraïssé's Theorem

Definition

An $\mathcal{L}$-structure $\mathcal{M}$ is homogeneous if isomorphisms beteween finite substructures extend to automorphisms of $\mathcal{M}$.

Theorem (Fraïssé's Theorem)

There is a one-to-one correspondence between amalgamation classes and countable homogeneous structures, called the Fraïssé limit

$$\begin{tikzcd} \boxed{\text{Countable Homogeneous structures } \mathcal{M}} \arrow[d, leftrightarrow, swap, "\mathcal{C} = \{ \text{ all finite substructure of } \mathcal{M}\}"] \arrow[d, phantom, "\mathcal{M} \text{ embeds every structure in } \mathcal{C}", near start, shift left=20ex] \\ \boxed{\text{Amalgamation classes } \mathcal{C}} \end{tikzcd}$$

Semi-free amalgamation

Definition

$\mathcal{C}$ is a semi-free amalgamation class if there exists $\mathcal{L}' \subsetneq \mathcal{L}$ such that for any finite structures $A, B, C \in \mathcal{C}$ and embeddings $f_1 : B \to A, f_2 : B \to C$, there exist $D \in \mathcal{C}$ and embeddings $g_1 : A \to D, g_2 : C \to D$ such that $g_1 f_1(B) = g_2 f_2(B) = g_1(A) \cap g_2(C)$ and for any $a \in g_1(A) \setminus g_1 f_1(B), c \in g_2(C) \setminus g_2 f_2(B)$, if a, c are related by some $R \in \mathcal{L}$, then $R \in \mathcal{L}'$.

Note this is a generalisation of the free amalgamation.

Cherlin's List

Language: $\{R, G, X\}$

#1	RXX GGX XXX

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#27	RRY RRX GYX GXX GYY YYX YXX XXX YYY

- S a set of the forbidden triangles
- $Forb_c(S)$ the set of all finite completely edge-labelled $\mathcal{L}$-structures that do not embed any triangle in S.
- $\mathcal{M}_S$ the Fraïssé limit of $Forb_c(S)$.

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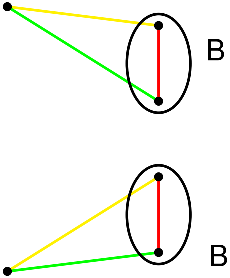

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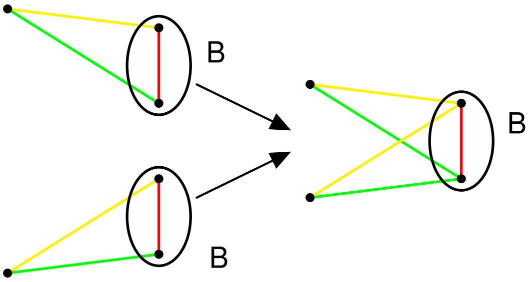

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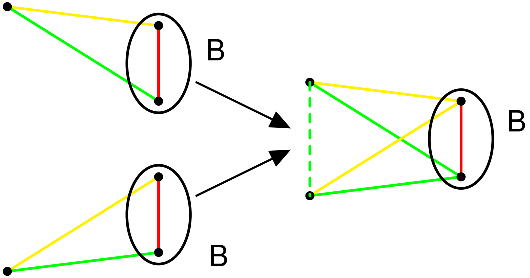

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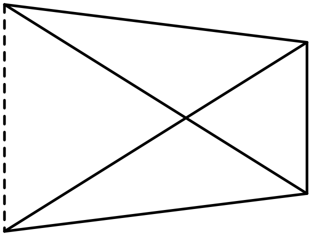

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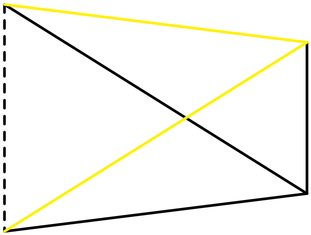

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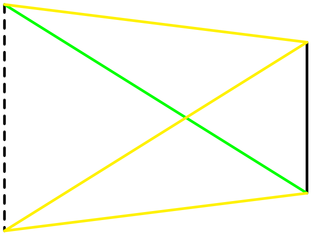

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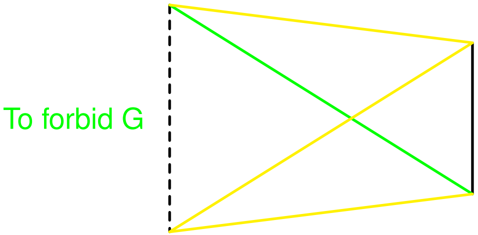

Therefore, this is a semi-free amalgamation class :)

Stationary Independence Relation

Definition

Let $\mathcal{M}$ be a structure. $\mathop{\smile\!\!\!\!\!\!|}\,$ is a stationary independence relation if the following is satisfied for any substructure $A, B, C, D \subseteq \mathcal{M}$:

- *Invariance: A, C independence over B depends only on the type of ABC*
- *Monotonicity: $A \mathop{\smile\!\!\!\!\!\!|}_B CD \Rightarrow A \mathop{\smile\!\!\!\!\!\!|}_B C$, $A \mathop{\smile\!\!\!\!\!\!|}_{BC} D$*
- *Transitivity: $A \mathop{\smile\!\!\!\!\!\!|}_B C$, $A \mathop{\smile\!\!\!\!\!\!|}_{BC} D \Rightarrow A \mathop{\smile\!\!\!\!\!\!|}_B D$*
- *Symmetry: $A \mathop{\smile\!\!\!\!\!\!|}_B C \Rightarrow C \mathop{\smile\!\!\!\!\!\!|}_B A$*
- *Existence: If p is a type over B and C is a finite set, then p has a realisation that is independent from C over B*
- *Stationarity: If $\bar{a}$ and $\bar{a}'$ have the same type over B and are both independent from C over B, then $\bar{a}$ and $\bar{a}'$ have the same type over BC.*

Proposition

Let $\mathcal{C}$ be the set of all finite graphs and $\mathcal{M}$ be its Fraïssé limit. For substructure A, B, C of $\mathcal{M}$, let $A \mathop{\smile\!\!\!\!\!\!|}_B C$ if the substructure $ABC \subseteq \mathcal{M}$ agrees with the free amalgamation of AB and BC over B. Then this is a stationary independence relation on $\mathcal{M}$.

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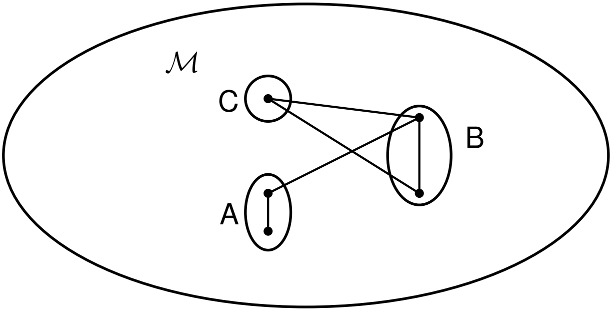

Definition

$g \in Aut(\mathcal{M})$ moves almost maximally if for every 1-type over a finite set X has a realisation a such that $a \mathop{\smile\!\!\!\!\!\!\;|}_X ga$.

Theorem (Tent and Ziegler, 2012, Corollary 5.4)

Let $\mathcal{M}$ be a countable homogeneous structure with a stationary independence relation. If $g \in Aut(\mathcal{M})$ moves almost maximally, then any element of $Aut(\mathcal{M})$ is the product of sixteen conjugates of g.

- we want to find a stationary independence relation on the Fraïssé limit $\mathcal{M}_S$ of the semi-free amalgamation classes. The key is to find a 'unique' amalgam for every A, C over B.
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Invariance and Stationarity

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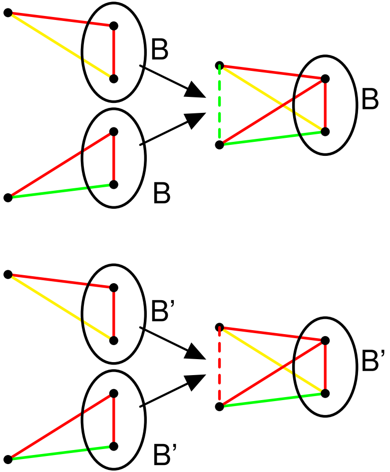

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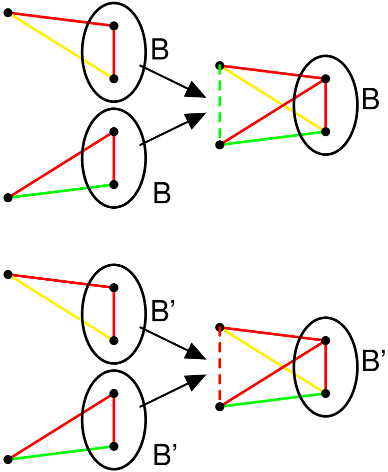

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Prioritised Semi-Free Amalgamation

We can put an order on $\mathcal{L}'$!

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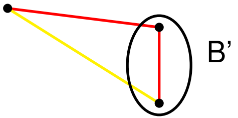

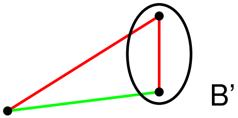

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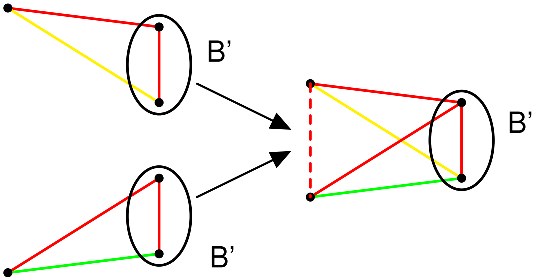


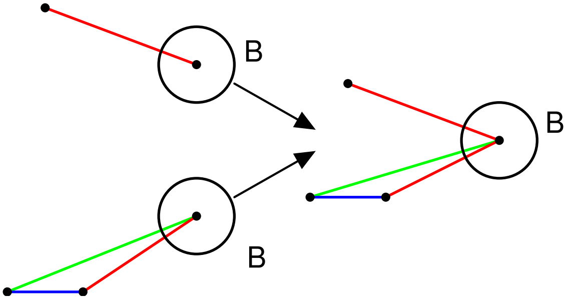
Prioritised Semi-Free Amalgamation

Definition

Suppose $\mathcal{L}' = \{R_1, ..., R_m\}$*. We can order the set as* $R_1 > \cdots > R_m$ *and for every* $A, B, C \in Forb_c(S)$*, where* $B \subseteq A, C$*, define the following way to amalgamate* A *and* C *over* B*: for each* $a \in A \setminus B, c \in C \setminus B$*, first check whether abc form a forbidden triangle for any* $b \in B$ *if* $(a, c) \in R_1$*. If* $B = \emptyset$ *or colouring* (a, c) *by* R_1 *does not form any forbidden triangle, we let* $(a, c) \in R_1$*. Otherwise, we check the same thing for* $(a, c) \in R_2$ *and so on so forth. In other word,* $(a, c) \in R_i$ *where* i *is the smallest possible integer such that* $(a, b)(b, c)R_i \notin S$*.*

Example

$\mathcal{L} = \{R, G, Y, X\}$ with forbidden triangles
$S = \{RXX, GGX, YXX, XXX\}$ and $\mathcal{L}' = \{R, G\}$ with order $G > R$.



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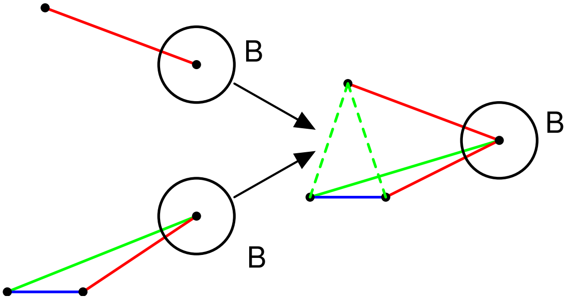

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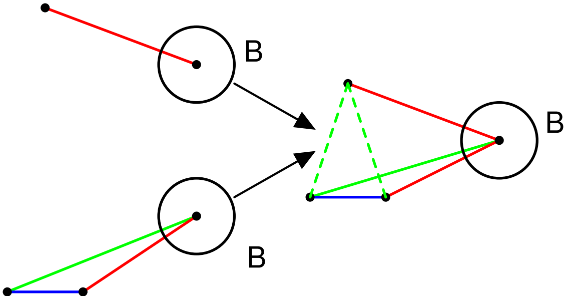

Condition on S

- *S does not contain any constraint of the form R_iR_jR' where $R_i, R_j \in \mathcal{L}', R' \in \mathcal{L}$*
- *Let $\mathcal{L}' = \{R_1, R_2\}$ with $R_1 > R_2$ and for some subset $\mathcal{L}^\star \subseteq \mathcal{L} \backslash \mathcal{L}'$ and S contains all triangles of the form $R'R''R'''$, R_2R_2R'' where $R' \in \mathcal{L} \setminus \{R_2\}$, $R'', R''' \in \mathcal{L}^\star$ and S contains no other triangle involving R_1 or triangle of the form R_2R_2R', $R' \in \mathcal{L}$.*

Proposition

Let S be a set of forbidden triangles satisfying either one of the above conditions. For substructure A, B, C of $\mathcal{M}_S$, let $A \downarrow_B C$ if the substructure $ABC \subseteq \mathcal{M}$ agrees with the prioritised semi-free amalgamation of AB and BC over B. Then this is a stationary independence relation on $\mathcal{M}$.

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- ✓ we want to find a stationary independence relation on $\mathcal{M}_S$
- we want to find an automorphism of $\mathcal{M}$ that moves almost maximally.

Theorem

Let $\mathcal{M}$ be the Fraïssé limit of a free amalgamation classes and there is a stationary independence relation on $\mathcal{M}$, then for any non-trivial automorphism $g \in Aut(\mathcal{M})$, there exist $h \in Aut(M)$ such that $[h, g]$ moves almost maximally.

Lemma

- *any 1-type over some finite subset X has either exactly one realisation in X or infinitely many.*
- *for any non-trivial automorphism $g \in Aut(\mathcal{M})$, there does not exist a type with infinite realisations such that g fixes the set of realisations pointwise.*
- *for any $A, B, C \subseteq \mathcal{M}$ such that $A \mathop{\smile\!\!\!\!\!\!|}_B C$, $A \mathop{\smile\!\!\!\!\!\!|}_{B'} C$ for any subset $B' \subseteq B$*

Proof.

Idea: We build h by a back-and-forth argument.
Sketch: Suppose we already have $h' : A \to B$, a partial isomorphism between finite substructure of $\mathcal{M}$. Let p be a type over some finite set X. We may assume $X \subseteq A$ by extending h'. We want to extend h' so that X has a realisation a such that $a \mathop{\smile\!\!\!\!\!\!|}_X [g, h']a$.

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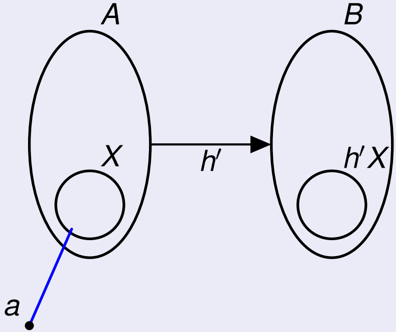

$a \models p$ and $a \downarrow_X A$

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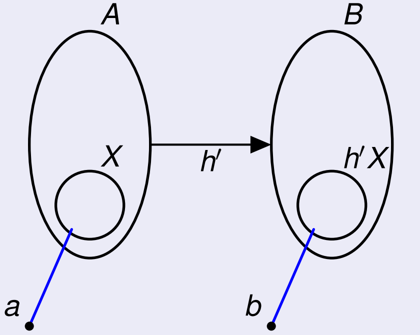
Sketch: Suppose we already have $h' : A \to B$, a partial isomorphism between finite substructure of $\mathcal{M}$. Let p be a type over some finite set X. We may assume $X \subseteq A$ by extending h'. We want to extend h' so that X has a realisation a such that $a \mathop{\smile\hspace{-0.9em}\mid}_X [g, h']a$.



$a \models p$ and $a \mathop{\smile\hspace{-0.9em}\mid}_X A$

$b \models h' \cdot tp(a/A)$ and $b \mathop{\smile\hspace{-0.9em}\mid}_X g^{-1}B$

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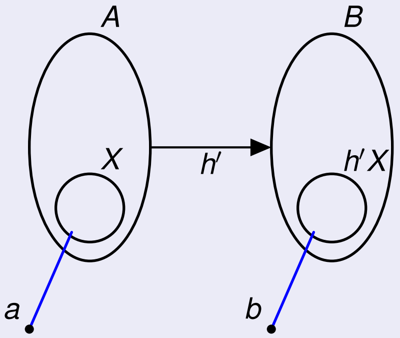

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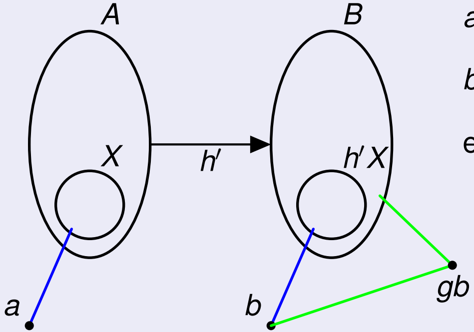

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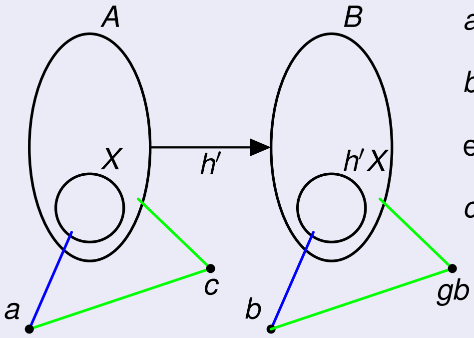

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$b \models h' \cdot tp(a/A)$ and $b \mathop{\smile\!\!\!\!\!|}_X g^{-1}B$

extend h' by sending a to b

$c \models h'^{-1} \cdot tp(gB/bB)$

and $c \mathop{\smile\!\!\!\!\!|}_{aA} ga$

Theorem

Let S be a set of forbidden triangles satisfying one of the conditions. Let $\mathcal{M}_S$ be the Fraïssé limit of Forb(S), then for any non-trivial automorphism $g \in Aut(\mathcal{M}_S)$, there exist $k, h \in Aut(\mathcal{M}_S)$ such that $[h, [g, k]]$ moves almost maximally.

Corollary

Let S be a set of forbidden triangles satisfying one of the conditions. Let $\mathcal{M}_S$ be the Fraïssé limit of Forb(S), then $Aut(\mathcal{M}_S)$ is simple. In particular, if S is any set in Cherlin's list, with the exception of # 26, then $Aut(\mathcal{M}_S)$ is simple.

Reference

- *G. L. Cherlin. The Classification of Countable Homogeneous Directed Graphs and Countable Homogeneous n-Tournaments. American Mathematical Soc., 1998.*
- *K. Tent and M. Ziegler. "On the isometry group of the Urysohn space". In: Journal of the London Mathematical Society 87.1 (2013), pp. 289-303.*

Thank you :)

Reference

- *G. L. Cherlin. The Classification of Countable Homogeneous Directed Graphs and Countable Homogeneous n-Tournaments. American Mathematical Soc., 1998.*
- *K. Tent and M. Ziegler. "On the isometry group of the Urysohn space". In: Journal of the London Mathematical Society 87.1 (2013), pp. 289-303.*

Thank you :)