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Coloured Graphs Determined by Forbidden Triangles

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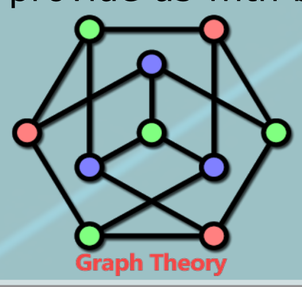
What is Graph Theory?

A graph consists of a set of dots (nodes), connected by lines (edges). Graphs appear in our daily life in different forms. We can think of the tube map as a graph. Our social networks can also be modelled as graphs if we represent each person as a node. Studies of algebraic properties of these graphs provide us with better understandings of their underlying structures.



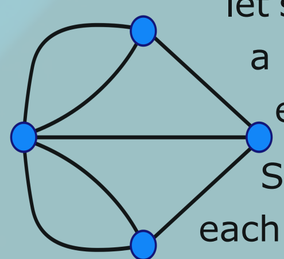
Graph Theory

History and Current Application

Königsberg bridge problem

In the early 18th century, there were seven bridges in Königsberg (now Kaliningrad, Russia). The residents tried to walk around the town in a way that they only cross each bridge once.

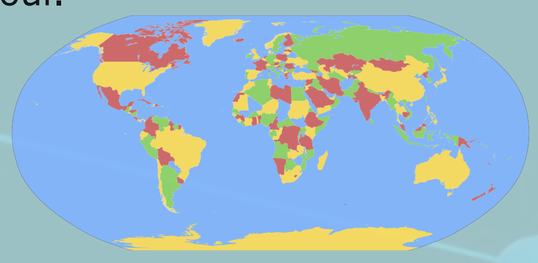
In 1736, Euler showed that such a walk is not possible there. Why? let's represent each landmass as a node and each bridge as an edge connecting two nodes.



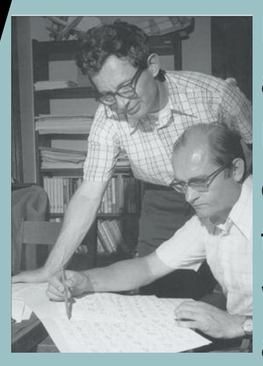
Since we can only cross each bridge once, we have to walk into a landmass by one bridge and then come out by a different one. So all nodes except two (the starting/ending node) should have an even number of edges connected to it.

Four colour theorem

The four colour theorem states that any geographic map can be coloured using no more than four colours in a way that no two adjacent regions share the same colour.



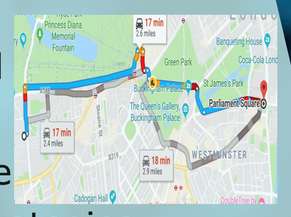
The theorem was proved in 1976 by Kenneth Appel and Wolfgang Haken. Their proof combined graph theory techinque with computer-aided calculations.



Kenneth Appel and Wolfgang Haken in the 1970s

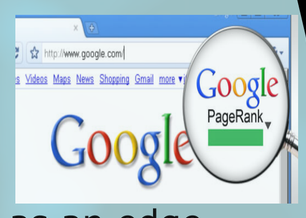
Path optimisation

Research in path optimisation tries to find the optimal path using graph theory. It is applied in map navigation, warehouse pick path planning and even trading strategies.



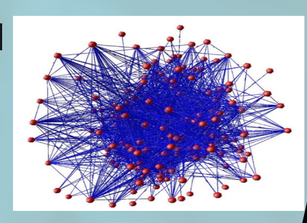
Google PageRank

PageRank is an algorithm that assigns a weight to each website, measuring its relative importance. It represents each website as a dot and hyperlink as an edge.



Biological neural network

In biology, graphs are used to model activations between neurons. Each node is a neuron and edge represents the connection between neurons.



Machine learning (AI)

Many machine learning algorithms, for example the well-known AlphaGo from DeepMind, depend on probabilistic graph theory and tree (a special type of graph) search.

My Research

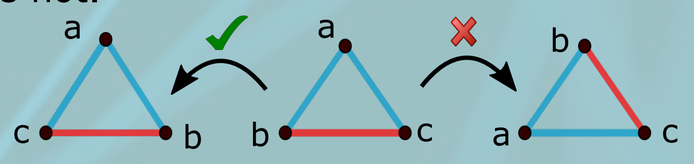
You are given:
* A piece of paper
* Three pens: red, green and blue
* A list of forbidden triangles, e.g.  , that players cannot create in the graph

Players alternately
* Add a node and connect this node with every but one existing node using one of the pens.
* Let the next player complete the remaining edge.
* Player loses if s/he cannot draw an edge without creating a forbidden triangle in the list. So your aim is to create a graph that the next player cannot complete.

Did you know that there is a winning strategy for some lists of forbidden triangles? For example, for  , you can always complete every graph with either red or green.

We call this property of being able to continue forever (i.e. using either green or red) the amalgamation property. I study mathematical structures with this property and their symmetries.

Symmetry of a graph can be considered as a rearrangement (called a permutation) of nodes that preserves the colours. For example, swapping b,c in the following graph preserves the colour whereas swapping a,b does not.



The collection of symmetries is called a group. Groups are applied in Physics and Chemistry to model and classify objects such as crystals, molecules, hydrogen atoms, etc.

Because of the amalgamation property, I can find coloured graphs that grow infinitely large and I study the groups of these graphs using methods from logic.

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