Understanding Longitudinal Modified Treatment Policies in `lmtp`

Dynamic Treatment Strategies Based on Propensity Score Thresholds

Based on Díaz et al. (2021)

August 7, 2025

Outline

The Research Question

Clinical Scenario

How do we estimate the effect of a dynamic treatment strategy where:

- Treatment continues as long as the cumulative propensity score remains above a threshold α
- We have longitudinal data with time-varying confounders
- Outcome: probability of surviving event-free

Example

- $A_1 = 1$ if $g_1(1|L_1) > \alpha$
- $A_2 = 1$ if $g_1 \times g_2 > \alpha$ (but which policy?)
- In our code: $Y_t = 1$ means event occurred (note the coding!)
- Estimate: $P(Y_3 = 0) = P(\text{no event by time 3})$ under the policy

What are Modified Treatment Policies?

Definition (Modified Treatment Policy)

An intervention where the post-intervention treatment can depend on:

- The natural (observed) value of treatment
- The patient's history
- A random component

$$A_t^d = d(A_t, H_t, \epsilon_t)$$

Examples

- Static: $d(a_t, h_t) = 1$ (always treat)
- Dynamic: $d(a_t, h_t) = \mathbb{I}(L_t > c)$ (treat if covariate exceeds threshold)
- Modified: $d(a_t, h_t) = \min(a_t + \delta, u_{\max})$ (increase dose by δ)

Two Fundamentally Different Policies

Policy 1: Sequential Adherence

"Treat at time 2 if currently on treatment AND cumulative propensity $> \alpha$"

```
return(ifelse(
  data$A1 * g1 * g2 > alpha,
  1,
  data$A2
))
```

- More conservative
- Respects treatment continuity
- Realistic for clinical protocols

Policy 2: Cumulative Propensity

Treat at time 2 if cumulative propensity $> \alpha$ regardless of current treatment

```
return(ifelse(
  g1 * g2 > alpha,
  1,
  data$A2
))
```

- More aggressive
- Based purely on propensity
- Theoretical optimal

Example: When Policies Differ

Consider a patient with:

- $g_1 = 0.4 > \alpha = 0.3$ (high propensity at time 1)
- Observed $A_1 = 0$ (not treated despite high propensity)
- $g_2 = 0.8$
- $g_1 \times g_2 = 0.32 > \alpha$

Policy 1

$A_1 \times g_1 \times g_2 = 0 \times 0.32 = 0$

$0 < 0.3 \Rightarrow A_2 =$ natural value

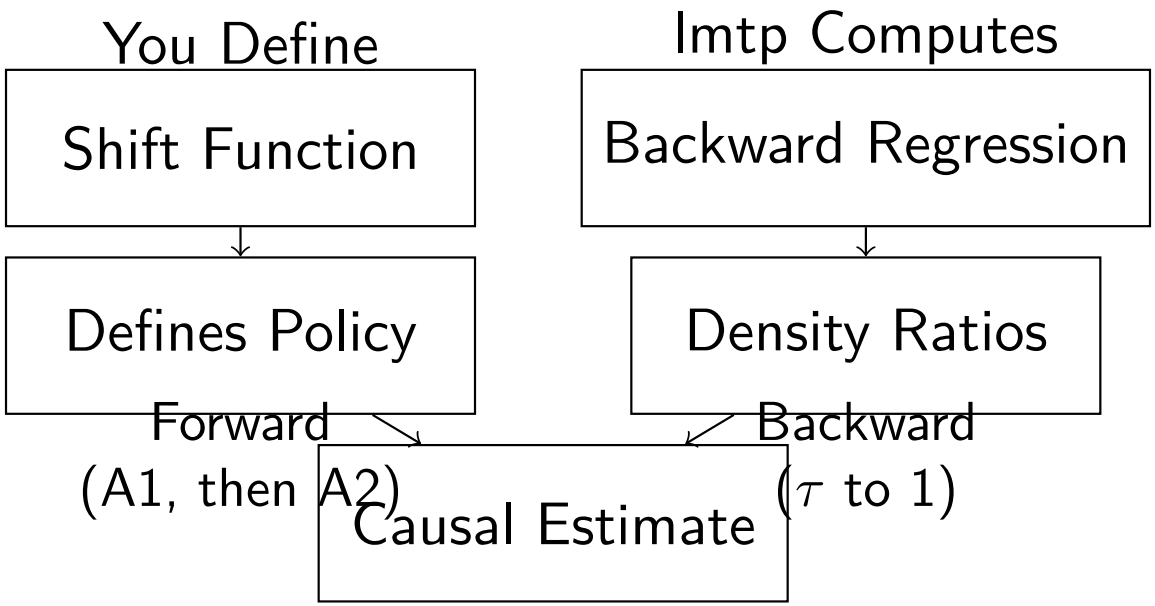
Policy 2

$g_1 \times g_2 = 0.32$

$0.32 > 0.3 \Rightarrow A_2 = 1$

Key insight: These define different causal questions!

The lmtp Framework



Backward Sequential Regression (Our Setting)

The Sequential Regression Process

Starting from the end and working backward:

$$q_{\tau+1} = Y_{\tau+1} \text{ (final outcome)} \tag{1}$$

$$q_t(a_t, h_t) = \mathbb{E}[R_{t+1} \times q_{t+1}(A^d_{t+1}, H_{t+1}) | R_t = 1, A_t = a_t, H_t = h_t] \tag{2}$$

- R_t: at-risk indicator (no event by time t)
- $Y_t = 1$: event occurred (our coding)
- Regression only uses observations still at risk
- Final estimate: $\theta = 1 - \mathbb{E}[q_1(A^d_1, L_1)]$ estimates $P(Y_3 = 0)$

Important: Outcome Coding

In our R code, $Y_t = 1$ means event **occurred**. The lmtp package handles this internally when `outcome_type = "survival"`. Different papers use different codings!

Understanding Different Outcome Codings

Our R Code

- $Y_t = 1$: Event **occurred** by time t
- $Y_t = 0$: Event **not occurred**
- Estimating: $P(Y_3 = 0)$ (survival)
- No competing risks

Sequential regression:

$$q_t = \mathbb{E}[R_{t+1} \times q_{t+1} | \cdots]$$

Simple and clean!

Díaz et al. Papers

- $Y_t = 1$: Event **not occurred**
- $Y_t = 0$: Event **occurred**
- With competing risks:
  - $Z_t = 0$: No competing event
  - $Z_t = 1$: Competing event occurred

Their formula:

$$\phi_t = \mathbb{E}[R_{t+1}\{R_{t+2}\phi_{t+2} + Z_{t+1}\} | \cdots]$$

The "$+Z_{t+1}$" makes sense with their coding!

Key insight: Different coding conventions lead to different formulas, but both estimate the same thing: survival probability under the LMTP!

Why the Competing Risks Formula Works

Understanding the Díaz et al. Formula

$$\phi_t = \mathbb{E}[R_{t+1}\{R_{t+2}\phi_{t+2} + Z_{t+1}\}|\cdots]$$

With their coding ($Y_t = 1$ means no event, $Z_t = 1$ means no competing event):

- **If** $R_{t+1} = 0$: Person had an event by $t+1$, contributes 0
- **If** $R_{t+1} = 1$: Person is still at risk at $t+1$
  - **If** $Z_{t+1} = 1$: Competing event occurred $\rightarrow$ contributes $1 \times 1 = 1$
  - **If** $Z_{t+1} = 0$: No competing event $\rightarrow$ continues recursion

The Beauty of the Formula

The "$+Z_{t+1}$" term elegantly handles competing risks:

- When competing event occurs: $Z_{t+1} = 1$, stops recursion
- When no competing event: $Z_{t+1} = 0$, but $R_{t+2} = 0$ if main event occurs
- The formula naturally tracks both types of events!

Density Ratios and Compatibility

The Key Mechanism

$$w_t = \frac{g^d(a_t|h_t)}{g(a_t|h_t)}$$

where:

- $g^d(a_t|h_t)$: density under your intervention
- $g(a_t|h_t)$: observed density

Key Insight

$g^d(a_t|h_t)$ can be **ZERO** when history is incompatible with the policy!

How Weights Enforce Compatibility

For a deterministic policy "treat if $g_1 > \alpha$":

Scenario	$g^d(a\|h)$	$g(a\|h)$	**Weight**
$g_1 > \alpha$, $A_1 = 1$	1	g_1	$1/g_1 > 1$
$g_1 > \alpha$, $A_1 = 0$	0	$1 - g_1$	0
$g_1 \leq \alpha$, any A_1	$g(a\|h)$	$g(a\|h)$	1

Result

- Compatible observations: weighted appropriately
- Incompatible observations: zero weight
- Natural values: unit weight

Example: Weight Calculation

Patient with $g_1 = 0.4 > \alpha = 0.3$ but observed $A_1 = 0$:

1. Policy requires: $A_1 = 1$ (since $g_1 > \alpha$)
2. But observed: $A_1 = 0$
3. Density under policy: $g^d(0|h_1) = 0$ (impossible!)
4. Observed density: $g(0|h_1) = 1 - 0.4 = 0.6$
5. Weight: $w_1 = 0/0.6 = 0$

Consequence

This patient contributes **nothing** to the estimate because their history is incompatible with the policy being evaluated!

R Implementation

```r
# Important: In our data generation, Y=1 means event OCCURRED
# Y2 = as.numeric(U.Yt2 > plogis((L1+(A1*2))))
# Y3 = ifelse(Y2==0, as.numeric(U.Yt3+1 < plogis(...)), 1)

# Define shift function for Policy 2
dynamic_threshold_shift <- function(data, trt) {
  # Compute propensity scores
  fit1 <- glm(A1 ~ L1, data = data, family = binomial())
  g1 <- predict(fit1, type = "response")

  if (trt == "A1") {
    return(ifelse(g1 > alpha, 1, data$A1))
  } else if (trt == "A2") {
    fit2 <- glm(A2 ~ L1 + A1 + L2, data = data,
                family = binomial())
    g2 <- predict(fit2, type = "response")

    # Policy 2: Cumulative propensity
    return(ifelse(g1 * g2 > alpha, 1, data$A2))
  }
}

# Run lmtp - it handles our Y coding when outcome_type = "survival"
# Estimates P(Y3 = 0) = P(no event by time 3)
result <- lmtp_tmle(
  data = data,
  trt = c("A1", "A2"),
  outcome = c("Y2", "Y3"),
  shift = dynamic_threshold_shift,
  outcome_type = "survival"
)
```

What the Shift Function Does NOT Do

Common Misconception

"The shift function assumes previous histories satisfy the protocol"

Reality

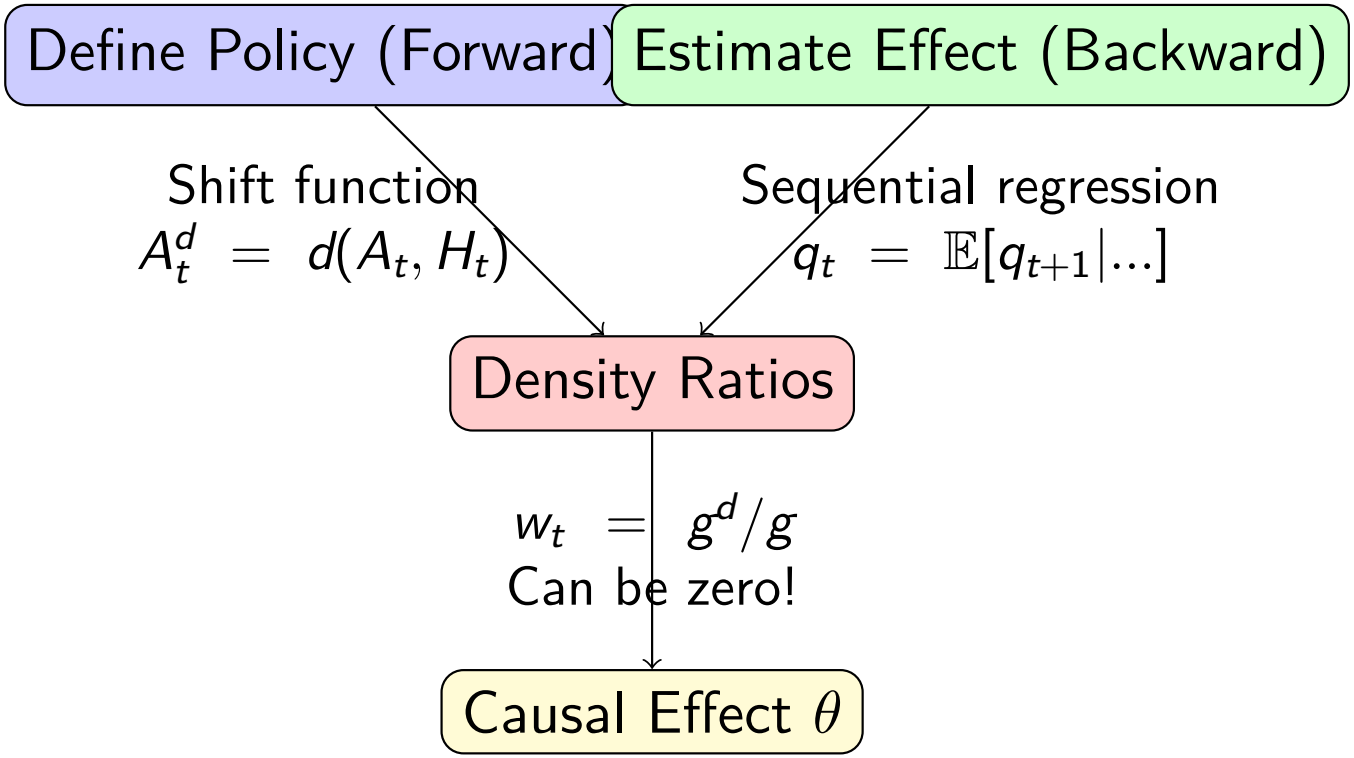
The shift function **defines a policy**, it doesn't make assumptions!

- The shift function is called with **observed data**
- It defines what treatment **should be** under your policy
- The weighting mechanism handles incompatible histories
- Zero weights effectively filter out incompatible observations

Key Takeaways

1. **Shift functions define policies**, not assumptions about data
2. **Two distinct policies** answer different causal questions:
   - Sequential adherence: realistic, conservative
   - Cumulative propensity: theoretical, aggressive
3. **lmtp elegantly handles incompatibility**:
   - Density ratios can be zero
   - Incompatible histories get zero weight
   - Correct counterfactual estimation
4. **Outcome coding matters!**
   - Our code: $Y = 1$ means event occurred
   - Some papers: $Y = 1$ means event NOT occurred
   - Different codings $\rightarrow$ different formulas
   - lmtp's `outcome_type = "survival"` handles this
5. **Backward sequential regression** is flexible for:
   - Different outcome codings
   - Competing risks (when present)
   - Time-varying confounders

The Complete Picture



Practical Guidelines

When to Use Policy 1 (Sequential Adherence)

- Clinical protocols require treatment continuity
- Stopping and restarting treatment is problematic
- You want to respect observed treatment patterns

When to Use Policy 2 (Cumulative Propensity)

- You want the theoretically optimal policy
- Treatment can be started/stopped freely
- You're interested in propensity-based decisions

Remember

Different policies answer different causal questions!

Thank You

Questions?

References

- Díaz et al. (2021). Nonparametric causal effects based on longitudinal modified treatment policies. JASA.
- Díaz et al. (2024). Causal survival analysis under competing risks using longitudinal modified treatment policies. Lifetime Data Analysis.
- Williams & Díaz (2023). lmtp: An R Package for Estimating the Causal Effects of Modified Treatment Policies. Observational Studies.

Key Insight from This Talk

Understanding outcome coding is crucial! Always check whether $Y = 1$ means event occurred or event-free. This affects the formulas but not the underlying concepts.