

## Programming Assignment 1

### Introduction

In this programming assignment, parallel computing is used to sum  $N$  random numbers with  $P$  processors, assuming  $N$  is a multiplier of  $P$ . The program is coded in C++ with basic MPI commands. This document reports runtime and speedup achieved at different  $N$  and  $P$ .

### Theory

Theoretically, the total runtime for a parallel system is expected to be

$$T_p(n, p) = T_{rv, p}(n, p) + T_{sum, p}(n, p) = \Theta\left(\frac{n}{p}\right) + \Theta\left(\frac{n}{p} + \log(p)\right) = \Theta\left(\frac{n}{p} + \log(p)\right) \quad (1)$$

Comparing to a serial system, of which the total runtime is computed at  $P = 1$ ,

$$T_s(n, 1) = T_{rv, s}(n, 1) + T_{sum, s}(n, 1) = \Theta(n) + \Theta(n) = \Theta(n) \quad (2)$$

the speed up achieved is

$$S = \frac{T_s(n, 1)}{T_p(n, p)} = \frac{\Theta(n)}{\Theta\left(\frac{n}{p} + \log(p)\right)} \quad (3)$$

### Results

1. Runtime while changing the number of processors  $P$  from 1 up to 16.

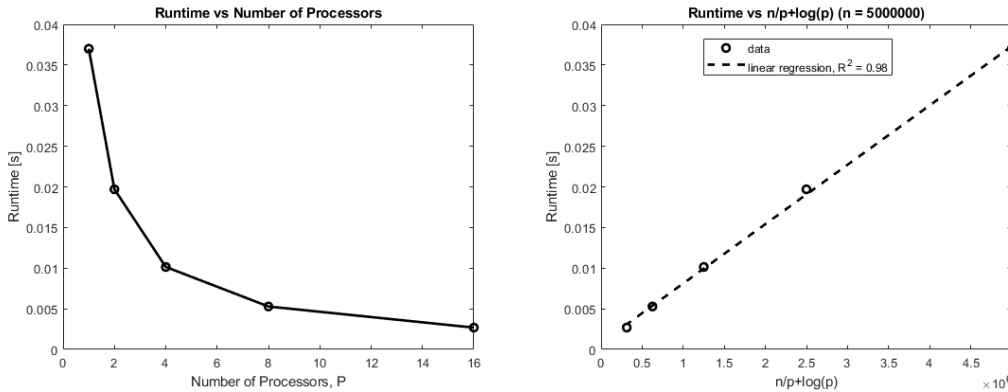


Figure 1: Runtime vs.  $P$  and Runtime vs.  $\frac{N}{P} + \log(P)$  ( $N = 5 \times 10^6$ )

The run time decreases with  $P$ . The right figure shows that this decrease follows  $T_n(n, p) \propto \frac{n}{p} + \log(p)$  for different  $P$ , with  $R^2 = 0.98$ . It matches the theory in Equation (1).

2. Runtime while changing  $N$  from  $5 \times 10^6$  to  $1.6 \times 10^8$

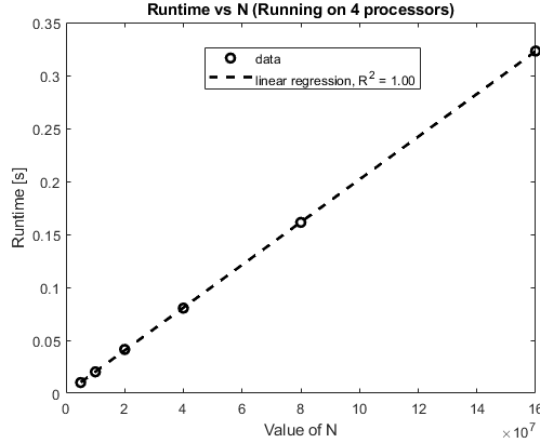


Figure 2: Runtime vs. N (P = 4)

The run time increases with  $N$ . Assuming no overflow, the run time  $T_n(n, p) \propto n$  with  $R^2 = 1.00$ . This is consistent with Equation (1) because as  $N \gg p$ ,  $N/P \gg \log(p)$ , which approximates Equation (1) to  $T_p(n, p) \approx \Theta(\frac{n}{p} + \log(p))$ .

3. Speedup while changing  $P$  and  $N$ .

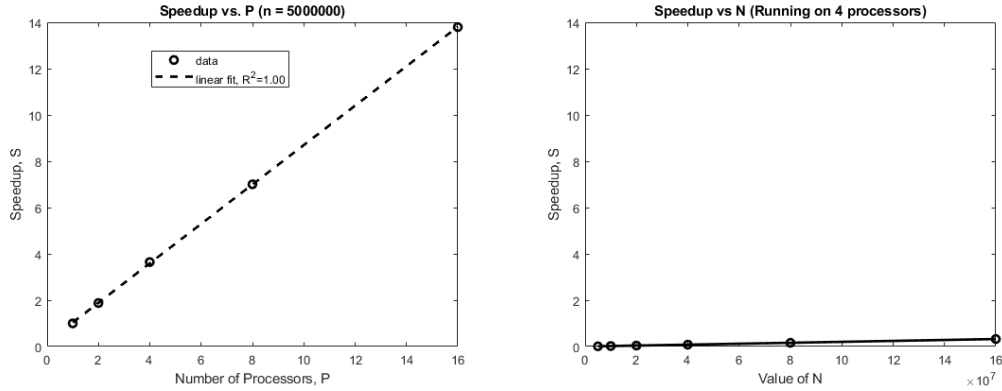


Figure 3: Speedup vs.  $P$  and Speedup vs.  $N$

The speedup increases linearly with  $P$ , but seems to be independent of  $N$ . This is consistent with Equation (3) because as  $N \gg p$ ,  $S \approx \frac{\Theta(n)}{\Theta(\frac{n}{p})} = \Theta(p)$ .

## Summary

In summary, the runtime using parallel computing can be approximated by  $T_p(n, p) = \Theta(\frac{n}{p})$ , and speedup can be described by  $\Theta(p)$  in this assignment, given  $N \gg P$ . This indicates parallel computing is highly effective when  $N \gg P$ .