# Econ 613 - Applied Econometrics - 2022 Spring
# Homework 3

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# 1

## 1.1

Please see my R code for the calculation for these statistics. There are 340,823 students, 775 schools, and 32 programs in this dataset.

## 1.2

There are altogether 3,086 unique pairs of (school, program).

## 1.3

There are 284,658 students who are applying for the senior high schools which are located at the same district to home.

## 1.4

See the following figure for the number of students each senior high school admitted. In the R code, I firstly generated the variable which indicates the admitted (school, program) for each students rank by rank. Then, I count the number of students admitted for each school (or *schoolcode*).

```
   schoolcode num.students
1       10101          398
2       10102          248
3       10103          443
4       10104          220
5       10105          346
6       10106          395
7       10107          306
8       10108          318
9       10109          300
10      10110          535
11      10111          600
12      10112          300
13      10114          350
14      10115          238
15      10116          446
16      10117          471
17      10118          539
18      10119          398
```

## 1.5

Please see the following figure for the cutoff of each school (or *schoolcode*). I use *aggregate* function in R.

```
   schoolcode cutoff
1       10101    284
2       10102    343
3       10103    316
4       10104    245
5       10105    260
6       10106    293
7       10107    281
8       10108    248
9       10109    257
10      10110    343
11      10111    371
12      10112    316
13      10114    319
14      10115    274
15      10116    205
16      10117    330
```

## 1.6

Please see the following figure for the quality of each school (or *schoolcode*). I use *aggregate* function in R.

```
   schoolcode   quality
1       10101 320.2312
2       10102 394.1492
3       10103 353.8330
4       10104 296.9182
5       10105 351.2139
6       10106 340.1013
7       10107 311.9542
8       10108 303.9025
9       10109 281.8233
10      10110 408.0785
11      10111 412.5100
12      10112 375.6133
13      10114 346.2229
14      10115 316.3361
15      10116 289.9574
16      10117 369.3163
17      10118 315.1206
```

## 2

In the data *data.schpgm* in my R code, I record this dataset. See the following figure for some rows in this dataset.

```
schoolcode       choicepgm size  quality cutoff       sssdistrict      ssslong   ssslat
     10101       Agriculture   49 310.1429    288 Accra Metropolitan -0.19711526 5.607396
     10101          Business  100 324.8600    305 Accra Metropolitan -0.19711526 5.607396
     10101      General Arts  100 330.0900    316 Accra Metropolitan -0.19711526 5.607396
     10101   General Science   50 329.1000    299 Accra Metropolitan -0.19711526 5.607396
     10101    Home Economics   49 300.5714    284 Accra Metropolitan -0.19711526 5.607396
     10101       Visual Arts   50 311.5400    296 Accra Metropolitan -0.19711526 5.607396
     10102      General Arts   88 404.9773    388 Accra Metropolitan -0.19711526 5.607396
     10102   General Science   70 406.4143    389 Accra Metropolitan -0.19711526 5.607396
     10102    Home Economics   45 377.1111    363 Accra Metropolitan -0.19711526 5.607396
     10102       Visual Arts   45 370.9333    343 Accra Metropolitan -0.19711526 5.607396
     10103       Agriculture   38 333.1316    316 Accra Metropolitan -0.19711526 5.607396
     10103          Business  119 357.9664    341 Accra Metropolitan -0.19711526 5.607396
     10103      General Arts  117 362.5812    349 Accra Metropolitan -0.19711526 5.607396
     10103   General Science   80 353.5625    335 Accra Metropolitan -0.19711526 5.607396
     10103    Home Economics   49 336.0408    320 Accra Metropolitan -0.19711526 5.607396
     10103       Visual Arts   40 357.9500    343 Accra Metropolitan -0.19711526 5.607396
     10104      General Arts   55 320.1273    302 Accra Metropolitan -0.19711526 5.607396
```

# 3

In the dataset for student information *data.datstu*, I record the six distances for each choice of each students. See the following figure for some rows of this distance.

| distance1 | distance2 | distance3 | distance4 | distance5 | distance6 |
|---|---|---|---|---|---|
| 2.577169 | 2.577169 | 16.935744 | 16.935744 | 2.526762 | 15.350763 |
| 0.000000 | 17.838288 | 0.000000 | 0.000000 | 17.838288 | NA |
| 0.000000 | 0.000000 | 3.968300 | 0.000000 | 10.519267 | 0.000000 |
| 0.000000 | NA | 69.920385 | 22.945949 | 69.920385 | 8.664879 |
| 91.765769 | 42.224017 | 25.342021 | 25.342021 | 42.224017 | 25.342021 |
| 96.602382 | 0.000000 | 2.576629 | 14.535318 | 4.268067 | 14.322724 |
| 26.816957 | 0.000000 | 26.816957 | 0.000000 | 23.152323 | 22.957162 |
| 33.864677 | 0.000000 | 0.000000 | 33.864677 | 15.085830 | 33.864677 |
| 0.000000 | 0.000000 | 7.088005 | 7.088005 | 2.409313 | 10.799306 |
| NA | 43.162568 | 13.071302 | 0.000000 | 13.071302 | 13.071302 |
| 23.047843 | 25.344831 | 0.000000 | 25.344831 | 25.344831 | 13.078815 |
| 107.597001 | 0.000000 | NA | 0.000000 | 50.171823 | 29.188865 |
| 16.986853 | 16.986853 | 0.000000 | 0.000000 | 16.986853 | 0.000000 |
| NA | NA | NA | NA | 66.295911 | 87.107300 |
| 88.643584 | 60.069095 | NA | 82.935074 | 0.000000 | 0.000000 |
| 20.066721 | 20.066721 | 20.066721 | 0.000000 | 0.000000 | 65.218485 |
| NA | NA | NA | NA | 0.000000 | 0.000000 |
| 62.640244 | 86.105477 | 0.000000 | 8.998180 | 33.725703 | 27.912259 |
| 27.086959 | 35.435313 | 35.435313 | 27.086959 | 21.875942 | 21.875942 |
| 11.718883 | 0.000000 | 39.405900 | 0.000000 | NA | 141.407771 |

# 4

Please see my R code for such dimension reduction. In the following figure, I show several rows in the dataset after dimension reduction.



# 5

The model I use is a multinomial model. The model specification is:

$$\Pr(Y_i = k) = \frac{\exp(\beta_k \cdot \text{score}_i)}{\sum_{j=1}^{m} \exp(\beta_j \cdot \text{score}_i)}$$

In our setting, there are altogether 246 cases. I select the first choice, (school, program) = (100, Art), as the benchmark, and I estimate the parameters, $\beta_k$, for the rest of these choices. Each $\beta_k$

include one constant as well as one coefficient. The likelihood function is

$$\text{Likelihood} = \prod_{i=1}^{N} \frac{\exp(\beta_{ki} \cdot \text{score}_i)}{\sum_{j=1}^{m} \exp(\beta_j \cdot \text{score}_i)}$$

There are altogether $N$ individuals, and $\beta_{ki}$ is the estimated coefficient for the choice which individual $i$ selected. Intuitively, this likelihood is the production of the estimated probability for each of students' choices.

In my R code, I construct a function, *maximum.likelihood*, to estimate the likelihood (negative value of likelihood). Then, I use *optim* to estimate the coefficient of this model. The results are show in the following figure,[1]

```
Coefficients:
                 (Intercept)         score
100 Economics   1.159890e-01  1.129448e-03
100 Others     -5.900832e-03 -1.500258e-03
100 Science     2.314422e-01  3.570387e-03
101 Arts        1.201838e+00  6.753548e-03
101 Economics   1.335635e+00  2.681114e-03
101 Others     -9.099388e-05  2.726235e-03
101 Science    -9.254632e+00  3.454959e-02
102 Arts        5.382385e-01  2.654350e-03
102 Economics   4.378082e-01  3.451623e-03
102 Others      1.958375e-03  3.775480e-04
102 Science     1.976038e-01  2.703180e-03
103 Arts        2.208058e-02 -3.313528e-03
```

To estimate the marginal effects, I use the following methodology,

$$p_{ij} = \frac{\exp(\text{score}_i \beta_j)}{\sum_{l=1}^{m} \exp(\text{score}_i \beta_l)}$$

and the marginal effects for student $i$ on choice $j$ is

$$\text{marginal effects}_{i,j} = p_{ij}(\beta_j - \bar{\beta}_i)$$

where $\bar{\beta}_i = \sum_l p_{il} \beta_l$. In my R code, I write a function, *marginal.effect*, which record the calculation of marginal effects. In the following figure, I show the marginal effects of the four top students on the first 39 choices.[2]

```
      100 Economics   100 Others 100 Science    101 Arts 101 Economics  101 Others   101 Science   102 Arts 102 Economics   102 Others 102 Science    103 Arts 103 Economics
[1,]  -0.062829395 -0.010094181 -0.16965580 -5.2752947  -1.1617140 -0.49299656 -456.17211087 -0.32770914  -0.41336441 -0.044710307 -0.28886804 -0.013050935  -0.011537190
[2,]  -0.062567940 -0.010084779 -0.16868901 -5.2175359  -1.1571333 -0.48839159 -427.96702268 -0.32595867  -0.41070663 -0.044527842 -0.28683341 -0.013033841  -0.011523926
[3,]  -0.062298576 -0.010073932 -0.16770351 -5.1596643  -1.1524042 -0.48375976 -401.44780028 -0.32417074  -0.40800701 -0.044339718 -0.28477197 -0.013014863  -0.011509016
[4,]  -0.062298576 -0.010073932 -0.16770351 -5.1596643  -1.1524042 -0.48375976 -401.44780028 -0.32417074  -0.40800701 -0.044339718 -0.28477197 -0.013014863  -0.011509016
      103 Others  103 Science     104 Arts 104 Economics  104 Others  104 Science   105 Arts 105 Economics  105 Others  105 Science    201 Arts 201 Economics  201 Others
[1,] -0.017275197 -0.015076589 -0.021792320  -0.032242185 -0.011289651 -0.019873228 -0.055620682  -0.042555019 -0.018387405 -0.025389720 -0.9151315  -1.32782684 -0.24176115
[2,] -0.017237211 -0.015050797 -0.021740797  -0.032145068 -0.011276005 -0.019830392 -0.055422758  -0.042434231 -0.018343259 -0.025336683 -0.9120030  -1.31426317 -0.24059365
[3,] -0.017196824 -0.015022880 -0.021686264  -0.032043617 -0.011260749 -0.019784792 -0.055217564  -0.042307676 -0.018296577 -0.025280106 -0.9087539  -1.30065020 -0.23939721
[4,] -0.017196824 -0.015022880 -0.021686264  -0.032043617 -0.011260749 -0.019784792 -0.055217564  -0.042307676 -0.018296577 -0.025280106 -0.9087539  -1.30065020 -0.23939721
      201 Science      202 Arts 202 Economics   202 Others  202 Science    203 Arts 203 Economics  203 Others  203 Science    204 Arts 204 Economics   204 Others 204 Science
[1,] -1.58805470 -0.012821861  -0.012268297  0.012176021 -0.012726795 -1.4595293  -2.25804438 -0.012460664 -2.08544711 -0.35536999  -0.25395243 -0.13669827 -0.14525009
[2,] -1.56768429 -0.012802823  -0.012252758 -0.012158668 -0.012710405 -1.4514933  -2.22330864 -0.012443858 -2.04527644 -0.35430709  -0.25236475 -0.13587953 -0.14439538
[3,] -1.54735167 -0.012781968  -0.012235472  0.012139585 -0.012692203 -1.4432931  -2.18879106 -0.012425280 -2.00558981 -0.35319637  -0.25075078 -0.13504619 -0.14352498
[4,] -1.54735167 -0.012781968  -0.012235472  0.012139585 -0.012692203 -1.4432931  -2.18879106 -0.012425280 -2.00558981 -0.35319637  -0.25075078 -0.13504619 -0.14352498
```

# 6

Similarly, in this section, I use the similar model as Question 5. The model is,

$$\Pr(Y_i = k) = \frac{\exp(\beta_k \cdot \text{quality}_i)}{\sum_{j=1}^{m} \exp(\beta_j \cdot \text{quality}_i)}$$

---

[1]I use the R package, *multinom*, to help my estimation. I initialize the starting point as the coefficients estimated by *multinom*, and I estimate the marginal effects by the coefficients estimated form this function.

[2]These four students earns the same highest test score, so the estimated marginal effects are the same across them four.

where quality$_i$ is the quality or average score of (school, program) which the student select. The likelihood is also the production of all estimated probability of each choices by the students. That is,

$$\text{Likelihood} = \prod_{i=1}^{N} \frac{\exp(\beta_{ki} \cdot \text{quality}_i)}{\sum_{j=1}^{m} \exp(\beta_j \cdot \text{quality}_i)}$$

In the following figure, I record the estimated value for coefficients

```
100 Economics    0.807840234       -2.452042e-03
100 Science      1.552112079        1.035820e-03
101 Arts        -0.253021094        1.220328e-02
101 Economics    5.791865784       -7.120379e-03
101 Others       1.717823709        4.654651e-04
101 Science    -17.796238123        5.604263e-02
102 Arts         2.794782847       -1.334127e-03
102 Economics    2.024979083        2.150056e-04
102 Others       0.450065457       -2.372324e-03
102 Science      1.594832528       -1.972448e-05
103 Arts         0.174868102       -3.288997e-03
103 Economics    0.098018030        2.699227e-03
103 Others       0.059533973       -2.494252e-02
103 Science      0.193033539       -4.084781e-03
104 Arts         0.484551175       -1.148757e-03
104 Economics    0.264866124        1.659634e-04
104 Others       0.029645064        3.084018e-03
104 Science      0.191671233       -1.967634e-03
105 Arts         0.871331544        4.626644e-04
105 Economics    0.817325379       -1.302000e-03
105 Others       0.010702312       -1.311466e-02
105 Science      0.303252595        1.398802e-03
```

The marginal effect of this model is similar to the previous section, where I estimate,

$$p_{ij} = \frac{\exp(\text{quality}_i \beta_j)}{\sum_{l=1}^{m} \exp(\text{quality}_i \beta_l)}$$

and the marginal effects for student $i$ on choice $j$ is

$$\text{marginal effects}_{i,j} = p_{ij}(\beta_j - \bar{\beta}_i)$$

where $\bar{\beta}_i = \sum_l p_{il} \beta_l$. The marginal utility is recorded in figure below

```
       100 Economics    100 Science    101 Arts 101 Economics    101 Others    101 Science       102 Arts 102 Economics    102 Others    102 Science       103 Arts 103 Economics
[1,] -7.042202e-02 -0.6998003871 -1.656288e+01 -1.2884956808 -0.6407966335 -1.180884e+02 -0.8446266861 -0.7793177630 -5.101929e-02 -4.566069e-01 -2.576737e-02 -3.427078e-01
[2,] -6.035281e-02 -0.5689555540 -1.137541e+01 -1.1849563978 -0.5254926573 -4.182108e+01 -0.7117359256 -0.6415113507 -4.367173e-02 -3.772006e-01 -2.236406e-02 -2.717153e-01
[3,] -6.035281e-02 -0.5689555540 -1.137541e+01 -1.1849563978 -0.5254926573 -4.182108e+01 -0.7117359256 -0.6415113507 -4.367173e-02 -3.772006e-01 -2.236406e-02 -2.717153e-01
[4,] -7.042202e-02 -0.6998003871 -1.656288e+01 -1.2884956808 -0.6407966335 -1.180884e+02 -0.8446266861 -0.7793177630 -5.101929e-02 -4.566069e-01 -2.576737e-02 -3.427078e-01
       103 Others    103 Science    104 Arts 104 Economics    104 Others    104 Science       105 Arts 105 Economics    105 Others    105 Science       201 Arts 201 Economics
[1,] -1.501128e-06 -1.841506e-02 -9.102630e-02 -1.311686e-01 -3.798318e-01 -4.717610e-02 -2.745062e-01 -1.185972e-01 -2.760295e-01 -2.359109e-01 -2.9827064567 -2.7584827147
[2,] -1.807064e-06 -1.617614e-02 -7.649001e-02 -1.080541e-01 -2.994033e-01 -4.013588e-02 -2.251215e-01 -9.988904e-02 -2.779106e-04 -1.907526e-01 -2.3836962666 -2.1694510571
[3,] -1.807064e-06 -1.617614e-02 -7.649001e-02 -1.080541e-01 -2.994033e-01 -4.013588e-02 -2.251215e-01 -9.988904e-02 -2.779106e-04 -1.907526e-01 -2.3836962666 -2.1694510571
[4,] -1.501128e-06 -1.841506e-02 -9.102630e-02 -1.311686e-01 -3.798318e-01 -4.717610e-02 -2.745062e-01 -1.185972e-01 -2.760295e-04 -2.359109e-01 -2.9827064567 -2.7584827147
       201 Others    201 Science    202 Arts 202 Economics    202 Others    202 Science       203 Arts 203 Economics    203 Others    203 Science       204 Arts 204 Economics
[1,] -0.6072625525 -3.5451690969 -3.527996e-03 -3.034725e-02 -4.082474e-02 -1.773917e-02 -3.129965e+00 -6.877079e+00 -8.088224e-02 -4.661056e+00 -2.4596074800 -7.425662e-01
[2,] -0.5077417858 -2.5352675226 -3.258664e-03 -2.615733e-02 -3.480512e-02 -1.560227e-02 -2.132553e+00 -4.328763e+00 -6.723744e-02 -2.980861e+00 -1.8608014713 -5.826708e-01
[3,] -0.5077417858 -2.5352675226 -3.258664e-03 -2.615733e-02 -3.480512e-02 -1.560227e-02 -2.132553e+00 -4.328763e+00 -6.723744e-02 -2.980861e+00 -1.8608014713 -5.826708e-01
[4,] -0.6072625525 -3.5451690969 -3.527996e-03 -3.034725e-02 -4.082474e-02 -1.773917e-02 -3.129965e+00 -6.877079e+00 -8.088224e-02 -4.661056e+00 -2.4596074800 -7.425662e-01
       204 Others    204 Science    205 Arts 205 Economics    205 Others    205 Science       206 Arts 206 Economics    206 Science       207 Arts    207 Others    207 Science
[1,] -3.383728e-01 -4.568885e-01 -1.402459e-02 -3.077453e-02 -3.563648e-02 -3.493144e-02 -1.737287e-01 -6.035724e-02 -2.597581e-01 -2.929785e-01 -8.756257e-03 -9.556780e-03
[2,] -2.720529e-01 -3.585049e-01 -1.243308e-02 -2.657379e-02 -3.045025e-02 -2.996007e-02 -1.448092e-01 -5.151176e-02 -2.082818e-01 -2.332843e-01 -7.856850e-03 -8.576756e-03
[3,] -2.720529e-01 -3.585049e-01 -1.243308e-02 -2.657379e-02 -3.045025e-02 -2.996007e-02 -1.448092e-01 -5.151176e-02 -2.082818e-01 -2.332843e-01 -7.856850e-03 -8.576756e-03
[4,] -3.383728e-01 -4.568885e-01 -1.402459e-02 -3.077453e-02 -3.563648e-02 -3.493144e-02 -1.737287e-01 -6.035724e-02 -2.597581e-01 -2.929785e-01 -8.756257e-03 -9.556780e-03
       208 Arts 208 Economics    208 Science    209 Others    209 Science       210 Arts 210 Economics    210 Others    210 Science       211 Arts 211 Economics    211 Science
[1,] -3.868647e-02 -6.023288e-02 -8.285755e-02 -1.198364e-01 -3.549793e-02 -3.6161069014 -2.5961951275 -7.446639e-01 -1.388398e+01 -5.173334e+00 -9.395027e-01 -4.403381e+00
[2,] -3.319261e-02 -5.096612e-02 -6.882493e-02 -9.836904e-02 -3.035268e-02 -2.4893186717 -1.9384963849 -5.728867e-01 -6.917816e+00 -3.327456e+00 -7.105872e-01 -2.893885e+00
[3,] -3.319261e-02 -5.096612e-02 -6.882493e-02 -9.836904e-02 -3.035268e-02 -2.4893186717 -1.9384963849 -5.728867e-01 -6.917816e+00 -3.327456e+00 -7.105872e-01 -2.893885e+00
[4,] -3.868647e-02 -6.023288e-02 -8.285755e-02 -1.198364e-01 -3.549793e-02 -3.6161069014 -2.5961951275 -7.446639e-01 -1.388398e+01 -5.173334e+00 -9.395027e-01 -4.403381e+00
```

# 7

I use the second model, because in the second model, the quality of school is alternative variant. That is, quality of school depend on the choice the students make, instead of depend on the students themselves.

I write a new function to estimate the choice probability, *choice.prob*, in my R code. The choice probability is

```
     100 Economics  100 Science    101 Arts 101 Economics 101 Science    102 Arts 102 Economics 102 Science    103 Arts 103 Economics 103 Science    104 Arts 104 Economics
[1,]   5.435235e-06 2.444929e-05  0.001157779  6.675093e-05 0.017171995 4.605568e-05  4.380221e-05 3.574306e-05 7.596959e-07  1.274495e-06 2.930017e-06 4.280467e-06  4.051763e-06
[2,]   5.961361e-06 2.677044e-05  0.001255158  7.360005e-05 0.017887073 5.048792e-05  4.794990e-05 3.910932e-05 8.356945e-07  1.400236e-06 3.213986e-06 4.694256e-06  4.442041e-06
[3,]   6.536850e-06 2.930493e-05  0.001360401  8.113250e-05 0.018627469 5.533345e-05  5.247777e-05 4.278237e-05 9.190759e-07  1.538015e-06 3.524633e-06 5.146813e-06  4.868747e-06
[4,]   6.536850e-06 2.930493e-05  0.001360401  8.113250e-05 0.018627469 5.533345e-05  5.247777e-05 4.278237e-05 9.190759e-07  1.538015e-06 3.524633e-06 5.146813e-06  4.868747e-06
[5,]   7.166086e-06 3.207127e-05  0.001474097  8.941326e-05 0.019393612 6.062872e-05  5.741870e-05 4.678856e-05 1.010521e-06  1.688924e-06 3.864330e-06 5.641575e-06  5.335094e-06
     104 Science    105 Arts 105 Economics 105 Science    201 Arts 201 Economics  201 Science    202 Arts 202 Economics  202 Science    203 Arts 203 Economics  203 Science
[1,]  3.001307e-06 1.393233e-05  1.399397e-05 4.586859e-06 0.0002021099  0.0001712673 0.0002545005 1.841246e-06  2.374611e-06 4.237128e-07 0.0002344925  0.0005430642 0.0004309604
[2,]  3.292020e-06 1.525255e-05  1.531824e-05 5.027745e-06 0.0002208330  0.0001869677 0.0002762156 2.021006e-06  2.605619e-06 4.666975e-07 0.0002538683  0.0005849761 0.0004644242
[3,]  3.610029e-06 1.669387e-05  1.676383e-05 5.509688e-06 0.0002412328  0.0002040586 0.0002997117 2.217786e-06  2.858416e-06 5.139198e-07 0.0002747793  0.0006299718 0.0005003667
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