

1 Kinematics Analysis

Table 1: DHtable

i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	135
2	-90°	0	θ_2	0
3	0	135	θ_3	0
4	-90°	38	θ_4	120
5	90°	0	θ_5	0
6	-90°	0	θ_6	70

$${}^0_6T = {}^0_1T \cdot {}^1_2T \cdot {}^2_3T \cdot {}^3_4T \cdot {}^4_5T \cdot {}^5_6T = \begin{bmatrix} {}^0_6R_{3 \times 3} & {}^0\vec{P}_{6org} \\ 0_{1 \times 3} & 1 \end{bmatrix} = \begin{bmatrix} {}^0_6T_{00} & {}^0_6T_{01} & {}^0_6T_{02} & {}^0_6T_{03} \\ {}^0_6T_{10} & {}^0_6T_{11} & {}^0_6T_{12} & {}^0_6T_{13} \\ {}^0_6T_{20} & {}^0_6T_{21} & {}^0_6T_{22} & {}^0_6T_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} {}^0_6T_{00} &= -S_6(C_4S_1 + S_4(C_1S_3 - C_2 - C_1C_3S_2)) - C_6(C_5(S_1S_4 - C_4(C_1S_3 - C_2 - C_1C_3S_2)) \\ &\quad - S_5(C_1C_3 - C_2 + C_1S_2S_3)) \\ {}^0_6T_{01} &= S_6(C_5(S_1S_4 - C_4(C_1S_3 - C_2 - C_1C_3S_2)) - S_5(C_1C_3 - C_2 + C_1S_2S_3)) - C_6(C_4S_1 \\ &\quad + S_4(C_1S_3 - C_2 - C_1C_3S_2)) \\ {}^0_6T_{02} &= -S_5(S_1S_4 - C_4(C_1S_3 - C_2 - C_1C_3S_2)) - C_5(C_1C_3 - C_2 + C_1S_2S_3) \\ {}^0_6T_{03} &= 135C_1S_2 - 70S_5(S_1S_4 - C_4(C_1S_3 - C_2 - C_1C_3S_2)) - 70C_5(C_1C_3 - C_2 + C_1S_2S_3) \\ &\quad - 120C_1C_3 - C_2 - 120C_1S_2S_3 - 38C_1S_3 - C_2 + 38C_1C_3S_2 \\ {}^0_6T_{10} &= S_6(C_1C_4 + S_4(C_3S_2S_1 - S_1S_3 - C_2)) + C_6(C_5(C_1S_4 - C_4(C_3S_2S_1 - S_1S_3 - C_2)) \\ &\quad + S_5(C_3S_1 - C_2 + S_2S_1S_3)) \\ {}^0_6T_{11} &= C_6(C_1C_4 + S_4(C_3S_2S_1 - S_1S_3 - C_2)) - S_6(C_5(C_1S_4 - C_4(C_3S_2S_1 - S_1S_3 - C_2)) \\ &\quad + S_5(C_3S_1 - C_2 + S_2S_1S_3)) \\ {}^0_6T_{12} &= S_5(C_1S_4 - C_4(C_3S_2S_1 - S_1S_3 - C_2)) - C_5(C_3S_1 - C_2 + S_2S_1S_3) \\ {}^0_6T_{13} &= 135S_2S_1 + 70S_5(C_1S_4 - C_4(C_3S_2S_1 - S_1S_3 - C_2)) - 70C_5(C_3S_1 - C_2 + S_2S_1S_3) \\ &\quad + 38C_3S_2S_1 - 120C_3S_1 - C_2 - 120S_2S_1S_3 - 38S_1S_3 - C_2 \\ {}^0_6T_{20} &= C_6(S_5(C_3S_2 - S_3 - C_2) + C_4C_5(C_3 - C_2 + S_2S_3)) - S_4S_6(C_3 - C_2 + S_2S_3) \\ {}^0_6T_{21} &= -S_6(S_5(C_3S_2 - S_3 - C_2) + C_4C_5(C_3 - C_2 + S_2S_3)) - C_6S_4(C_3 - C_2 + S_2S_3) \\ {}^0_6T_{22} &= C_4S_5(C_3 - C_2 + S_2S_3) - C_5(C_3S_2 - S_3 - C_2) \\ {}^0_6T_{23} &= 120S_3 - C_2 - 120C_3S_2 - 38C_3 - C_2 - 38S_2S_3 - 135 - C_2 - 70C_5(C_3S_2 - S_3 - C_2) \\ &\quad + 70C_4S_5(C_3 - C_2 + S_2S_3) + 135 \end{aligned}$$

$$\begin{aligned}
\begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} &= {}^0_1 \text{R} \begin{bmatrix} 0 \\ 0 \\ \theta_1 \end{bmatrix} + {}^0_1 \text{R} \begin{bmatrix} 0 \\ 0 \\ \theta_2 \end{bmatrix} + {}^0_1 \text{R} \begin{bmatrix} 0 \\ 0 \\ \theta_0 \end{bmatrix} + {}^4_1 \text{R} \begin{bmatrix} 0 \\ 0 \\ \theta_0 \end{bmatrix} + {}^0_1 \text{R} \begin{bmatrix} 0 \\ 0 \\ \theta_2 \end{bmatrix} + {}^2_1 \text{R} \begin{bmatrix} 0 \\ 0 \\ \theta_2 \end{bmatrix} \\
\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{bmatrix} &= \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ w_x \\ w_y \\ w_z \end{bmatrix} = J_g \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} \\
J_g = \begin{bmatrix} J_{\vec{v}} \\ J_{\vec{w}} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} &= J_a \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = J_{we} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} \\
J_{we} = \begin{bmatrix} 1 & 0 & S_\beta \\ 0 & C_\alpha & -S_\alpha C_\beta \\ 0 & S_\alpha & C_\alpha C_\beta \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} &= \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} + R_x(\alpha) \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix} + R_x(\alpha) R_y(\beta) \\
\begin{bmatrix} 0 \\ 0 \\ \gamma \end{bmatrix} &= \begin{bmatrix} 1 & 0 & S_\beta \\ 0 & C_\alpha & -S_\alpha C_\beta \\ 0 & S_\alpha & C_\alpha C_\beta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \\
\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ w_x \\ w_y \\ w_z \end{bmatrix} &= \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & J_{we} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} \\
Jg &= \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & J_{we} \end{bmatrix} Ja
\end{aligned}$$

$$\begin{aligned}
J_{g0,00} &= 120S_1S_2S_3 - 38C_2S_1S_3 - 38C_3S_1S_2 - 70C_1S_4S_5 - 135S_1S_2 - 120C_2C_3S_1 \\
&\quad - 70C_2C_3C_5S_1 + 70C_5S_1S_2S_3 + 70C_2C_4S_1S_3S_5 + 70C_3C_4S_1S_2S_5 \\
J_{g0,01} &= -C_1(120C_2S_3 - 38C_2C_3 - 135C_2 + 120C_3S_2 + 38S_2S_3 + 70C_2C_5S_3 \\
&\quad + 70C_3C_5S_2 + 70C_2C_3C_4S_5 - 70C_4S_2S_3S_5) \\
J_{g0,02} &= -2C_1(60C_2S_3 - 19C_2C_3 + 60C_3S_2 + 19S_2S_3 + 35C_2C_5S_3 + 35C_3C_5S_2 \\
&\quad + 35C_2C_3C_4S_5 - 35C_4S_2S_3S_5) \\
J_{g0,03} &= 70S_5(C_1C_2S_3S_4 - C_4S_1 + C_1C_3S_2S_4) \\
J_{g0,04} &= -70C_5(S_1S_4 + C_1C_2C_4S_3 + C_1C_3C_4S_2) - 70C_{23}C_1S_5 \\
J_{g0,05} &= 0 \\
J_{g0,10} &= 135C_1S_2 - 120C_1S_2S_3 - 70S_1S_4S_5 + 120C_1C_2C_3 + 38C_1C_2S_3 + 38C_1C_3S_2 \\
&\quad + 70C_1C_2C_3C_5 - 70C_1C_5S_2S_3 - 70C_1C_2C_4S_3S_5 - 70C_1C_3C_4S_2S_5 \\
J_{g0,11} &= -S_1(120C_2S_3 - 38C_2C_3 - 135C_2 + 120C_3S_2 + 38S_2S_3 + 70C_2C_5S_3 \\
&\quad + 70C_3C_5S_2 + 70C_2C_3C_4S_5 - 70C_4S_2S_3S_5) \\
J_{g0,12} &= -2S_1(60C_2S_3 - 19C_2C_3 + 60C_3S_2 + 19S_2S_3 + 35C_2C_5S_3 + 35C_3C_5S_2 \\
&\quad + 35C_2C_3C_4S_5 - 35C_4S_2S_3S_5) \\
J_{g0,13} &= 70S_5(C_1C_4 + C_2S_1S_3S_4 + C_3S_1S_2S_4) \\
J_{g0,14} &= -70C_5(C_2C_4S_1S_3 - C_1S_4 + C_3C_4S_1S_2) - 70C_{23}S_1S_5 \\
J_{g0,15} &= 0 \\
J_{g0,20} &= 0 \\
J_{g0,21} &= 120S_2S_3 - 120C_2C_3 - 38C_2S_3 - 38C_3S_2 - 135S_2 + 70C_5S_2S_3 - 70C_2C_3C_5 \\
&\quad + 70C_2C_4S_3S_5 + 70C_3C_4S_2S_5 \\
J_{g0,22} &= 120S_2S_3 - 38C_2S_3 - 38C_3S_2 - 120C_2C_3 + 70C_5S_2S_3 - 70C_2C_3C_5 + 70C_2C_4S_3S_5 \\
&\quad + 70C_3C_4S_2S_5 \\
J_{g0,23} &= 70C_{23}S_4S_5 \\
J_{g0,24} &= 70S_{23}S_5 - 70C_{23}C_4C_5 \\
J_{g0,25} &= 0 \\
J_{g0,30} &= \theta_4S_1S_2S_3 - \theta_3C_1 - \theta_5C_1C_4 - \theta_4C_2C_3S_1 - \theta_6C_1S_4S_5 - \theta_2C_1 - \theta_6C_2C_3C_5S_1 \\
&\quad - \theta_5C_2S_1S_3S_4 - \theta_5C_3S_1S_2S_4 + \theta_6C_5S_1S_2S_3 + \theta_6C_2C_4S_1S_3S_5 + \theta_6C_3C_4S_1S_2S_5 \\
J_{g0,31} &= \theta_5C_1C_2C_3S_4 - \theta_4C_1C_2S_3 - \theta_4C_1C_3S_2 - S_1 - \theta_6C_1C_2C_5S_3 - \theta_6C_1C_3C_5S_2 \\
&\quad - \theta_5C_1S_2S_3S_4 - \theta_6C_1C_2C_3C_4S_5 + \theta_6C_1C_4S_2S_3S_5 \\
J_{g0,32} &= \theta_5C_1C_2C_3S_4 - \theta_4C_1C_2S_3 - \theta_4C_1C_3S_2 - S_1 - \theta_6C_1C_2C_5S_3 - \theta_6C_1C_3C_5S_2 \\
&\quad - \theta_5C_1S_2S_3S_4 - \theta_6C_1C_2C_3C_4S_5 + \theta_6C_1C_4S_2S_3S_5 \\
J_{g0,33} &= \theta_5(S_1S_4 + C_1C_2C_4S_3 + C_1C_3C_4S_2) - C_1S_2S_3 + C_1C_2C_3 + \theta_6S_5(C_1C_2S_3S_4 \\
&\quad - C_4S_1 + C_1C_3S_2S_4) \\
J_{g0,34} &= -C_4S_1 - \theta_6(C_5(S_1S_4 - C_4(C_1S_3 - C_2 - C_1C_3S_2))) - S_5(C_1C_3 - C_2 + C_1S_2S_3) \\
&\quad - S_4(C_1S_3 - C_2 - C_1C_3S_2) \\
J_{g0,35} &= C_{23}C_1C_5 - S_5(S_1S_4 + C_1C_2C_4S_3 + C_1C_3C_4S_2)
\end{aligned}$$

$$\begin{aligned}
J_{90,40} &= \theta_4 C_1 C_2 C_3 - \theta_3 S_1 - \theta_5 C_4 S_1 - \theta_4 C_1 S_2 S_3 - \theta_6 S_1 S_4 S_5 + \theta_6 C_1 C_2 C_3 C_5 \\
&\quad + \theta_5 C_1 C_2 S_3 S_4 + \theta_5 C_1 C_3 S_2 S_4 - \theta_6 C_1 C_5 S_2 S_3 - \theta_6 C_1 C_2 C_4 S_3 S_5 - \theta_6 C_1 C_3 C_4 S_2 S_5 \\
J_{90,41} &= C_1 - \theta_4 C_2 S_1 S_3 - \theta_4 C_3 S_1 S_2 + \theta_5 C_2 C_3 S_1 S_4 - \theta_6 C_2 C_5 S_1 S_3 - \theta_6 C_3 C_5 S_1 S_2 \\
&\quad - \theta_5 S_1 S_2 S_3 S_4 - \theta_6 C_2 C_3 C_4 S_1 S_5 + \theta_6 C_4 S_1 S_2 S_3 S_5 \\
J_{90,42} &= C_1 - \theta_4 C_2 S_1 S_3 - \theta_4 C_3 S_1 S_2 + \theta_5 C_2 C_3 S_1 S_4 - \theta_6 C_2 C_5 S_1 S_3 - \theta_6 C_3 C_5 S_1 S_2 \\
&\quad - \theta_5 S_1 S_2 S_3 S_4 \\
&\quad - \theta_6 C_2 C_3 C_4 S_1 S_5 + \theta_6 C_4 S_1 S_2 S_3 S_5 \\
J_{90,43} &= \theta_5 (C_2 C_4 S_1 S_3 - C_1 S_4 + C_3 C_4 S_1 S_2) + \theta_6 S_5 (C_1 C_4 + C_2 S_1 S_3 S_4 + C_3 S_1 S_2 S_4) \\
&\quad - S_1 S_2 S_3 + C_2 C_3 S_1 \\
J_{90,44} &= C_1 C_4 + S_{23} S_1 S_4 - \theta_6 C_{23} S_1 S_5 + \theta_6 C_1 C_5 S_4 - \theta_6 C_2 C_4 C_5 S_1 S_3 - \theta_6 C_3 C_4 C_5 S_1 S_2 \\
J_{90,45} &= C_{23} C_5 S_1 - S_5 (C_2 C_4 S_1 S_3 - C_1 S_4 + C_3 C_4 S_1 S_2) \\
J_{90,50} &= 1 \\
J_{90,51} &= \theta_6 S_{23} C_4 S_5 - \theta_6 C_{23} C_5 - \theta_5 S_{23} S_4 - \theta_4 C_{23} \\
J_{90,52} &= \theta_6 S_{23} C_4 S_5 - \theta_6 C_{23} C_5 - \theta_5 S_{23} S_4 - \theta_4 C_{23} \\
J_{90,53} &= \theta_5 C_2 C_3 C_4 - C_3 S_2 - C_2 S_3 - \theta_5 C_4 S_2 S_3 + \theta_6 C_2 C_3 S_4 S_5 - \theta_6 S_2 S_3 S_4 S_5 \\
J_{90,54} &= \theta_6 (S_{23} S_5 - C_{23} C_4 C_5) + C_{23} S_4 \\
J_{90,55} &= -S_{23} C_5 - C_{23} C_4 S_5 \\
J_{96,00} &= 135 C_4 S_2 S_6 + 120 C_2 C_3 C_4 S_6 + 70 C_2 C_3 C_6 S_4 + 38 C_2 C_4 S_3 S_6 + 38 C_3 C_4 S_2 S_6 \\
&\quad + 135 C_5 C_6 S_2 S_4 - 120 C_4 S_2 S_3 S_6 - 70 C_6 S_2 S_3 S_4 - 70 C_2 S_3 S_5 S_6 - 70 C_3 S_2 S_5 S_6 \\
&\quad + 70 C_2 C_3 C_4 C_5 S_6 + 120 C_2 C_3 C_5 C_6 S_4 + 38 C_2 C_5 C_6 S_3 S_4 + 38 C_3 C_5 C_6 S_2 S_4 \\
&\quad - 70 C_4 C_5 S_2 S_3 S_6 - 120 C_5 C_6 S_2 S_3 S_4 \\
J_{96,01} &= 70 C_4 C_6 - 38 C_6 S_5 - 120 S_4 S_6 - 70 C_5 S_4 S_6 + 135 S_3 S_4 S_6 + 120 C_4 C_5 C_6 - 135 C_3 C_6 S_5 \\
&\quad - 135 C_4 C_5 C_6 S_3 \\
J_{96,02} &= 70 C_4 C_6 - 38 C_6 S_5 - 120 S_4 S_6 - 70 C_5 S_4 S_6 + 120 C_4 C_5 C_6 \\
J_{96,03} &= 70 S_5 S_6 \\
J_{96,04} &= 70 C_6 \\
J_{96,05} &= 0 \\
J_{96,10} &= 135 C_4 C_6 S_2 + 120 C_2 C_3 C_4 C_6 + 38 C_2 C_4 C_6 S_3 + 38 C_3 C_4 C_6 S_2 - 70 C_2 C_3 S_4 S_6 \\
&\quad - 120 C_4 C_6 S_2 S_3 - 70 C_2 C_6 S_3 S_5 - 70 C_3 C_6 S_2 S_5 - 135 C_5 S_2 S_4 S_6 + 70 S_2 S_3 S_4 S_6 \\
&\quad + 70 C_2 C_3 C_4 C_5 C_6 - 120 C_2 C_3 C_5 S_4 S_6 - 70 C_4 C_5 C_6 S_2 S_3 - 38 C_2 C_5 S_3 S_4 S_6 \\
&\quad - 38 C_3 C_5 S_2 S_4 S_6 + 120 C_5 S_2 S_3 S_4 S_6 \\
J_{96,11} &= 38 S_5 S_6 - 120 C_6 S_4 - 70 C_4 S_6 + 135 C_6 S_3 S_4 + 135 C_3 S_5 S_6 - 120 C_4 C_5 S_6 - 70 C_5 C_6 S_4 \\
&\quad + 135 C_4 C_5 S_3 S_6 \\
J_{96,12} &= 38 S_5 S_6 - 120 C_6 S_4 - 70 C_4 S_6 - 120 C_4 C_5 S_6 - 70 C_5 C_6 S_4 \\
J_{96,13} &= 70 C_6 S_5 \\
J_{96,14} &= -70 S_6 \\
J_{96,15} &= 0
\end{aligned}$$

$$\begin{aligned}
J_{g6,20} &= S_4 S_5 (135 S_2 + \sqrt{23961} \cos(\theta_2 + \theta_3 - \tan^{-1}(19/60))) \\
J_{g6,21} &= 38 C_5 + 135 C_3 C_5 + 120 C_4 S_5 - 135 C_4 S_3 S_5 \\
J_{g6,22} &= 38 C_5 + 120 C_4 S_5 \\
J_{g6,23} &= J_{g6,24} = J_{g6,25} = 0 \\
J_{g6,30} &= C_2 C_3 S_4 S_6 + C_2 C_6 S_3 S_5 + C_3 C_6 S_2 S_5 - S_2 S_3 S_4 S_6 + \theta_2 C_2 C_3 C_6 S_5 + \theta_3 C_2 C_3 C_6 S_5 \\
&\quad + \theta_4 C_2 C_3 C_4 S_6 + \theta_5 C_2 C_5 C_6 S_3 + \theta_5 C_3 C_5 C_6 S_2 + \theta_6 C_2 C_3 C_6 S_4 - \theta_2 C_2 S_3 S_4 S_6 \\
&\quad - \theta_2 C_3 S_2 S_4 S_6 - \theta_2 C_6 S_2 S_3 S_5 - \theta_3 C_2 S_3 S_4 S_6 - \theta_3 C_3 S_2 S_4 S_6 - \theta_3 C_6 S_2 S_3 S_5 \\
&\quad - \theta_4 C_4 S_2 S_3 S_6 - \theta_6 C_6 S_2 S_3 S_4 - \theta_6 C_2 S_3 S_5 S_6 - \theta_6 C_3 S_2 S_5 S_6 - C_2 C_3 C_4 C_5 C_6 \\
&\quad + C_4 C_5 C_6 S_2 S_3 + \theta_2 C_2 C_4 C_5 C_6 S_3 + \theta_2 C_3 C_4 C_5 C_6 S_2 + \theta_3 C_2 C_4 C_5 C_6 S_3 \\
&\quad + \theta_3 C_3 C_4 C_5 C_6 S_2 + \theta_4 C_2 C_3 C_5 C_6 S_4 + \theta_5 C_2 C_3 C_4 C_6 S_5 + \theta_6 C_2 C_3 C_4 C_5 S_6 \\
&\quad - \theta_4 C_5 C_6 S_2 S_3 S_4 - \theta_5 C_4 C_6 S_2 S_3 S_5 - \theta_6 C_4 C_5 S_2 S_3 S_6 \\
J_{g6,31} &= C_4 S_6 + \theta_6 C_4 C_6 - \theta_4 S_4 S_6 + C_5 C_6 S_4 + \theta_4 C_4 C_5 C_6 - \theta_5 C_6 S_4 S_5 - \theta_6 C_5 S_4 S_6 \\
J_{g6,32} &= C_4 S_6 + \theta_6 C_4 C_6 - \theta_4 S_4 S_6 + C_5 C_6 S_4 + \theta_4 C_4 C_5 C_6 - \theta_5 C_6 S_4 S_5 - \theta_6 C_5 S_4 S_6 \\
J_{g6,33} &= \theta_6 S_5 S_6 - \theta_5 C_5 C_6 - C_6 S_5 \\
J_{g6,34} &= S_6 + \theta_6 C_6 \\
J_{g6,35} &= 0 \\
J_{g6,40} &= C_2 C_3 C_6 S_4 - C_6 S_2 S_3 S_4 - C_2 S_3 S_5 S_6 - C_3 S_2 S_5 S_6 + \theta_4 C_2 C_3 C_4 C_6 - \theta_2 C_2 C_6 S_3 S_4 \\
&\quad - \theta_2 C_3 C_6 S_2 S_4 - \theta_2 C_2 C_3 S_5 S_6 - \theta_3 C_2 C_6 S_3 S_4 - \theta_3 C_3 C_6 S_2 S_4 - \theta_3 C_2 C_3 S_5 S_6 \\
&\quad - \theta_4 C_4 C_6 S_2 S_3 - \theta_5 C_2 C_5 S_3 S_6 - \theta_5 C_3 C_5 S_2 S_6 - \theta_6 C_2 C_3 S_4 S_6 - \theta_6 C_2 C_6 S_3 S_5 \\
&\quad - \theta_6 C_3 C_6 S_2 S_5 + \theta_2 S_2 S_3 S_5 S_6 + \theta_3 S_2 S_3 S_5 S_6 + \theta_6 S_2 S_3 S_4 S_6 + C_2 C_3 C_4 C_5 S_6 \\
&\quad - C_4 C_5 S_2 S_3 S_6 + \theta_6 C_2 C_3 C_4 C_5 C_6 - \theta_2 C_2 C_4 C_5 S_3 S_6 - \theta_2 C_3 C_4 C_5 S_2 S_6 \\
&\quad - \theta_3 C_2 C_4 C_5 S_3 S_6 - \theta_3 C_3 C_4 C_5 S_2 S_6 - \theta_4 C_2 C_3 C_5 S_4 S_6 - \theta_5 C_2 C_3 C_4 S_5 S_6 \\
&\quad - \theta_6 C_4 C_5 C_6 S_2 S_3 + \theta_4 C_5 S_2 S_3 S_4 S_6 + \theta_5 C_4 S_2 S_3 S_5 S_6 \\
J_{g6,41} &= C_4 C_6 - C_5 S_4 S_6 - \theta_4 C_6 S_4 - \theta_6 C_4 S_6 - \theta_4 C_4 C_5 S_6 - \theta_6 C_5 C_6 S_4 + \theta_5 S_4 S_5 S_6 \\
J_{g6,42} &= C_4 C_6 - C_5 S_4 S_6 - \theta_4 C_6 S_4 - \theta_6 C_4 S_6 - \theta_4 C_4 C_5 S_6 - \theta_6 C_5 C_6 S_4 + \theta_5 S_4 S_5 S_6 \\
J_{g6,43} &= S_5 S_6 + \theta_5 C_5 S_6 + \theta_6 C_6 S_5 \\
J_{g6,44} &= C_6 - \theta_6 S_6 \\
J_{g6,45} &= 0 \\
J_{g6,50} &= \theta_2 C_5 S_2 S_3 - C_3 C_5 S_2 - \theta_2 C_2 C_3 C_5 - \theta_3 C_2 C_3 C_5 - C_2 C_5 S_3 + \theta_3 C_5 S_2 S_3 + \theta_5 C_2 S_3 S_5 \\
&\quad + \theta_5 C_3 S_2 S_5 - C_2 C_3 C_4 S_5 + C_4 S_2 S_3 S_5 - \theta_5 C_2 C_3 C_4 C_5 + \theta_2 C_2 C_4 S_3 S_5 + \theta_2 C_3 C_4 S_2 S_5 \\
&\quad + \theta_3 C_2 C_4 S_3 S_5 + \theta_3 C_3 C_4 S_2 S_5 + \theta_4 C_2 C_3 S_4 S_5 + \theta_5 C_4 C_5 S_2 S_3 - \theta_4 S_2 S_3 S_4 S_5 \\
J_{g6,51} &= S_4 S_5 + \theta_4 C_4 S_5 + \theta_5 C_5 S_4 \\
J_{g6,52} &= S_4 S_5 + \theta_4 C_4 S_5 + \theta_5 C_5 S_4 \\
J_{g6,53} &= C_5 - \theta_5 S_5 \\
J_{g6,54} &= 0 \\
J_{g6,55} &= 1
\end{aligned}$$

2 Tool Center Point

$$\begin{aligned}
{}^B_H\mathbf{T} &= {}^B_F\mathbf{T} {}^F_H\mathbf{T} \\
\Rightarrow \begin{bmatrix} {}^B_H\mathbf{R} & {}^B\vec{P}_{Horg} \\ 0 & 1 \end{bmatrix} &= \begin{bmatrix} {}^B_F\mathbf{R} & {}^B\vec{P}_{Forg} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^F_H\mathbf{R} & {}^F\vec{P}_{Horg} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^B_F\mathbf{R} {}^F_H\mathbf{R} & {}^B_F\mathbf{R} {}^F\mathbf{P}_{Horg} + {}^B\vec{P}_{Forg} \\ 0 & 1 \end{bmatrix} \\
{}^B\vec{P}_{Horg} &= {}^B_F\mathbf{R} \cdot {}^F\vec{P}_{Horg} + {}^B\vec{P}_{Forg} \\
\Rightarrow {}^B\vec{P}_{Horg} &= {}^B_F\mathbf{R}^1 \cdot {}^F\vec{P}_{Horg} + {}^B\vec{P}_{Forg}^1 \\
{}^B\vec{P}_{Horg} &= {}^B_F\mathbf{R}^2 \cdot {}^F\vec{P}_{Horg} + {}^B\vec{P}_{Forg}^2 \\
{}^B\vec{P}_{Horg} &= {}^B_F\mathbf{R}^3 \cdot {}^F\vec{P}_{Horg} + {}^B\vec{P}_{Forg}^3 \\
{}^B\vec{P}_{Horg} &= {}^B_F\mathbf{R}^4 \cdot {}^F\vec{P}_{Horg} + {}^B\vec{P}_{Forg}^4 \\
\begin{bmatrix} {}^B_F\mathbf{R}^1 - {}^B_F\mathbf{R}^2 \\ {}^B_F\mathbf{R}^1 - {}^B_F\mathbf{R}^3 \\ {}^B_F\mathbf{R}^1 - {}^B_F\mathbf{R}^4 \end{bmatrix} \cdot {}^F\vec{P}_{Horg} &= \begin{bmatrix} {}^B\vec{P}_{Forg}^2 - {}^B\vec{P}_{Forg}^1 \\ {}^B\vec{P}_{Forg}^3 - {}^B\vec{P}_{Forg}^1 \\ {}^B\vec{P}_{Forg}^4 - {}^B\vec{P}_{Forg}^1 \end{bmatrix}
\end{aligned}$$

3 Gravity Compensation

$$\begin{aligned}
&\begin{cases} G_x = F_x - F_{x0} \\ G_z = F_y - F_{y0} \\ G_y = F_z - F_{z0} \\ M_{gx} = M_x - F_{x0} \\ M_{gy} = M_y - F_{y0} \\ M_{gz} = M_z - F_{z0} \end{cases} \\
&\begin{cases} M_{gx} = G_z \cdot y - G_y \cdot z \\ M_{gy} = G_x \cdot z - G_z \cdot x \\ M_{gz} = G_y \cdot x - G_x \cdot y \end{cases} \\
\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} &= \begin{bmatrix} 0 & F_z & -F_y & 1 & 0 & 0 \\ -F_z & 0 & F_x & 0 & 1 & 0 \\ F_y & -F_x & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ k_1 \\ k_2 \\ k_3 \end{bmatrix} \\
&\begin{cases} k_1 = M_{x0} + F_{y0} \cdot z - F_{z0} \cdot y \\ k_2 = M_{y0} + F_{z0} \cdot x - F_{x0} \cdot z \\ k_3 = M_{z0} + F_{x0} \cdot y - F_{y0} \cdot x \end{cases}
\end{aligned}$$

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{00} & r_{01} & r_{02} \\ r_{10} & r_{11} & r_{12} \\ r_{20} & r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -G \end{bmatrix} + \begin{bmatrix} F_{x0} \\ F_{y0} \\ F_{z0} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

$$\begin{bmatrix} -r_{02} & -r_{12} & 0 & 1 & 0 & 0 \\ -r_{12} & r_{02} & 0 & 0 & 1 & 0 \\ 0 & 0 & -r_{22} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} G\cos\theta \\ G\sin\theta \\ G \\ F_{x0} \\ F_{y0} \\ F_{z0} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

$$\begin{cases} F_{ex} = F_x - F_{x0} - G_x \\ F_{ey} = F_y - F_{y0} - G_y \\ F_{ez} = F_z - F_{z0} - G_z \\ M_{ex} = M_x - M_{x0} - M_{gx} \\ M_{ey} = M_y - M_{y0} - M_{gy} \\ M_{ez} = M_z - M_{z0} - M_{gz} \end{cases}$$

$$\begin{cases} G_x = -G\cos\theta r_{13} - G\sin\theta r_{23} \\ G_y = G\sin\theta r_{13} - G\cos\theta r_{23} \\ G_z = -Gr_{33} \end{cases}$$

$$\theta = \arccos\left(\frac{G\cos\theta}{G}\right) \text{ or } \theta = \arcsin\left(\frac{G\sin\theta}{G}\right)$$

4 Admittance Control based on F/T Sensor

5 Reference Frame Changing

$$M_q = r_q F$$

$$M_p = r_p F + \vec{q}\vec{p} \times F_p$$

$$M_q = M_p + \vec{q}\vec{p} \times F_p$$

$$\begin{bmatrix} F_q \\ M_q \end{bmatrix}_{S_2} = \begin{bmatrix} I_{3 \times 3} & 0 \\ S_2[S_{1org} - S_{2org}]_x & I_{3 \times 3} \end{bmatrix} \begin{bmatrix} {}^{S_1}_2R & 0 \\ 0 & {}^{S_1}_2R \end{bmatrix} \begin{bmatrix} F_p \\ M_p \end{bmatrix}_{S_1}$$

$$M_q = r_q F$$

$$M_p = r_p F + {}^{S_2}[S_{1org} - S_{2org}]_x \times F_p$$

$$M_q = M_p + {}^{S_2}[S_{1org} - S_{2org}]_x \times F_p$$

$${}^{S_2}_1T = \begin{bmatrix} {}^{S_2}_1R & {}^{S_2}\vec{P}_{s1org} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & {}^{S_2}\vec{P}_{s1org} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{S_2}_1R & 0 \\ 0 & 1 \end{bmatrix}$$

$${}^{S_2}\vec{P}_{s1org} = -{}^{S_2}_1R \cdot {}^{S_1}\vec{P}_{s2org}$$