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First of all, this project was harder than I expected. It was very hard to wrap my head around the recursive nature of the project at first. I finally sat down and played Towers of Hanoi for a bit, until it made sense to me. There was a pattern to the optimal movement of disks that could be translated into a recursive code. Eventually, I realized that the code was very simple. You started from the bottom and worked upward, doing the same things for the n-1th disk.

The harder thing was trying to figure out how to do it iteratively. It wasn't possible to apply the recursive method to the iterative version, since there was no way to keep of which step it was on. For example, it was hard to know if the n-1 disk had completed all its movements so that the nth disk could be moved to its final destination. However, I realized that there was a trick to this as well. The cycles rotated between movements of 3 steps. For an odd number of disks, the first movement was always between the source and the destination, the second was always between the source and the auxiliary tower, and the last was between the destination and the auxiliary tower. The movement was different, since the smaller disk is always moved on top of the larger one, but it always involved the same towers. The even one started with the source and aux, with the 2<sup>nd</sup> step being the source and destination, and the last step remaining the same. It still followed the general pattern however.

I could not figure out how to perform the iterative solution without a stack. The recursive solution could be performed easily without a stack, since the recursion cascades down and each of the tower steps are performed in order. For the iterative version, however, since I did not know the exact movement of the disks at each step, I had to keep track of their relative sizes using stacks. This allowed me to separate the logic of the disk movement from the logic of which towers to look at. However, this probably increased the run time of the iterative solution by a large factor.

Which brings me to the runtimes, and the space and cost efficiencies. I tried to run it as highly as possible, but due to space considerations, I could not go higher than 28. The space is a factor of O(2^n), since for every increase in n the size doubled. This can be seen in the screenshot of the file sizes in the appendix. The runtime cost is not as exact as the space considerations, but this could be due to the natural fluctuations of the processor. My computer was not doing just the calculations, so random confounding variables could have affected the results. That being said, at the higher levels (24-28), the runtime roughly doubled for each successive increase. I would estimate the cost of runtime for both the iterative version and the recursive version at (2^n). This is because the number of steps does not change for either version. The number of steps needed to be printed is still 2^n-1.

The iterative solution appears to be doing worse runtime wise compared to the recursive solution. This is true even at higher numbers of n. I am not particularly surprised, since the Towers of Hanoi problem lends itself so well to recursion. In addition, the stack calls probably increase the runtime of the iterative version compared to the recursive version, which only needs to write to the file. It is possible that the iterative version can work with large numbers of n than the recursive solution due to memory concerns, but the file size limits and the limits of my poor hard drive did not allow me to test further.

To improve this program, I would try to find a way to test the performance of each without having to write to a file. I tried many different methods, such as removing the outputting entirely. However, this seriously skewed the runtime to the recursive method, since it was basically doing nothing, while the iterative method still had to go through its stack pops and pushes. I also tried to find ways to suppress printing to a console while still having it go through the motions, but no such luck. I even tried deleting

files as they were made, but due to the fact that the file size doubled with each iteration, I found that it was still unwieldy, since the size of n is greater than the sum of n-1 files.

I would also try to find a way to do the iterative technique without using stacks. Perhaps there is a method that I have not thought of, that would be able to keep track of the disk movements without having to compare sizes. I think this could help the iterative function keep on par with the recursive function.

Before I tried the write.append() function, I had used a StringBuilder like with the last assignment. This worked for values up to n=23, after which I found a cryptic error. It turned out the I had run out of memory and had an array overflow for the StringBuilder. This was not something that I would have known about had it not been for this project. I also realized that printing to the console takes A LOT longer than writing to a file. The same thing that takes seconds to write to a file could take a minute to output to the console.

I added a try-parse to the beginning of the program to check to see if the user entered a valid integer or not. The program would return an error and quit if the user entered a string instead. I also created an array for the runtimes so that I could output it as a nice csv file and open it in Excel.

Another option I would like to do next time is to give the option of running just the n number of disks, or everything under it. To do this, I would probably use a Scanner function, which I did not have access to for this lab.

## Appendix

| Size | Recursive It     |                     |  |
|------|------------------|---------------------|--|
| 1    | 0.007665         | 0.035258            |  |
| 2    | 0.006387         | 0.015585            |  |
| 3    | 0.013796         | 0.016862            |  |
| 4    | 0.030658         | 0.03117             |  |
| 5    | 0.048543         | 0.076136            |  |
| 6    | 0.108839         | 0.173988            |  |
| 7    | 0.161213         | 0.466267            |  |
| 8    | 0.345165         | 0.893444            |  |
| 9    | 0.963448         | 1.169883            |  |
| 10   | 1.619032         | 2.304508            |  |
| 11   | 2.239358         | 2.033435            |  |
| 12   | 2.921003         | 3.170615            |  |
| 13   | 4.956993         | 7.414027            |  |
| 14   | 9.32866          | 9.429067            |  |
| 15   | 14.43716         | 11.8069             |  |
| 16   | 23.73797         | 21.74848<br>40.1733 |  |
| 17   | 35.34278         |                     |  |
| 18   | 67.75536         | 96.66953            |  |
| 19   | 121.5074         | 98.48171            |  |
| 20   | 175.9875         | 187.9603            |  |
| 21   | 408.1724         | 375.9471            |  |
| 22   | 704.9243         | 722.9442            |  |
| 23   | 1539.288         | 1547.222            |  |
| 24   | 3271.464         | 3218.481            |  |
| 25   | 5873.401         | 6386.057            |  |
| 26   | 10926.05         | 11800.71            |  |
| 27   | 23677.31         | 23834.95            |  |
| 28   | 44354.36 48246.2 |                     |  |
| -    |                  |                     |  |

Time is in number of milliseconds

## Space

| ░ ( | Output of size 6.txt  | 4/3/2018 9:42 PM | Text Document | 5 KB          |
|-----|-----------------------|------------------|---------------|---------------|
| ░ ( | Output of size 7.txt  | 4/3/2018 9:42 PM | Text Document | 10 KB         |
| ░ ( | Output of size 8.txt  | 4/3/2018 9:42 PM | Text Document | 19 KB         |
| ░ ( | Output of size 9.txt  | 4/3/2018 9:42 PM | Text Document | 38 KB         |
| ░ ( | Output of size 10.txt | 4/3/2018 9:42 PM | Text Document | 75 KB         |
| ░ ( | Output of size 11.txt | 4/3/2018 9:42 PM | Text Document | 149 KB        |
| ░ ( | Output of size 12.txt | 4/3/2018 9:42 PM | Text Document | 297 KB        |
| ░ ( | Output of size 13.txt | 4/3/2018 9:42 PM | Text Document | 593 KB        |
| ░ ( | Output of size 14.txt | 4/3/2018 9:42 PM | Text Document | 1,185 KB      |
| ░ ( | Output of size 15.txt | 4/3/2018 9:42 PM | Text Document | 2,369 KB      |
| ░ ( | Output of size 16.txt | 4/3/2018 9:42 PM | Text Document | 4,737 KB      |
| ░ ( | Output of size 17.txt | 4/3/2018 9:42 PM | Text Document | 9,473 KB      |
| ░ ( | Output of size 18.txt | 4/3/2018 9:42 PM | Text Document | 18,946 KB     |
| ░ ( | Output of size 19.txt | 4/3/2018 9:42 PM | Text Document | 37,891 KB     |
| ░ ( | Output of size 20.txt | 4/3/2018 9:42 PM | Text Document | 75,781 KB     |
| ░ ( | Output of size 21.txt | 4/3/2018 9:42 PM | Text Document | 151,561 KB    |
| ░ ( | Output of size 22.txt | 4/3/2018 9:42 PM | Text Document | 303,121 KB    |
| ░ ( | Output of size 23.txt | 4/3/2018 9:42 PM | Text Document | 606,241 KB    |
| ░ ( | Output of size 24.txt | 4/3/2018 9:42 PM | Text Document | 1,212,481 KB  |
| ░ ( | Output of size 25.txt | 4/3/2018 9:42 PM | Text Document | 2,424,961 KB  |
| ░ ( | Output of size 26.txt | 4/3/2018 9:43 PM | Text Document | 4,849,921 KB  |
| ░ ( | Output of size 27.txt | 4/3/2018 9:43 PM | Text Document | 9,699,841 KB  |
| ░ ( | Output of size 28.txt | 4/3/2018 9:45 PM | Text Document | 19,399,681 KB |
|     |                       |                  |               |               |