

## Yichen Dong Final

### Problem 2

```
syms x y z;
f = x^5 + y^4 + z^3 + x*y^2 + y*z + 1;
h = hessian(f);
x = 1;
y=1;
z = -1;
subs(h)
%The diagonals are not all positive or negative
[ 20, 2, 0]

[ 2, 14, 1]

[ 0, 1, -6]
```

### Problem 5

If X goes first, X will pay  $\inf_x \sup_y f(x,y)$ . If Y goes first, Y will receive  $\sup_y \inf_x f(x,y)$ . Since  $\sup_y \inf_x f(x,y) \leq \inf_x \sup_y f(x,y)$ , Y would rather that X goes first, and X would rather that Y goes first, meaning that they both would prefer to be second.

### Problem 6

I assume that  $f(x^*, y^*)$  means that there exists a saddle point in this function. If there is a saddle point within the function, then any other  $y$  would have a lower return and any other  $x$  would have a higher return for every  $x$  and  $y$  in  $f$ . Of course, this wouldn't work for every function  $f$ , but only for those with a saddle point.

### Problem 7

Since we know that  $y^*$  is a constant, if we fixed  $y$  at  $y^*$  and allowed  $x$  to vary, and the result was a convex function for fixed  $y^*$ , then we know that any  $x$  that we set that is not at the lowest  $x^*$  will be higher, and thus the convexity ensures that  $f(x^*, y^*) \leq f(x, y^*)$ .

### Problem 8

Similar logic, but since for fixed  $x$  it is a concave function, any  $y$  that is not  $y^*$  will be lower than  $f$  evaluated at  $y^*$ .

### Problem 9

Using the notes for module 3 and the pset for module 8, we see that if  $Ax = b$ , then  $y$  is unbounded. Also, since  $x \geq 0$ , the  $A'y \geq c$ . Since in this case we are still maximizing  $-b'y$ , this means that we switched  $y$  with  $-y$ , and carrying through the negative, we get  $A'y \leq -c$ .

### Problem 10

They are as stated by " $L(x,u)$  is convex for fixed  $u$ ", which means that the original  $c'x$  is convex.

### Problem 11

Yes, since when we hold  $u$  constant we are still adding  $g(x)$ , and that is still a convex function.

#### Problem 16-20

```
Q = [2 1;
     1 2];
A = [3 1;
     -3 4];
b = [5;3];
c = [0,0];
[x,fval] = quadprog(Q,c,A,b)
Qd = inv(Q);
cd = -c*inv(Q);
[xd,fvald] = quadprog(Qd,cd,A,b)
Q1 = [1 2;
      2 1];
Qd1 = inv(Q1);
[x1,fval1] = quadprog(Q1,c,A,b)
[xd1,fvald1] = quadprog(Qd1,c,A,b)
fval1 - fvald1
```

#### Problem 21

$F(x)$  could be not convex if  $Q$  did not have all positives on the diagonal.

#### Problem 22

I believe that this is convex, because the  $x_i^2$  are always a convex function no matter where the intercept is.

#### Problems 23-25

My logic here is that if we had a semidefinite positive  $Q$ , then  $x'Qx$  would always be positive or 0. Thus, if  $(M-Q)$  was positive semidefinite, we would have the same effect.