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Local singular value decomposition for signal enhancement of seismic data

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ABSTRACT

Singular value decomposition (SVD) is a coherency-based technique that provides both signal enhancement and noise suppression. It has been implemented in a variety of seismic applications — mostly on a global scale. In this paper, we use SVD to improve the signal-to-noise ratio of unstacked and stacked seismic sections, but apply it locally to cope with coherent events that vary with both time and offset. The local SVD technique is compared with *f-x* deconvolution and median filtering on a set of synthetic and real-data sections. Local SVD is better than *f-x* deconvolution and median filtering in removing background noise, but it performs less well in enhancing weak events or events with conflicting dips. Combining *f-x* deconvolution or median filtering with local SVD overcomes the main weaknesses associated with each individual method and leads to the best results.

INTRODUCTION

Enhancement of signal embedded in background noise is an important issue in seismic data processing. By improving the quality of the seismic images, the results of subsequent processing or interpretation are much facilitated. The quality of the seismic image can be improved by means of different methods. Here we discuss three of these: *f-x* predictive deconvolution filtering, median filtering, and local singular value decomposition (SVD). We illustrate the advantage and disadvantage of each of these techniques for signal-to-noise enhancement.

Signal enhancement in the *f-x* domain was introduced by Canales (1984). It is a widely accepted and used technique in the oil and gas industry. The idea behind *f-x* deconvolution is based on signal predictability. Events that are linear or quasi-linear in the *t-x* domain are equivalent to a superposition of harmonics in the *f-x* domain. In the presence of noise, autoregressive models (AR) are suitable to predict a superposition of harmonics. *F-x* deconvolution is effective in attenuating random noise. It can handle conflicting dips and does not require dip steering, i.e., alignment of events, to flatten them. On the other hand, if the noise level is high, it is known to distort signal levels significantly (Spitz and Deschizeaux, 1994).

Median filtering is another technique that is also widely accepted in the oil and gas industry (Bednar, 1983). It operates by selecting the middle value of a sequence of numbers ordered by ascending magnitude. These numbers are taking from a moving window applied to the data. Median filter is effective in removing glitches on data as well as enhancing discontinuities. It has a simple implementation; however, it requires dip steering prior to signal enhancement.

Singular value decomposition is a powerful tool to detect and enhance laterally coherent signals in multitrace recordings. It has been implemented in a variety of seismic applications like dip filtering, VSP up/down wavefield separation, and residual statics corrections (Ulrych et al., 1988). SVD is suitable for data where coherent events can be aligned laterally. For example, this includes NMO-corrected CMP (common-midpoint) gathers and stacked sections. Coherent signals in multitrace data are extracted using an eigenvalue decomposition of the data-covariance matrix, after initial alignment of events by means of dip steering. This is done by including the contribution of the largest singular values only, since these represent the laterally coherent signals, while the smallest singular values are related to the background noise.

SVD can be applied to enhance the signal-to-noise ratio S/N in data sections containing laterally coherent events. Instead of applying this technique globally, i.e., on the entire data section in one go, as is usually done with this method (Andrews and Patterson, 1976), we apply it using a local window sliding in space and time. Local SVD, contrary to global SVD, can cope with short and quickly varying events.

In this paper, we investigate the use of local SVD to enhance the S/N in seismic data. Data within a local window are first extracted. Dip steering is then applied to align any laterally coherent signal and SVD is used thereafter for signal enhancement. Finally, the output data of the SVD are shifted back to their original pre-aligned posi-

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tions. A window sliding in both time and space is employed to cover the entire section.

The outline of this paper is as follows. First, we present the principles of SVD for signal enhancement. We then show results for synthetic and real data (unstacked and stacked gathers). We end with a discussion on performance and parameter settings for the proposed technique.

THEORY

In the proposed coherency-based technique for S/N enhancement, a data window is first extracted from the noisy input section. Dip steering is then applied on the extracted data to align the coherent events laterally, and SVD is used to enhance them. The output data are shifted back to their original pre-alignment positions to construct a local cleaned-up section. This is repeated sequentially throughout the input section. The locally enhanced section is mapped, in the output section, to the same location as the extracted window in the global section. By using a sliding window while averaging all overlapping, locally enhanced sections, a new seismic section is created with an enhanced S/N. The window is moved along the time and space directions to cover the entire input section. A percentage overlap is defined in order to remove edge artifacts. The next subsections discuss this procedure in more detail.

Dip steering

Dip steering is applied on the windowed data to flatten events, so it displays a larger level of lateral coherency. This is essential for the SVD decomposition to work effectively. Dip steering is based on estimating the time delays between the different traces in the data window and a reference trace using crosscorrelation. The maximum of the crosscorrelation indicates the time delay. Each trace is then shifted by its corresponding time delay.

The choice of the reference trace is an important issue. In our case, it is obtained by stacking the traces in the data window. This choice turns out to be robust when the noise level is relatively high or when most events are already correctly aligned, for example, after an NMO correction has been applied on a CMP gather. The reference trace is updated after each crosscorrelation pass by stacking the resulting shifted traces. The process of crosscorrelation, shifting, and stacking is repeated until the process converges.

SVD

The data in the analysis window are represented with an $(m \times n)$ data matrix, $\mathbf{X}$, consisting of m traces and n time samples per trace (generally $m < n$). The SVD of a data matrix $\mathbf{X}$, assumed of rank $r \leq m$, leads to a linear orthogonal expansion of the data given by Golub and Loan (1996)

$$\mathbf{X} = \mathbf{U_r D_r V_r}^T = \sum_{k=1}^{r} \lambda_k \mathbf{u_k v_k}^T, \tag{1}$$

where the matrix $\mathbf{U_r} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u_r}]$ contains the r left singular vectors, $\mathbf{D_r} = \mathrm{diag}(\lambda_1, \lambda_2, \dots, \lambda_r)$ is a diagonal matrix containing the singular values λ_i and the matrix $\mathbf{V_r} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v_r}]$ represents the r right singular vectors. The vectors $\mathbf{u_k}$ are called the propagation vectors and the vectors $\mathbf{v_k}$ the eigen-wavelets (Vrabie et al., 2004). The positive quantities λ_k, sorted as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r$, can be shown to be the positive square roots of the eigenvalues of the data covariance matrix $\mathbf{XX}^{\mathrm{T}}$. The term $\mathbf{u_k v_k}^T$ is an $(m \times n)$ matrix of rank one called the kth eigenimage of $\mathbf{X}$. Orthogonality of the SVD expansion ensures that the propagation vectors and the eigen-wavelets are orthogonal,i.e., $\mathbf{v_i}^T\mathbf{v_j} = \delta(i - j)$ and $\mathbf{u_i}^T\mathbf{u_j} = \delta(i - j)$ with δ the Kronecker function.

Laterally coherent events in the data window create a linear dependence among the traces. The data then show a high degree of trace-to-trace correlation and can therefore be reconstructed from a few eigenimages only. The SVD-enhanced signal $\hat{\mathbf{X}}_{svd}$ is obtained by rank reduction, i.e., by taking only the contribution of the first p eigenimages into account (Freire and Ulrych, 1998). That is,

$$\hat{\mathbf{X}}_{svd} = \sum_{k=1}^{p} \lambda_k \mathbf{u_k v_k}^T. \tag{2}$$

SVD acts as a data-driven, low-pass filter by rejecting highly uncorrelated traces. In most of the applications where local SVD is used, p is set to 1 or 2. In our method we assume that there is a single event in the window, so only a single eigenimage is needed for the reconstruction and thus S/N enhancement.

COMPARISON OF METHOD

In this section, we compare the performance of local SVD, *f-x* deconvolution, and median filtering for signal enhancement. We consider three types of data: a synthetic gather, an NMO-corrected CMP gather, and a stacked section.

For *f-x* deconvolution, we use a short-time Fourier analysis with a sliding temporal window with an overlap of 50% to remove edge effects. The median filter is applied locally after dip steering, similar to SVD. It is set up in two sequential steps, where we apply a median filter of length 3 and 5, respectively. In each step, the median filtering is applied until convergence to obtain the so-called root signal (Arce, 2005). Because the median filter is applied after dip steering, the filter length cannot exceed the window width used in the dip steering.

Synthetic data

We consider a noiseless synthetic section that consists of 80 traces with 256 samples per trace (Figure 1a). This section contains some interesting features often encountered in real data: a lateral event with decreasing amplitude level (A), a dipping event (B), an isolated event (C), a discontinuity (D), and two events with different amplitudes and conflicting dips (E). Zero-mean Gaussian noise is added to create a more realistic section (Figure 1d).

The sliding window for dip steering, which is common to median filtering and local SVD, consists of $m = 20$ traces and $n = 32$ time samples. The SVD window has a 50% overlap to prevent edge artifacts. We set $p = 1$ for the rank reduction in all final sections. For *f-x* deconvolution, we use a temporal window of 32 samples and 50% overlap to cope with the nonstationarity of the data. Prediction filtering is done with an AR model of order four fitted with 20 samples. For the purpose of comparison, we investigate the use of global SVD where we set $p = 5$, i.e., five eigenimages are used in the reconstruction. A value of p larger than 2 is used in the global SVD to include more details in the reconstructed section.

Global SVD (Figure 1b) partially boosts those lateral events with a relatively high S/N. However, it completely fails to retrieve the isolated event and does not succeed in boosting any of the dipping events. On the other hand, local SVD (Figure 1e) is effective in removing most of the background noise and retrieving the isolated and dipping events. It is partially able to retrieve the conflicting dips, specially the stronger one. Though local SVD fails to retrieve events with low S/N (top right of Figure 1e), the boosted signal preserves the original amplitude variation.

F-x deconvolution performs best in terms of boosting the different events and retrieving the conflicting dips, except maybe for the isolated dip (Figure 1). Remarkably, it does not manage to preserve amplitude variation as well as local SVD does. On the other hand, it is the only method that interpolated the discontinuity. In this case, this is an unwanted artifact, but it is often seen as a desirable feature. It is not as effective as the local methods in removing the background noise. Median filtering (Figure 1f) also succeeds in retrieving the coherent events, but it did least well in terms of suppression of background noise. As for the conflicting dips, it is able to retrieve only the strongest one.

The main drawback of *f-x* deconvolution and median filtering, i.e., their tendency to retain some background noise, can be remedied by combining *f-x* deconvolution or median filtering with local SVD. Figure 2a and b shows the result of feeding the output of *f-x* deconvolution and median filtering to local SVD. Clearly, the background noise is largely suppressed as compared with Figure 1c and f.

Finally, to have more insight on the filtering effect of each method, we plot the residual sections (i.e., difference plots, section before minus section after filtering) for each technique as shown in Figure 3. Among all the techniques, local SVD (Figure 3a) has removed less signal, followed by *f-x* deconvolution. Remarkably, although applying local SVD after *f-x* deconvolution or median filtering reduces the background noise best, it has also a tendency to leak some signal into the residual sections as shown in Figure 3e and f.

It is evident that when applying local SVD after *f-x* deconvolution or median filtering, a trade-off between noise suppression and signal removal is taking place. This trade-off is determined by the number of eigenimages included in the local SVD computation. In Figure 3e and f, one eigenimage is used, which corresponds to a maximum noise-suppression configuration, and results consequently in the removal of signal components. By including more eigenimages, we potentially reduce this drawback, at the expense, however, of leaving some background noise.

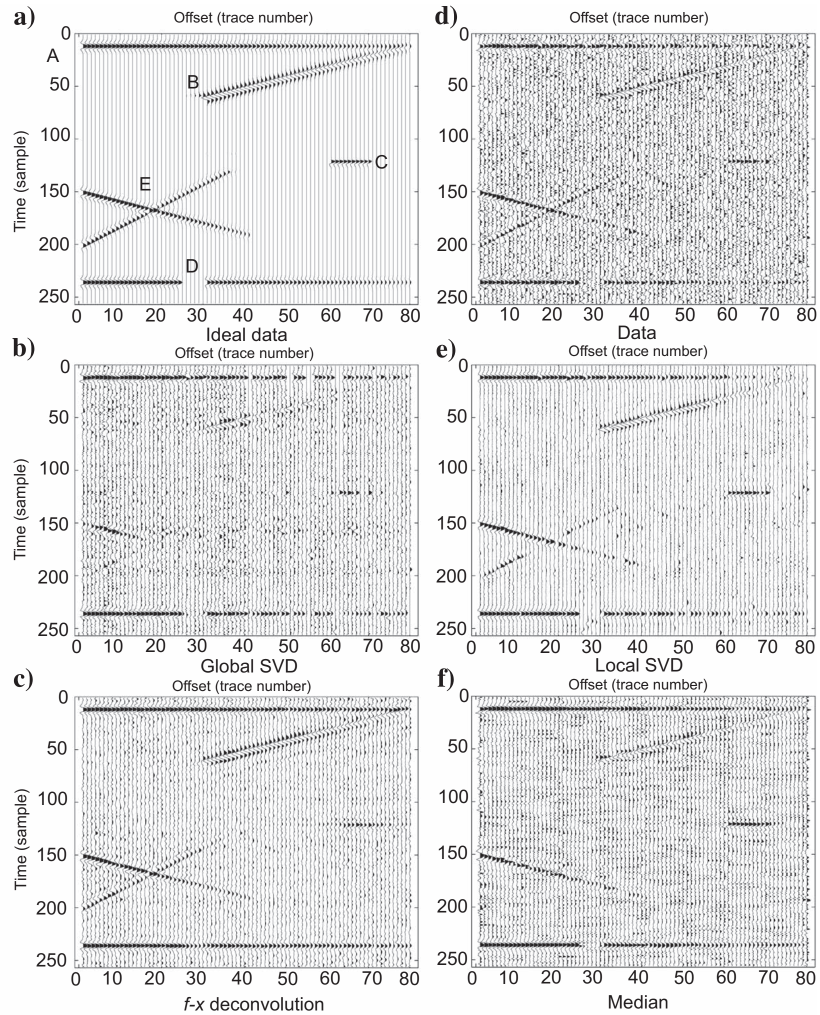

Figure 1. Signal enhancement of synthetic section: (a) noiseless data, (d) noisy data and results using different techniques: (b) global SVD, (c) *f-x* deconvolution, (e) local SVD, and (f) median filtering. Local SVD performs better than global SVD. All methods except global SVD boost the S/N. *F-x* deconvolution interpolates the discontinuous event (D), but performs well for the isolated event (C). Median filtering is the least effective in removing background noise.

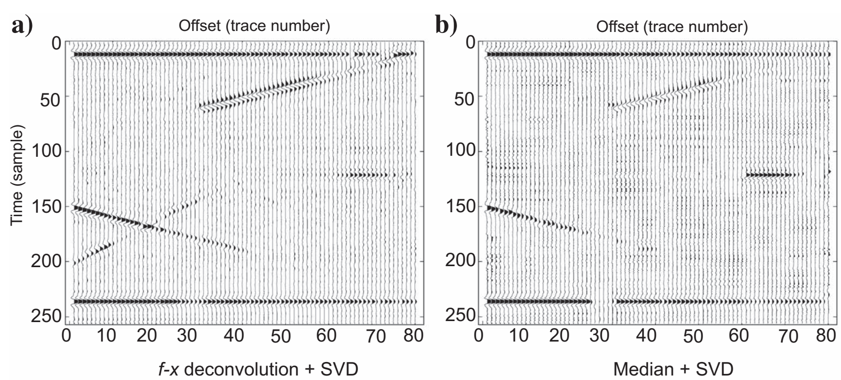

Figure 2. Signal enhancement of synthetic section: combination of classical techniques with local SVD. (a) *F-x* deconvolution + SVD. (b) Median filter + SVD. Most of the background noise left in Figure 1c and f is removed in (a) and (b), respectively.

Moveout-corrected CMP gather

Here we consider an unstacked NMO-corrected CMP gather consisting of 76 traces with 2500 samples per trace. The sliding window for the local method consists of $m = 10$ traces and $n = 100$ time samples, with a percentage overlap of 50%. We set again $p = 1$ for dimension reduction in all final sections. For the *f-x* deconvolution, we use a short-time Fourier analysis with a sliding temporal window of 100 samples and an overlap of 50%. We again use 20 samples to estimate an autoregressive filter of order 4. Figure 4a contains the original NMO-corrected CMP data while Figure 4b to d shows the results after signal enhancement using, respectively, local SVD, *f-x* deconvolution, and median filtering.

We obtain results similar to the synthetic data section. The local SVD method (Figure 4b) removes most of the background noise present in the raw data (Figure 4a). *F-x* deconvolution (Figure 4c) and median filtering (Figure 4d) perform quite similarly in terms of noise suppression and coherency boosting. *F-x* deconvolution actually maps some of the noiselike signal into the muted area (e.g., trace number 30 at time = 400 m sec). All methods perform similarly in terms of coherency boosting of the strong events. However, *f-x* deconvolution wins in terms of its ability to interpolate discontinuous events, along with boosting events with weak energy.

The tendency of *f-x* deconvolution and median filtering to retain some background noise can be largely prevented by combining them with local SVD as we have done for the synthetic data. Figure 5a and b shows the results of respectively feeding the output of *f-x* deconvolution and median filtering to the local SVD technique. Clearly, the background noise is largely suppressed as compared with Figure 4c and d. Masked and isolated events are also preserved along with the interpolated ones.

The residual plots for each technique are shown in Figure 6. Very similar remarks to the synthetic data example are observed. Local SVD shows the smallest amount of removed signal, followed this time by median filtering and then *f-x* deconvolution. Clearly, the noise suppression ability achieved by applying local SVD after median filtering or *f-x* deconvolution is balanced by the drawback of removing useful signal in addition to noise. This trade-off is more severe with median filtering (Figure 6d) than with *f-x* deconvolution (Figure 6c).

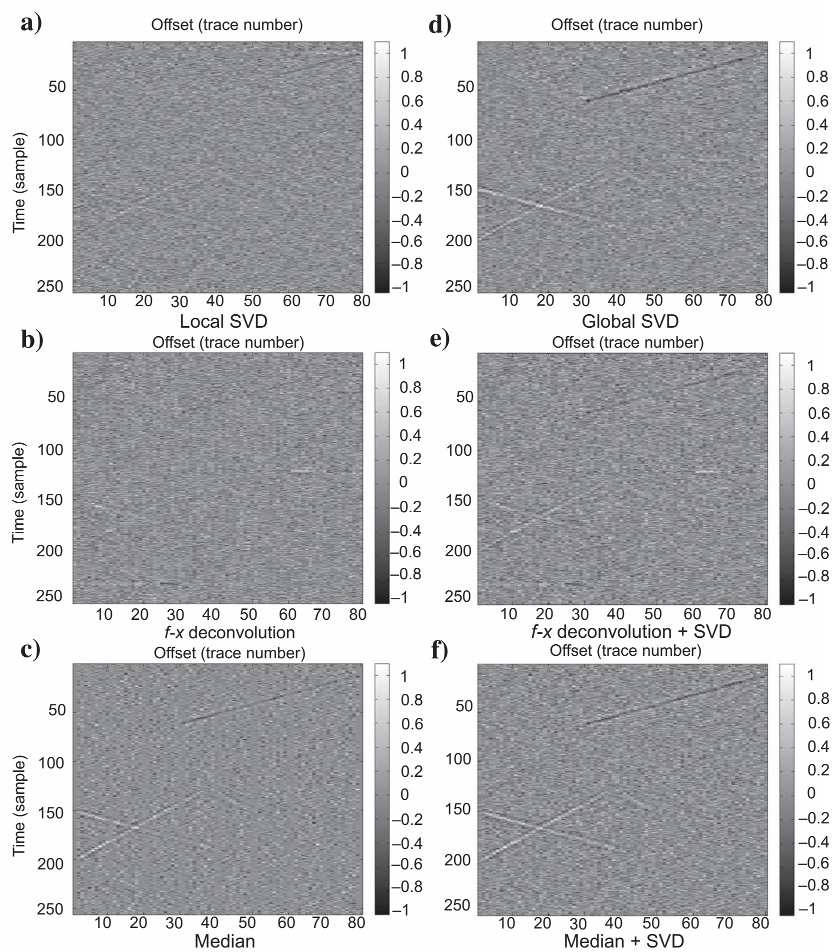

Figure 3. Residual plots for the different signal enhancement techniques (synthetic section): (a) local SVD, (b) *f-x* deconvolution, (c) median filtering, (d) global SVD, (e) *f-x* deconvolution + SVD, and (f) median + SVD. Little signal is found in the residual sections after local SVD (a). On the other hand, application of local SVD after *f-x* deconvolution (e) or median filtering (f) reduces the amount of background noise but also removes more signal.

Stacked section

In Figure 7a, we consider finally a stacked section. We apply the same methods and with the same parameter values, except that for the local window we use a length of 50 samples. The window length of the short-time Fourier analysis in the *f-x* deconvolution is 50 samples. The results are displayed in Figure 7b to d after respectively applying local SVD, *f-x* deconvolution, and median filtering.

Similar results are again obtained as before. Local SVD performs the best in terms of suppression of background noise (bottom of Figure 7b). The *f-x* deconvolution (Figure 7e) and median filtering (Figure 7d) show comparable performance, but *f-x* deconvolution has better interpolation capabilities. Combining *f-x* deconvolution and median filtering with local SVD again gives the best results (Figure 8a and b). Once more, a considerable amount of the background noise left by *f-x* deconvolution and median filtering is removed by applying local SVD afterwards.

Inspection of residual sections shows again that the enhanced suppression of background noise, obtained by combining local SVD with *f-x* deconvolution or median filtering, was counteracted by an increased smoothing of the reflections and an increased signal removal.

DISCUSSION

Parameter settings

How do the parameters of the local SVD method affect its performances?

- In most seismic applications, retaining only one or two eigenimages leads to satisfactory results. All our results were obtained using the first eigenimage only ($p = 1$). Increasing the number of eigenimages used can help reduce the amount of removed signal, in particular if SVD is applied after median filtering or *f-x* deconvolution, but it results in less noise reduction.
- Increasing the width m of the analysis window produces more noise suppression, but at the expense of slightly flattening events and missing some weak dips. The window length n is less crucial. It should be large enough to ensure an efficient computation of the eigenvalue decomposition, but not too large; otherwise, the analysis window becomes a global one, and it can deal no longer with rapidly changing events and waveforms.
- A zero percent overlap between sliding windows produces artifacts that look like discontinuities in the output section. Increasing the overlap provides more noise suppression and increases lateral continuity of events, but at the expense of increasing the computational load somewhat. A 50% overlap gave a satisfactory compromise between computation cost and creation of potential artifact, but sometimes a larger percentage overlap may be needed, depending on the data set.
- Median filtering can lead to a better suppression of background noise by increasing the window length (Arce, 2005). However, this may produce signal distortion. The same is true for *f-x* deconvolution if the AR order is reduced (Sacchi, 1999).

Advantage and disadvantage of each technique

- The SVD method is effective in removing the background noise. It is less capable of interpolating discontinuous events than *f-x* deconvolution. It can only handle confecting dips to a limited extent. On the other hand, it did seem to respect amplitude variations along reflectors better in synthetic data than the two other techniques.
- In the implementation of the local SVD method, not all the singular values are to be computed because we are only interested in the largest one. Algorithms exist that extract the largest singular value only. These are computationally very effective (Golub and Loan, 1996).
- Median filtering is an attractive alternative to *f-x* deconvolution as its implementation is much simpler and it requires less computational effort. However, it cannot deal with conflicting dips.
- *F-x* deconvolution can handle conflicting dips and interpolate discontinuous events. On the other hand, it reduces the background noise less well and may map energy to areas where none existed before (e.g., muted areas), indicating that it is more prone to boosting aliased energy.
- Like local SVD, median filtering requires dip steering. However, the performance of the median filter is less sensitive to a misalignment than local SVD.
- 3D versions of median filtering (Astola et al., 1990) and *f-x* deconvolution (Spitz and Deschizeaux, 1994) already exist. Similarly, global 3D-SVD has been investigated (Vrabie et al., 2006); therefore, a generalization to local 3D-SVD is straightforward.

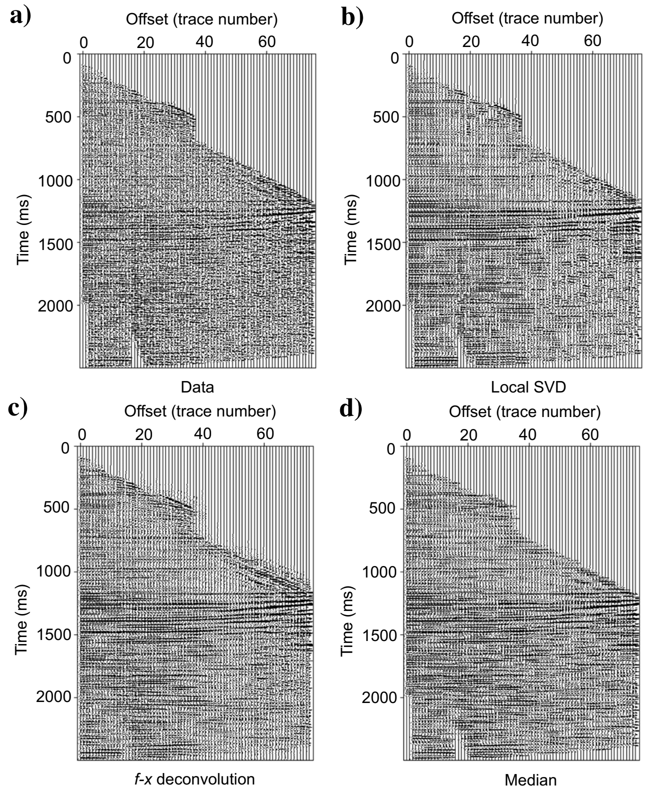

Figure 4. Signal enhancement of unstacked data. (a) Original NMO-corrected CMP gather, and results using different techniques: (b) local SVD, (c) *f-x* deconvolution, and (d) median filtering. All techniques boost the S/N. *F-x* deconvolution (c) interpolates discontinuous events and retrieves weak dips better, but suppresses the background noise less well than local SVD (b). Median filtering (d) yields a very similar performance to *f-x* deconvolution (c).

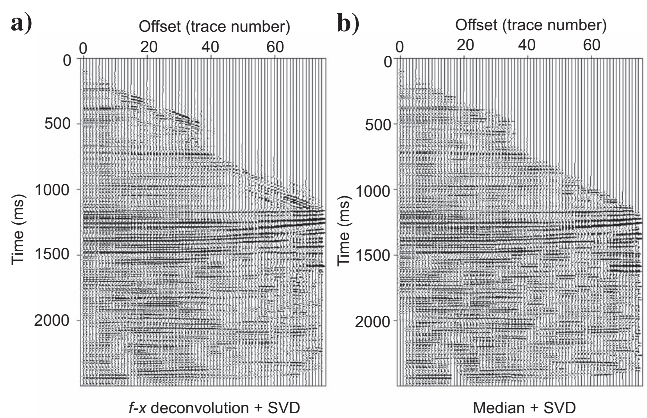

Figure 5. Signal enhancement of unstacked data using a combination of classical techniques with local SVD. (a) *F-x* deconvolution + SVD. (b) Median filter + SVD. Most of the background noise left in Figure 4d and e removed in (a) and (b), respectively.

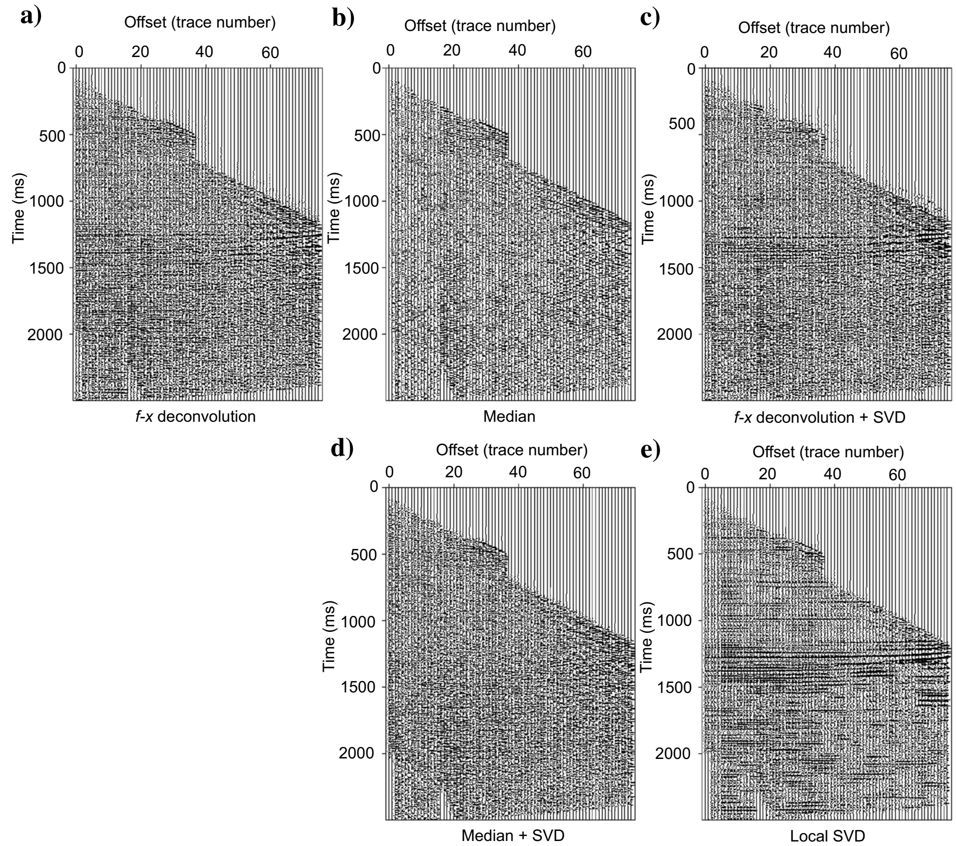

Figure 6. Residual plots for the different signal enhancement techniques (unstacked section). (a) *F-x* deconvolution, (b) median filtering, (c) *f-x* deconvolution + SVD, (d) median filtering, and (e) local SVD. Local SVD and median filtering lead to the least signal leakage into the residual sections. Combining median filtering or *f-x* deconvolution with local SVD reduces the background noise level but removes some signal component.

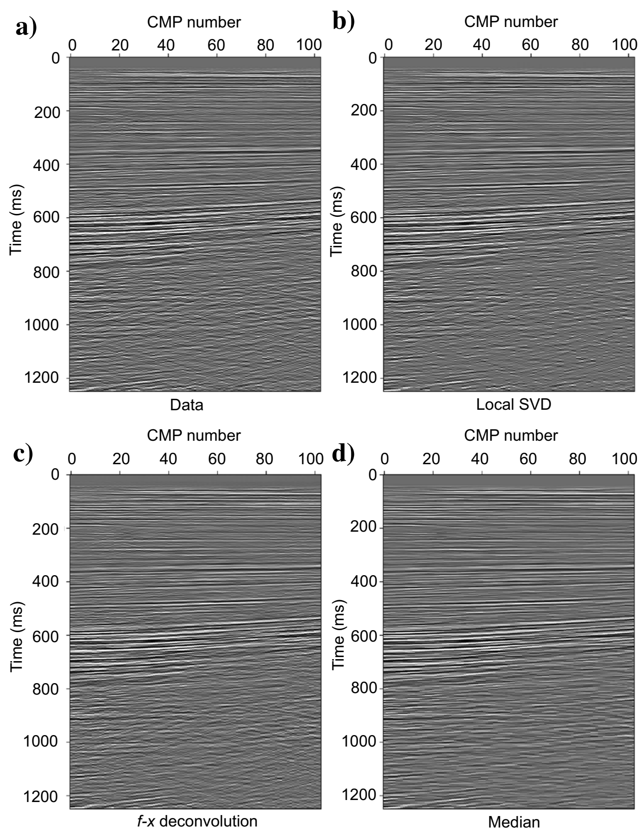

Figure 7. Signal enhancement of stacked section. (a) A stacked section. Results using different techniques: (b) local SVD, (c) *f-x* deconvolution, and (d) median filtering. All techniques boost the S/N. *F-x* deconvolution interpolates discontinuous events and retrieves weak dips better, but removes the background noise less well, than local SVD.

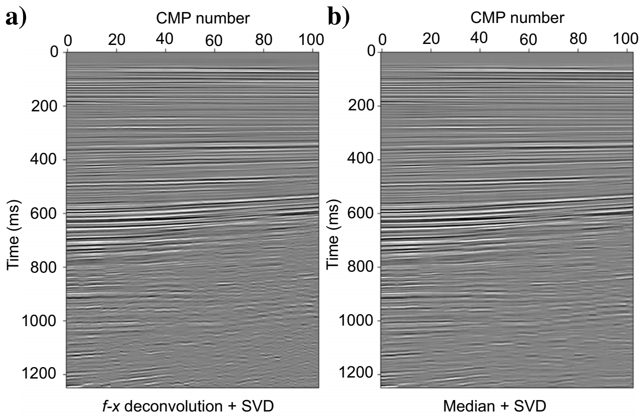

Figure 8. Signal enhancement of stacked section using a combination of classical techniques with local SVD. (a) *F-x* deconvolution + SVD. (b) Median filter + SVD. Most of the background noise left in Figure 7c and d is removed.

CONCLUSIONS

Local SVD is a powerful technique that can simultaneously boost coherent signals and suppress background noise in seismic sections. It is easy to implement and convenient to use because its performance depends on few parameters. The local SVD method is better than *f-x* deconvolution and median filtering in removing background noise, but it performs less well in boosting weak events or events with conflicting dips. *F-x* deconvolution and median filtering perform quite similarly, but *f-x* deconvolution is better in interpolating discontinuous events, and it can handle conflicting dips. Combining *f-x* deconvolution or median filtering with local SVD suppresses the background noise best, but may lead to overly smoothed seismic images with some signal recognized in the difference plots.

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REFERENCES

Andrews, H. C., and C. L. Patterson, 1976, Singular value decomposition and digital image processing: IEEE Transactions on Acoustics, Speech, and Signal Processing, **24**, 26–53.

Arce, G. R., 2005, Nonlinear signal processing: Wiley Interscience.

Astola, J., P. Haavisto, and Y. Neuvo, 1990, Vector median filter: IEEE Transactions on Acoustics, Speech, and Signal Processing, **78**, 678–689.

Bednar, J. B., 1983, Applications of median filtering to deconvolution, pulse estimation, and statistical editing of seismic data: Geophysics, **48**, 1598–1610.

Canales, L., 1984, Random noise reduction: 54th Annual International Meeting, SEG, Expanded Abstracts, 525–527.

Freire, S. L. M., and T. J. Ulrych, 1998, Application of singular value decomposition to vertical seismic profiling: Geophysics, **53**, 778–785.

Golub, G. H., and C. F. V. Loan, 1996, Matrix computations: Johns Hopkins University Press.

Sacchi, M. D., 1999, Statistical and transform methods for seismic signal processing: Lecture notes, Department of Physics, University of Alberta, http://www-geo.phys.ualberta.ca/saig/courses.html.

Spitz, S., and B. Deschizeaux, 1994, Random noise attenuation in data volumes — FXY predictive filtering versus a signal subspace technique: 56th Annual Conference and Exhibition, EAGE, Extended Abstracts, H033.

Ulrych, T. J., S. L. M. Freire, and P. Siston, 1988, Eigenimages processing of seismic section: 58th Annual International Meeting, SEG, Expanded Abstracts, 1261–1265.

Vrabie, V. D., N. Le-Bihan, and J. L. Mars, 2006, Multicomponent wave separation using HOSVD/unimodal ICA subspace method: Geophysics, **71**, no. 5, V133–V143.

Vrabie, V. D., J. L. Mars, and T. L. Lacoume, 2004, Modified SVD by means of ICA: Signal Processing, **84**, 645–652.