Application of singular value decomposition to extract seismic waveforms

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Earthquakes generate seismic waves that travel through the Earth and are recorded as time series by instruments placed near the Earth's surface (Figure 1). Extraction of seismic waveforms from the recordings is crucial for understanding the characteristics of earthquakes and the internal structures of the Earth. However, individual recording can be noisy (Figure 1b) and contain more than one type of wave at similar times (Figure 1c), which poses a challenge to extract the target waveform of interest. Alternatively, if there are multiple instruments in a closely spaced configuration, the same type of waves will be recorded as coherent signals (Figure 2), and the singular value decomposition (SVD) can be utilized to extract such a coherent trend in the recordings [e.g., Freire and Ulrych, 1988, Bekara and Van der Baan, 2007]. The goal of this project is to apply SVD to extract the target waveforms from the background noise and waveforms of other types.

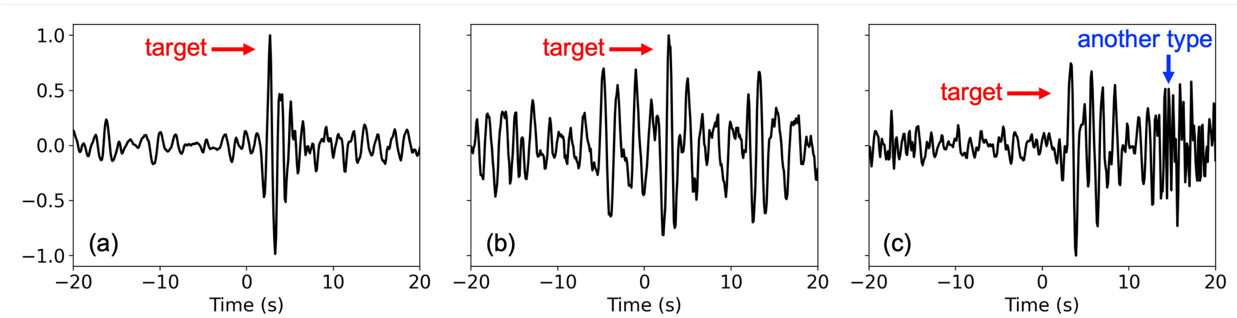

Figure 1: Examples of individual recording in a 40s time window. In each panel, the red arrow indicates the target waveform, which is centered at the predicted arrival times of 0s. (a) A clean waveform. (b) A waveform buried in the background noise. (c) A waveform followed by another waveform (neighboring) of a different type, which is indicated by the blue arrow.

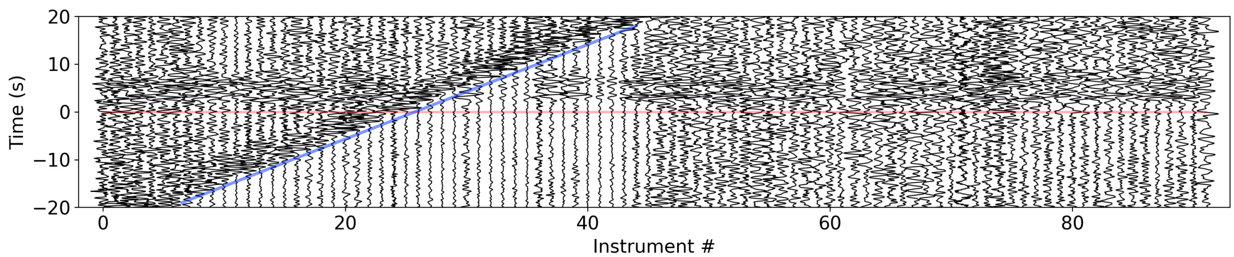

Figure 2: An example of recordings by multiple instruments. The x-axis indicates the instrument/recording number, and the y-axis indicates a 40s time window. The target waveforms are centered at the predicted arrival times of 0s, which is indicated by the transparent, red line. Note the coherency of the target waveforms among different recordings, despite a high level of background noise and the presence of neighboring waveforms of a different type, which is indicated by the transparent, blue line.

The basic idea is described as follows. First, time shifts are applied to individual recordings to align the target waveforms by cross-correlation. Next, a matrix $\boldsymbol{A}$ is constructed whose column represents a recording of length M with a total of N recordings. Depending on the length M (it is chosen such that it is long enough to capture the target waveforms of interest but short enough to exclude potential complexities in longer time series) and the number of instruments/recordings available, M can be smaller, equal to or larger than N. The SVD of $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \sum_{i=1}^{r} \sigma_i u_i v_i^T, \tag{1}$$

where r is the rank, σ_i is the i^{th} singular value, u_i is the i^{th} left singular vector, and v_i is the i^{th} right singular vector of $\boldsymbol{A}$. The target waveforms exhibit a high degree of coherency and hence can be approximated by a low-rank matrix $\boldsymbol{A_k}$, which is

$$\boldsymbol{A_k} = \sum_{i=1}^{k} \sigma_i u_i v_i^T, \tag{2}$$

where $k < r$, and the value of k is determined based on some cutoffs on the singular values σ_i.

The chapters of the textbook this project is mostly connected to are 3.6 Singular Value Decomposition and 4.7 Computing the Singular Value Decomposition.

References

M. Bekara and M. Van der Baan. Local singular value decomposition for signal enhancement of seismic data. *Geophysics*, 72(2):V59–V65, 2007.

S. L. Freire and T. J. Ulrych. Application of singular value decomposition to vertical seismic profiling. *Geophysics*, 53(6):778–785, 1988.