MAE 5510 Final Report

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In this report, I describe my glider design for this course, including basic dimensions and dynamic and static analysis. The design process is based on the baseline glider, improving lift characteristics while optimizing stability. Since we are focused on improving the glider's performance in simulated flights, we focus on the optimization of dynamic analysis. In this course I learned the ability to analyze glider performance, dynamic and static stability. I also learned how to analyze Handling Qualities. And be able to apply the above knowledge to design a better sliding box machine than the baseline

I. Nomenclature

b	=	wing span
c	=	chord
C_L	=	lift coefficient
C_D	=	drag coefficient
C_{D_0}	=	zero-lift drag coefficient
$C_{L,\alpha}$	=	lift slope
C_W	=	weight coefficient
$C_{n,\beta}$	=	yaw-stability derivativ
$C_{l,\beta}$	=	roll-stability derivative
C_m	=	moment coefficient
D	=	drag
e	=	Oswald efficiency
I	=	inertia tensor
L	=	lift
$(R_{G0})_{max}$	=	maximum zero-wind glide ratio
R_A	=	aspect ratio
S_w	=	planform area of main wing
t_{max}	=	max thickness of airfoil
V	=	velocity
V_{BG0}	=	zero-wind best glide airspeed
V_{min}	=	minimum airspeed
V_{MD}	=	minimum drag airspeed
V_{MDV}	=	minimum power airspeed
W	=	total weight of glider
W_w	=	wing weight
W_f	=	fuselage weight (dowel + ballast)
x_{ac}	=	x-coordinate of the aerodynamic center
y_{ac}	=	y-coordinate of the aerodynamic center
$\omega_{n_{sp}}$	=	undamped natural frequency of short-period mode
ρ	=	air density
σ_{max}	=	max stress
γ	=	specific weight
σ	=	static margin

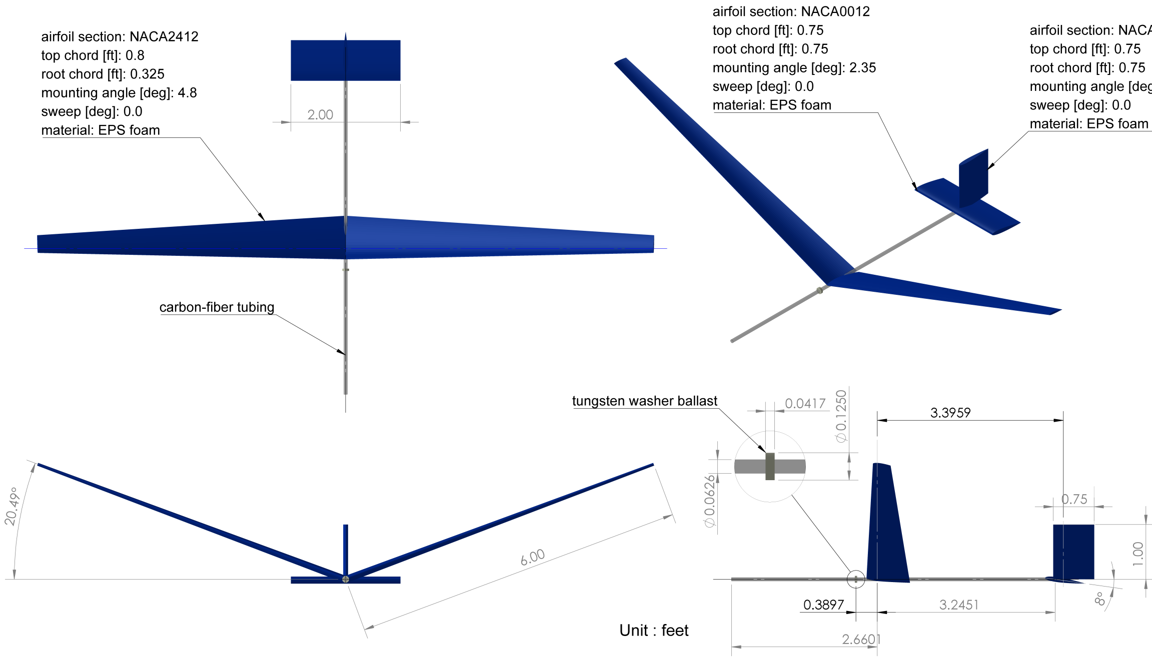


Fig. 1 The technical plot of the glider

II. Glider gemotery

A. Basic glider description and technical drawing

The glider design is a traditional configuration glider. The fuselage is a 6-feet long carbon-fiber tubing. The main wing is in front and the horizontal stabilizer is installed behind the main wing. Lastly, the vertical stabilizer is installed on the tip of the fuselage tube. All airfoils are made with EPS foam. All wings have no twist and sweep. Only main wing has 20.5 deg dihedral. A tungsten washer ballsat is installed on fuselage to balance the glider. Detailed dimensions and wing information are shown in Figure. 1.

B. Mass properties

The total weight of the glider is 1.032597 lbf. The fuselage weight is 0.65456 lbf. The CG location is in $x = -0.2980$, $y = 0$, $z = -0.2167$. The outer and inner dimensions of ballast is 1.5 inch and 0.75 inch and it locate in x=0.3896. The inertia tensor of this glider is

$$[\mathbf{I}] = \begin{bmatrix} 0.0615 & 0 & -0.0021 \\ 0 & 0.0526 & 0 \\ 0 & 0 & 0.1104 \end{bmatrix} \tag{1}$$

C. Planform area

The design limitation of this glider is that the wing area cannot exceed 9 ft^2. This glider uses a total of 8.25 ft^2, which meets the material restrictions. The total planform area is within the design limitation as Table. 1.

Property	main wing	horizontal stabilizer	vertical stabilizer	total
airfoil	NACA 2412	NACA 0012	NACA 0012	
semispan [ft]	6	1	1	
root chord [ft]	0.8	0.75	0.75	
tip chord [ft]	0.325	0.75	0.75	
planform area [ft^2]	6	1.5	0.75	8.25

Table 1 Wing planform table

D. Structure

For the wing structure calculation, the equation below is used;

$$R_A \leq \frac{16(t_{max}/c)\sigma_{max}W_w}{21W_f b\gamma} \tag{2}$$

Since the airfoil section use as main wing is NACA 2412, so $t_{max}/c = 0.12$, σ_{max} for EPS foam is 7200 lbf/ft^2 and γ is 0.804 lbf/ft^3 the max R_A allowed is 27.75. The glider have $R_A = 24$ is within the constraint.

E. Launch condition

The glider is design to flight with best no-wind glide velocity to get the maximum zero-wind glide ratio.

$$(R_{G0})_{max} = (C_L/C_D)_{max} = \frac{\sqrt{\pi e R_A}}{2\sqrt{C_{D_0}} + C_{D_1}\sqrt{\pi e R_A}} \tag{3}$$

$$V_{BG0} = V_{MD} \cong \frac{\sqrt{2}}{\sqrt[4]{\pi e R_A C_{D_0}}}\sqrt{\frac{W/S_w}{\rho}} \tag{4}$$

So the glider is trim with V_{BG0} = 13.89 ft/s, at this moment C_L = 0.886 and angle of attack is 0.

III. Design process

A. Bseline glider

1) Create all components of the glider in MachUp 6 to estimnate the weight property
2) Calculate the structural inteegrity of the main wing
3) Estimnate the maximum zero-glide ratio and corresponding lift coefficient and velocity. Here uses $e = 0.8$, and a zero-lift drag coefficient of $C_{D0} = 0.013$
4) Open a new file in MachUp 6, create the main wing and find the mounting angle that will be at the maximum zero-glide ratio lift coefficient with zero angle of attack.
5) Using MachUp 6 find the x location of the center of gravity such that the aircraft will trim at the design lift with zero angle of attack.
6) Create the horizontal stabilizer Place the horizontal stabilizer such that the glider has a static margin of $\sigma = 20\%$
7) Create the vertical stabilizer Place the vertical stabilizer such that the glider has a yaw-stability derivative of $C_{n,\beta} = 0.10$.
8) Add dihedral to the main wing until the glider has a roll-stability derivative of $C_{l,\beta}$ = -0.15
9) Using MachUp, Find the mounting angle of the horizontal stabilizer such that the aircraft is trim in pitch at zero degrees angle of attack (i.e. $C_m = 0$).
10) Add the dowel as a cylinder to your model in MachUp. Locate the dowel such that the aft end is coincident with the quarter-chord of the lifting surface that is furthest aft.
11) Add the ballast to your model in MachUp. Find the location of the ballast that results in the same x-location for center of gravity computed in step (5).

B. New glider

1) Modify the main wing based on the baseline to increase the aspect ratio to increase the gliding distance
2) Calculate the structural limits and confirm that the main wing is within the structural limits
3) Temporarily set the angle of attack of the horizontal stabilizer to 0 degrees
4) Calculate the maximum windless gliding speed and the corresponding CL, and find the corresponding main wing installation angle
5) Adjust the angle of attack of the horizontal stabilizer to trim the aircraft
6) Fine-tune the horizontal stabilizer, ballast position and dihedral angle so that all dynamic modes are positive and as large as possible

IV. Perfomance

A. Perfomance

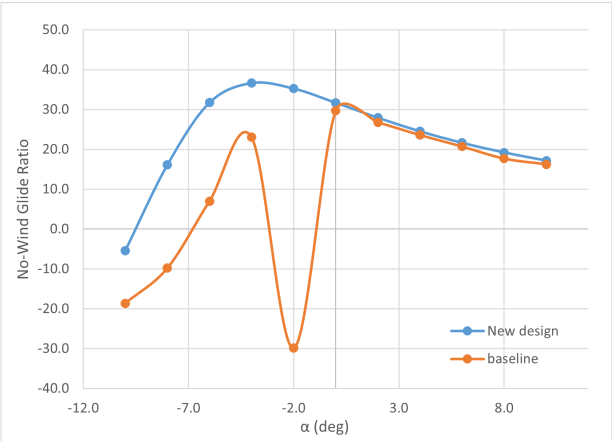
The basic perfomance C_L, C_D, C_m, L/D as Fig. 2. Compared with the baseline, the CL curve shifts upward, indicating that the lift performance increases, but the drag coefficient also increases. The newly designed Cm has the opposite sign to the angle of attack, indicating the increase in longitudinal stability, and L/D is significantly better than the baseline at negative angles of attack.

The drag and required power as Fig. 3. The newly designed curve between resistance and required power becomes flat at the negative angle of attack, and the resistance is slightly lower than the baseline at the positive angle of attack.

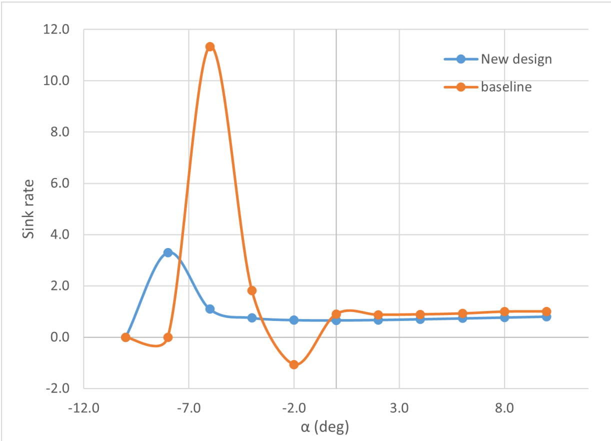
The static margin as Fig. 4. The newly designed Static Margin's negative angle of attack part is obviously inferior to the baseline, but Static Margin is not very important in dynamic analysis.

The no-wind glide ratio and sink rate as Fig. 5. No wind glide ratio is equal to L/D, Sink rate is equal to power required. The newly designed No wind glide ratio is obviously better than the baseline at negative angle of attack, and the Sink rate is also lower.

The Pitch, Roll, and Yaw stability derivatives as Fig. 6. Compared with the baseline, the newly designed pitch derivatives have a steeper slope and will be greater than 0 at -8 degrees, and the roll derivatives curve shifts downward, without much difference. Yaw derivatives become negative at an angle of attack of 6 degrees, which seems to affect stability.



(a) No-Wind Glide ratio

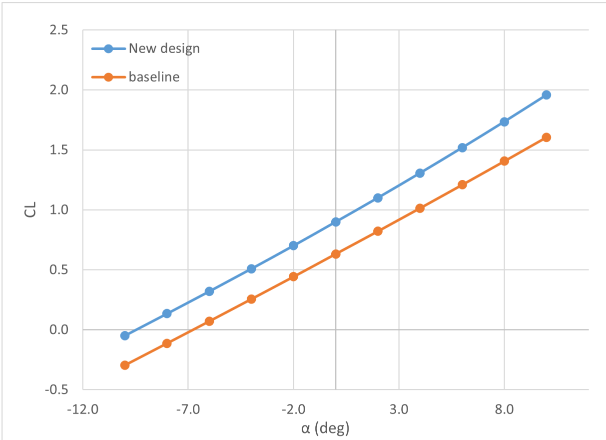


(b) Sink Rate

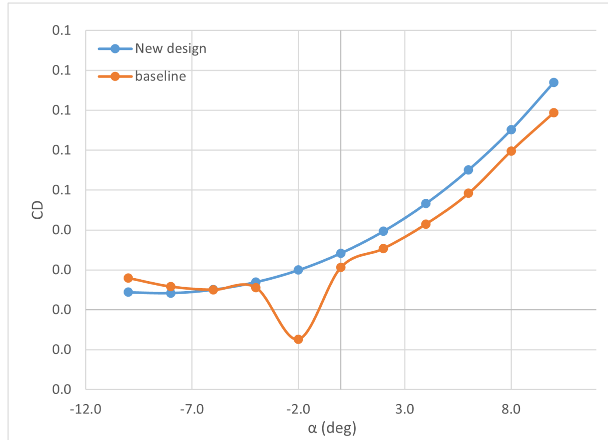
Fig. 5 Glide Perfomance

B. Change of aerodynamic centers

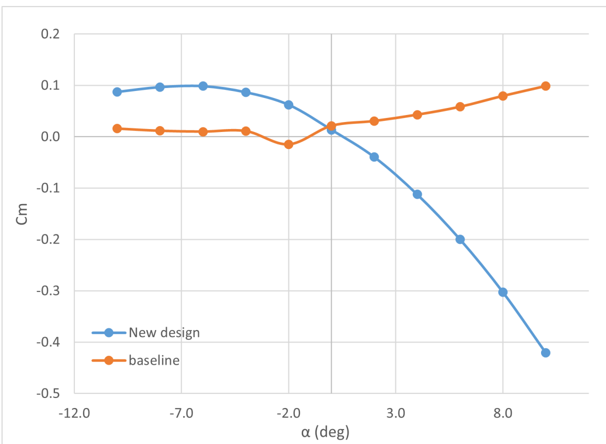
Compared with the baseline, the aerodynamic center of the new design changes drastically, as shown in Fig. 7.



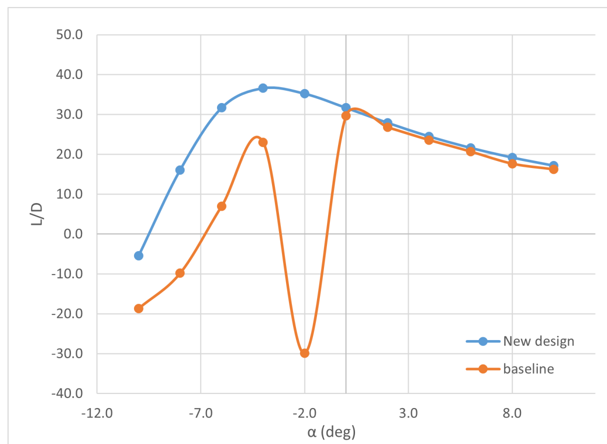
(a) C_L



(b) C_D

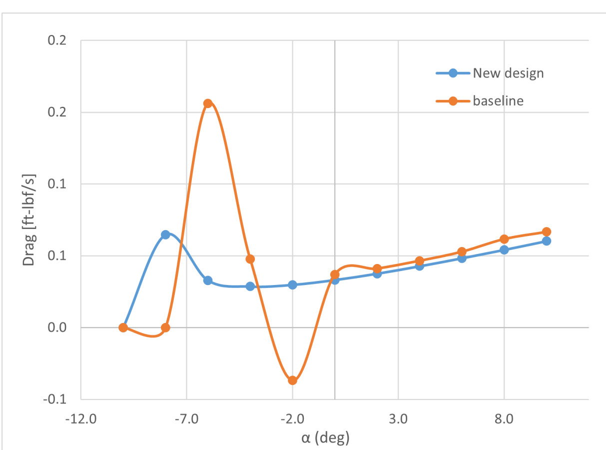


(c) C_m

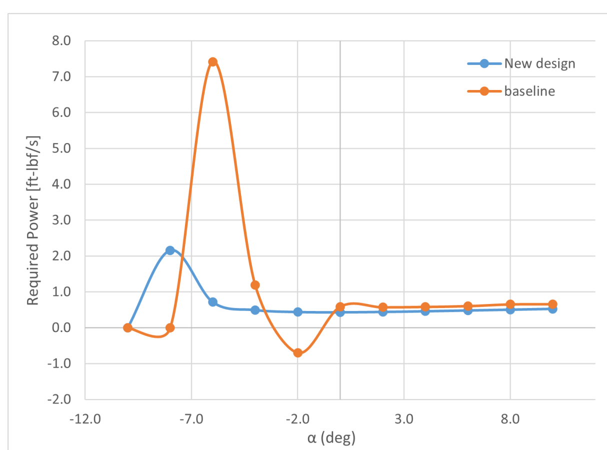


(d) L/D

Fig. 2 basic perfomance



(a) Drag



(b) Power Required

Fig. 3 Drag and Required Power

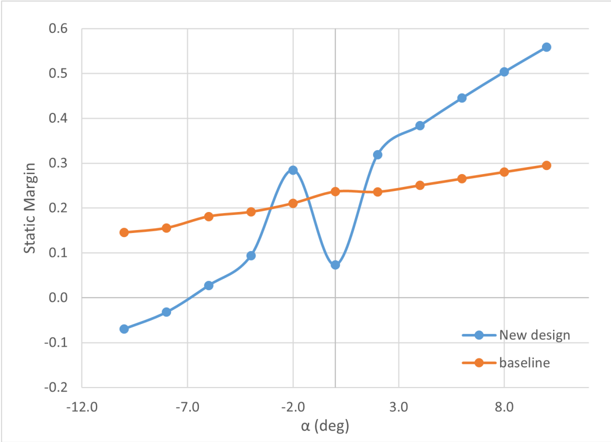


Fig. 4 Static Margin

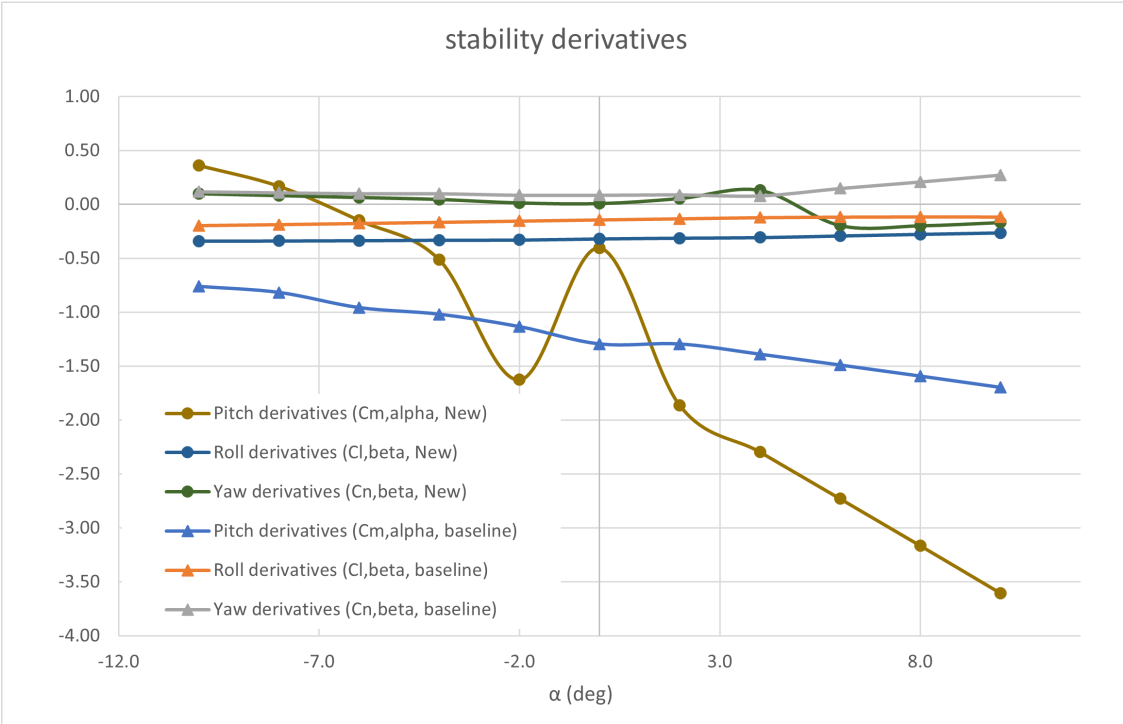
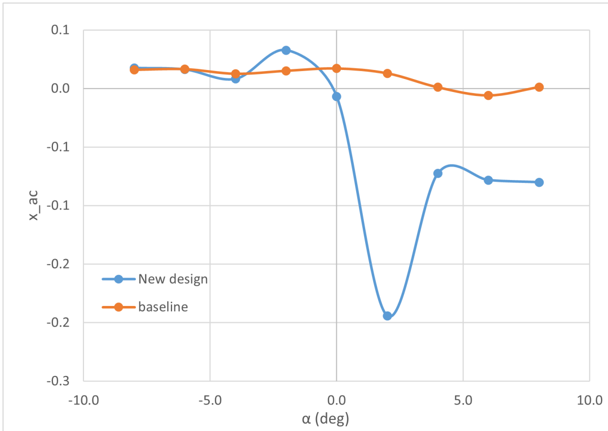
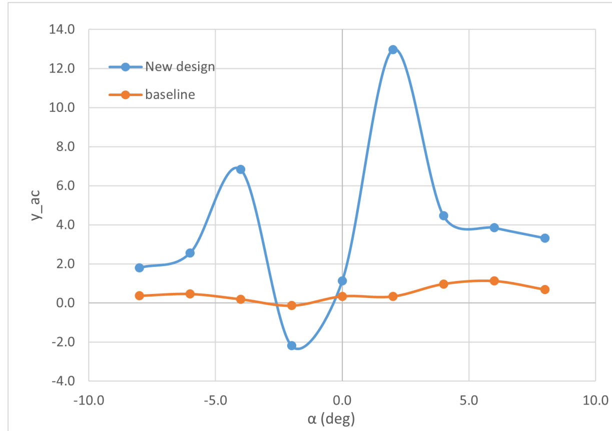


Fig. 6 Pitch, Roll, and Yaw stability derivatives



(a) x_{ac}



(b) y_{ac}

Fig. 7 Locus of aerodynamic centers

C. Stall characteristics

The minimum drag airspeed is equal to the no-wind glide velocity, so it can be calculated using eq.(4). The stall speed (V_{min}), and minimum power airspeed (V_{MDV}) are calculated as follows:

$$V_{min} = \sqrt{\frac{2}{C_{L_{max}}}}\sqrt{\frac{W/S_w}{\rho}} \tag{5}$$

$$V_{MDV} = \frac{2}{\pi e R_A C_{D_1} + \sqrt{(\pi e R_A C_{D_1})^2 + 12\pi e R_A C_{D_0}}}\sqrt{\frac{W/S_w}{\rho}} \tag{6}$$

The comparison of the new design with the baseline is listed in Table 2

	baseline	New design
CL_{max}	1.3768	1.5077
CD0	0.0104	0.0087
CD1	-0.0072	-0.0052
CD2	0.0389	0.0301
V_{min} [ft/s]	10.7515	10.0387
V_{MD} [ft/s]	17.5599	16.8234
V_{MDV} [ft/s]	14.0554	13.3865

Table 2 Stall characteristics compare to baseline glider

V. Dynamic analysis

A. Eigenvalues

The complex graph is shown in Figure 8. Compared with the baseline, the most obvious difference lies in the roll mode. The real part is much smaller than the baseline, so the stability is higher. The second is the Dutch roll mode. The sizes of both the real and imaginary parts have increased. Although it is difficult to see from the picture, the real part of the spiral mode has changed from positive to negative, which means that the spiral has changed from divergence to convergence.

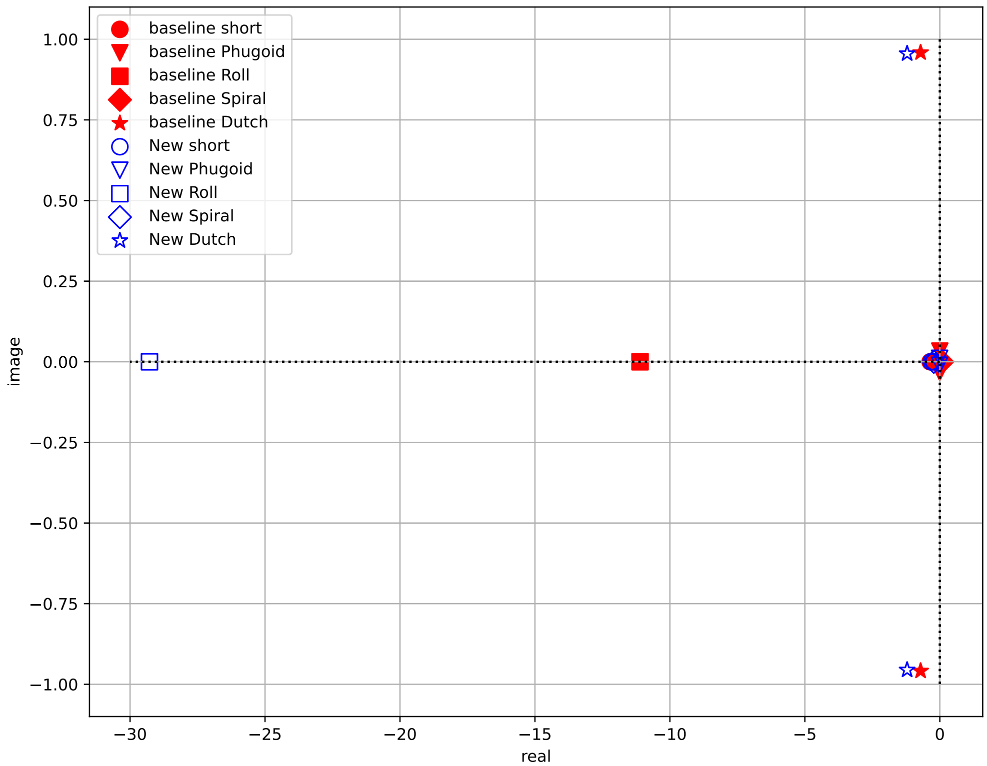


Fig. 8 Complex plot of eigenvalues

B. CAP

The Control Anticipation Parameter (CAP) is

$$CAP = \frac{\omega_{n_{sp}}^2}{C_{L,\alpha}/C_W} \tag{7}$$

The CAP of New design is 24.91329299. And baseline is 14.52559291.

C. Handling Qualities

Except for short period, all other modes are upgraded to level 1. It is significantly better than the baseline, as shown in Table 3.

Mode	Baseline Glider at Trim(V = 34.18 ft/s)	New Glider at Trim (V = 13.49 ft/s)
Short-Period	Level 3	Level 3
Phugoid	Level 2	Level 1
Roll	Level 1	Level 1
Spiral	Level 4	Level 1
Dutch Roll	Level 1	Level 1

Table 3 Handling Qualities of both gliders for Category B flight phases.

VI. Conclusion

All in all, the new glider is better than the baseline in basic performance. Although the static stability analysis is inferior to the baseline, we pay more attention to the dynamic analysis in simulated flight. In this part, the new design is better than the baseline. In this course, I learned how to analyze the static and dynamic stability of an aircraft, solve eigenvalue problems and analyze damping characteristics and frequencies, evaluate handling qualities. And combine the above knowledge to design a glider. Since the differences between the baseline and the new design will be compared in this report, it is recommended to introduce the concept of Python classes into the course, which can help students simplify and manage the program code and make it easier to compare the differences between the old and new designs.

Appendix

Appendix A: Json file

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{
    "tag": {},
    "simulation": {
        "constant_density": false,
        "time_step[sec]": 0.05,
        "total_time[sec]": 250,
        "ground_altitude[ft]": 4711
    },
    "aircraft": {
        "wing_area[ft^2]": 6.75,
        "wing_span[ft]": 12,
        "weight[lbf]": 1.0325971424482856,
        "Ixx[slug-ft^2]": 0.06047435477375984,
        "Iyy[slug-ft^2]": 0.06585068255662918,
        "Izz[slug-ft^2]": 0.11361377686262131,
        "Ixy[slug-ft^2]": -5.401947244440319e-18,
        "Ixz[slug-ft^2]": 0.0005467333248816431,
        "Iyz[slug-ft^2]": -3.890277595981199e-20,
        "hx[slug-ft^2/s]": 0,
        "hy[slug-ft^2/s]": 0,
        "hz[slug-ft^2/s]": 0,
        "name": "MyAirplane"
    },
    "initial": {
        "airspeed[ft/s]": 13.49,
        "altitude[ft]": 4761,
        "heading[deg]": 0,
        "type": "state",
        "state": {
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            "bank_angle[deg]": 0,
            "alpha[deg]": 0,
            "beta[deg]": 0,
            "p[deg/s]": 0,
            "q[deg/s]": 0,
            "r[deg/s]": 0,
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            "elevator[deg]": 0,
            "rudder[deg]": 0,
            "throttle": 0
        }
    },
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            "taper_ratio": 1
        },
        "gust_magnitude[ft/s]": 2,
        "stall": {
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            "alpha_blend[deg]": 10.9,
            "blending_factor": 40
        },
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            "0": 0.899892775482194,
            "alpha": 5.50494120529492,
            "alpha_hat": 0.7557,
            "qbar": -2.22539014739892,
            "de": 0
        },
        "CS": {
            "beta": -0.740659503113772,
            "pbar": -0.6333557267270769,
```

```
                "Lpbar": -0.07556077294121516,
                "rbar": 0.482928044737554,
                "da": 0,
                "dr": 0
            },
            "CD": {
                "L0": 0.012131549551287722,
                "L": -0.005316023766551325 5,
                "L2": 0.030156872900515802,
                "S2": 0.33167406643223796,
                "qbar": 0.09440715799018728,
                "Lqbar": 1.07844105175187,
                "L2qbar": -0.3566908395742603,
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                "Lde": 0,
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                "da": 0,
                "dr": 0
            },
            "Cm": {
                "0": 0.0135943526051724,
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                "de": 0
            },
            "Cn": {
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                "rbar": -0.0394895975025742,
                "da": 0,
                "Lda": 0,
                "dr": 0
            }
        }
    }
```


Listing 1 glider data json file

Appendix B Python code

```python
import numpy as np
import matplotlib.pyplot as plt
import json
import sys
from scipy.linalg import eig
import sympy as sp

g = 32.17
symbol_lo = ["delta mu   ", "delta alpla", "delta q    ", "delta zetax", "delta zetaz", "delta theta"]
symbol_la = ["delta beta ", "delta q    ", "delta rbar ", "delta zetay", "delta phi  ", "delta psi  "]
################################################################

# ======================================================================
def printMat(arr):
    shape = np.shape(arr)
    for i in range(shape[0]):
```

```python
        for j in range(shape[1]):
            print("{:> 17.12f}".format(arr[i, j]), end="")
        print("", end="\n")

def printMatToFile(arr, filename, variableName):
    with open(filename, "a") as f:
        shape = np.shape(arr)
        print("=================== ", variableName, " ================", end="\n", file=f)
        for i in range(shape[0]):
            for j in range(shape[1]):
                print("{:> 17.12f}".format(arr[i, j]), end="", file=f)
            print("", end="\n", file=f)

def error(A, B):
    err = (B - A) * 100 / A
    return err
# ======================================================================

class glider:
    def __init__(self, name):
        self.filename      = name
        self.vinf          = None
        self.Sw            = None
        self.bw            = None
        self.W             = None
        self.Ixx           = None
        self.Iyy           = None
        self.Izz           = None
        self.Ixy           = None
        self.Ixz           = None
        self.Iyz           = None
        self.rho           = None
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        self.CL_alpha      = None
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        self.CD_L0         = None
        self.CD_L          = None
        self.CD_L2         = None
        self.CD_qbar       = None
        self.CD_Lqbar      = None
        self.CD_L2qbar     = None
        self.CD_qbar       = None
        self.Cm_0          = None
        self.Cm_alpha      = None
        self.Cm_qbar       = None
        self.Cm_alpha_hat  = None
        self.CD_0          = None
        self.CD_1          = None
        self.CD_2          = None
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        self.phi_o         = None
        self.alpha_T0      = None
        self.V_o           = None
        self.CL_o          = None
        self.CY_beta       = None
        self.CY_pbar       = None
        self.CY_Lpbar      = None
        self.CY_rbar       = None
        self.CY_pbar       = None
        self.Cl_beta       = None
        self.Cl_pbar       = None
        self.Cl_rbar       = None
        self.Cl_Lrbar      = None
        self.Cl_rbar       = None
```

```
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        self.Cn_pbar      = None
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        self.Cn_pbar      = None
        self.cw           = None
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        self.CD_alpha     = None
        self.T_v          = None
        self.Z_T0         = None
        self.CD_alpha_hat = None
        self.CD_nu_hat    = None
        self.CL_mu_hat    = None
        self.Cm_mu_hat    = None
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        self.R_rhoy       = None
        self.R_xx         = None
        self.R_yy         = None
        self.R_zz         = None
        self.R_xz         = None
        self.CT_V         = None
        self.A_mat_lo     = None
        self.B_mat_lo     = None
        self.A_mat_la     = None
        self.B_mat_la     = None
        self.C_lo         = None
        self.C_la         = None
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        self.eigvecs_lo   = None
        self.eigvals_la   = None
        self.eigvecs_la   = None


        self.amps_lo           = None
        self.phase_lo          = None
        self.eigvals_lo_mag    = None
        self.sort_index        = None
        self.sigma_sp          = None
        self.sigma_sp99        = None
        self.sigma_sp_1        = None
        self.sigma_sp99_1      = None
        self.zeta_sp           = None
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        self.sigma_ph_1        = None
        self.sigma_ph99_1      = None
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        self.period_ph         = None
        self.zeta_ph           = None
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        self.B_sp              = None
        self.C_sp              = None
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        self.omega_d_sp_approx = None
        self.omega_n_sp_approx = None
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        self.sigma_q           = None
        self.R_ps              = None
        self.sigma_phi         = None
        self.sigma_ph_approx   = None
        self.omega_d_ph_approx = None
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        self.llambda_p         = None

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        self.phase_la      = None
```

```python
        self.dump_max      = None
        self.non_zero_real = None
        self.dump_min      = None
        self.index_roll    = None
        self.index_spiral  = None
        self.index_Dutch   = None
        self.eig_roll      = None
        self.sigma_roll    = None
        self.sigma99_roll  = None
        self.eig_spiral    = None
        self.sigma_spiral  = None
        self.doubling_time = None
        self.eig_Dutch     = None
        self.sigma_Dutch   = None
        self.sigma99_Dutch = None
        self.omega_d_Dutch = None
        self.omega_n_Dutch = None
        self.zeta_Dutch    = None

        self.llambda_r              = None
        self.sigma_roll_appox       = None
        self.llambda_s              = None
        self.sigma_spiral_approx    = None
        self.R_Ds                   = None
        self.omega_d_Dutch_approx   = None
        self.sigma_Dutch_approx     = None
        self.llambda_DR             = None

        self.CW = None
        self.CAP = None

    def open(self):
        with open(self.filename, "r", encoding="utf-8") as f:
            data = json.load(f)

        self.vinf          = data["initial"]["airspeed[ft/s]"]
        self.Sw            = data["aircraft"]["wing_area[ft^2]"]
        self.bw            = data["aircraft"]["wing_span[ft]"]
        self.W             = data["aircraft"]["weight[lbf]"]
        self.Ixx           = data["aircraft"]["Ixx[slug-ft^2]"]
        self.Iyy           = data["aircraft"]["Iyy[slug-ft^2]"]
        self.Izz           = data["aircraft"]["Izz[slug-ft^2]"]
        self.Ixy           = data["aircraft"]["Ixy[slug-ft^2]"]
        self.Ixz           = data["aircraft"]["Ixz[slug-ft^2]"]
        self.Iyz           = data["aircraft"]["Iyz[slug-ft^2]"]

        try:
            self.rho       = data["analysis"]["density[slugs/ft^3]"]
        except:
            print("No density data in file, using the default value instead")
            self.rho           = 0.0020664

        self.alpha         = data["initial"]["state"]["alpha[deg]"]

        self.CL_0          = data["aerodynamics"]["CL"]["0"]
        self.CL_alpha      = data["aerodynamics"]["CL"]["alpha"]
        self.CL_qbar       = data["aerodynamics"]["CL"]["qbar"]
        self.CL_alpha_hat  = data["aerodynamics"]["CL"]["alpha_hat"]
        self.CL_1          = self.CL_0  + self.CL_alpha * self.alpha

        self.CD_L0         = data["aerodynamics"]["CD"]["L0"]
        self.CD_L          = data["aerodynamics"]["CD"]["L"]
        self.CD_L2         = data["aerodynamics"]["CD"]["L2"]
        self.CD_qbar       = data["aerodynamics"]["CD"]["qbar"]
        self.CD_Lqbar      = data["aerodynamics"]["CD"]["Lqbar"]
        self.CD_L2qbar     = data["aerodynamics"]["CD"]["L2qbar"]
        self.CD_qbar       = self.CD_L2qbar * self.CL_1**2 + self.CD_Lqbar * self.CL_1 + self.
    CD_qbar
```

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        self.Cm_0         = data["aerodynamics"]["Cm"]["0"]
        self.Cm_alpha     = data["aerodynamics"]["Cm"]["alpha"]
        self.Cm_qbar      = data["aerodynamics"]["Cm"]["qbar"]
        self.Cm_alpha_hat = data["aerodynamics"]["Cm"]["alpha_hat"]

        self.CD_0         = self.CD_L0
        self.CD_1         = self.CD_L
        self.CD_2         = self.CD_L2
        self.Cm_o         = self.Cm_0

        self.theta_o      = 0
        self.phi_o        = 0
        self.alpha_T0     = 0
        self.V_o          = (self.W / (0.5 * self.rho * self.Sw * self.CL_0))**0.5
        self.CL_o         = self.W * np.cos(self.theta_o) / (0.5 * self.rho * self.V_o**2 * self.Sw * np.cos(self.phi_o))
        self.CY_beta      = data["aerodynamics"]["CS"]["beta"]
        self.CY_pbar      = data["aerodynamics"]["CS"]["pbar"]
        self.CY_Lpbar     = data["aerodynamics"]["CS"]["Lpbar"]
        self.CY_rbar      = data["aerodynamics"]["CS"]["rbar"]
        self.CY_pbar      = self.CY_Lpbar * self.CL_1 + self.CY_pbar
        self.Cl_beta      = data["aerodynamics"]["Cl"]["beta"]
        self.Cl_pbar      = data["aerodynamics"]["Cl"]["pbar"]
        self.Cl_rbar      = data["aerodynamics"]["Cl"]["rbar"]
        self.Cl_Lrbar     = data["aerodynamics"]["Cl"]["Lrbar"]
        self.Cl_rbar      = self.Cl_Lrbar * self.CL_1 + self.Cl_rbar
        self.Cn_beta      = data["aerodynamics"]["Cn"]["beta"]
        self.Cn_pbar      = data["aerodynamics"]["Cn"]["pbar"]
        self.Cn_Lpbar     = data["aerodynamics"]["Cn"]["Lpbar"]
        self.Cn_rbar      = data["aerodynamics"]["Cn"]["rbar"]
        self.Cn_pbar      = self.Cn_Lpbar * self.CL_1 + self.Cn_pbar
        self.cw           = self.Sw / self.bw
        self.CD_o         = self.CD_0 + self.CD_1 * self.CL_0 + self.CD_2 * self.CL_0**2
        self.CD_alpha     = self.CD_1 * self.CL_alpha + 2 * self.CD_2 * self.CL_o * self.CL_alpha
        self.T_v          = 0
        self.Z_T0         = 0
        self.CD_alpha_hat = 0
        self.CD_nu_hat    = 0
        self.CL_mu_hat    = 0
        self.Cm_mu_hat    = 0
        self.R_gx         = g * self.cw / (2 * self.V_o**2)
        self.R_gy         = g * self.bw / (2 * self.V_o**2)
        self.R_rhox       = 4 * self.W / (g * self.rho * self.Sw * self.cw)
        self.R_rhoy       = 4 * self.W / (g * self.rho * self.Sw * self.bw)
        self.R_xx         = 8 * self.Ixx / (self.rho   * self.Sw * self.bw**3)
        self.R_yy         = 8 * self.Iyy / (self.rho   * self.Sw * self.cw**3)
        self.R_zz         = 8 * self.Izz / (self.rho   * self.Sw * self.bw**3)
        self.R_xz         = 8 * self.Ixz / (self.rho   * self.Sw * self.bw**3)
        self.CT_V         = self.T_v / 0.5 * self.rho * self.V_o * self.Sw

    def getMatrix(self):
        self.A_mat_lo = np.array([
            [- 2 * self.CD_o + self.CT_V * np.cos(self.alpha_T0),  self.CL_o - self.CD_alpha, -self.CD_qbar              ,  0,     0, -self.R_rhox * self.R_gx * np.cos(self.theta_o)],
            [- 2 * self.CL_o + self.CT_V * np.sin(self.alpha_T0), -self.CL_alpha - self.CD_o, -self.CL_qbar + self.R_rhox,  0,     0, -self.R_rhox * self.R_gx * np.sin(self.theta_o)],
            [  2 * self.Cm_o + self.CT_V * self.Z_T0 / self.cw  ,  self.Cm_alpha            , self.Cm_qbar               ,  0,     0, 0                                              ],
            [np.cos(self.theta_o)                               ,  np.sin(self.theta_o)     , 0                    ,  0,     0, -np.sin(self.theta_o)                   ],
            [- np.sin(self.theta_o)                             ,  np.cos(self.theta_o)     , 0                    ,  0,     0, -np.cos(self.theta_o)                   ],
            [0                                                  ,                         0 , 1                    ,  0,     0, 0                                       ],
        ])


```

```
        self.B_mat_lo = np.array([
            [self.R_rhox + self.CD_nu_hat, self.CD_alpha_hat              ,         0,   0, 0, 0],
            [self.CL_mu_hat              , self.R_rhox + self.CL_alpha_hat,         0,   0, 0, 0],
            [-self.Cm_mu_hat             , -self.Cm_alpha_hat             , self.R_yy,   0, 0, 0],
            [0                           ,                      0         ,         0,   1, 0, 0],
            [0                           ,                      0         ,         0,   0, 1, 0],
            [0                           ,                      0         ,         0,   0, 0, 1],

        ])


        self.A_mat_la = np.array([
            [self.CY_beta, self.CY_pbar, self.CY_rbar - self.R_rhoy     ,  0, self.R_rhoy * self.R_gy * np.cos(self.theta_o),      0,],
            [self.Cl_beta, self.Cl_pbar, self.Cl_rbar                   ,  0, 0 , 0                    , ],
            [self.Cn_beta, self.Cn_pbar, self.Cn_rbar                   ,  0, 0 , 0                    , ],
            [1           ,             0,      0                        ,  0, 0 , np.cos(self.theta_o), ],
            [0           ,             1,      np.tan(self.theta_o)     ,  0, 0 , 0                    , ],
            [0           ,             0,      1 / np.cos(self.theta_o)  ,  0, 0 , 0                    , ],
        ])


        self.B_mat_la = np.array([
            [self.R_rhoy,     0,          0,   0, 0, 0],
            [0     ,  self.R_xx, -self.R_xz,   0, 0, 0],
            [0     , -self.R_xz,  self.R_zz,   0, 0, 0],
            [0     ,          0,          0,   1, 0, 0],
            [0     ,          0,          0,   0, 1, 0],
            [0     ,          0,          0,   0, 0, 1],

        ])
        self.C_lo = np.matmul(np.linalg.inv(self.B_mat_lo), self.A_mat_lo)
        self.C_la = np.matmul(np.linalg.inv(self.B_mat_la), self.A_mat_la)

    def geteig(self):
        self.eigvals_lo, self.eigvecs_lo = eig(self.C_lo)
        self.eigvals_la, self.eigvecs_la = eig(self.C_la)

    def longitudinal(self):
        self.amps_lo            = (self.eigvecs_lo.real**2 + self.eigvecs_lo.imag**2)**0.5
        self.phase_lo           = np.arctan2(self.eigvecs_lo.imag, self.eigvecs_lo.real)
        self.eigvals_lo_mag     = np.abs(self.eigvals_lo)
        self.sort_index         =  np.argsort(np.abs(self.eigvals_lo))
        self.sigma_sp           = -self.eigvals_lo[self.sort_index[-1]].real * 2 * self.V_o / self.cw
        self.sigma_sp99         = np.log(0.01) /  (- self.sigma_sp)
        self.sigma_sp_1         = -self.eigvals_lo[self.sort_index[-2]].real * 2 * self.V_o / self.cw
        self.sigma_sp99_1       = np.log(0.01) /  (- self.sigma_sp_1)
        self.zeta_sp            = - (self.eigvals_lo[self.sort_index[-1]] + self.eigvals_lo[self.sort_index[-2]]) / (2 * (self.eigvals_lo[self.sort_index[-1]] * self.eigvals_lo[self.sort_index[-2]])**0.5)
        self.sigma_ph           = -self.eigvals_lo[self.sort_index[-3]].real * 2 * self.V_o / self.cw
        self.sigma_ph99         = np.log(0.01) /  (- self.sigma_ph)
        self.sigma_ph_1         = -self.eigvals_lo[self.sort_index[-4]].real * 2 * self.V_o / self.cw
```

```
        self.sigma_ph99_1      = np.log(0.01) /  (- self.sigma_ph)
        self.omega_d_ph        = np.abs(self.eigvals_lo[self.sort_index[-3]].imag) * 2 * self.V_o / self.cw
        self.period_ph         = 2 * np.pi / self.omega_d_ph
        self.zeta_ph           = - (self.eigvals_lo[self.sort_index[-3]] + self.eigvals_lo[self.sort_index[-4]]) / (2 * (self.eigvals_lo[self.sort_index[-3]] * self.eigvals_lo[self.sort_index[-4]])**0.5)
        self.A_sp              = self.R_yy * (self.R_rhox + self.CL_alpha_hat)
        self.B_sp              = self.R_yy * (self.CL_alpha + self.CD_o) - self.Cm_qbar * (self.R_rhox + self.CL_alpha_hat)  - self.Cm_alpha_hat * (self.R_rhox - self.CL_qbar)
        self.C_sp              = - self.Cm_qbar * (self.CL_alpha + self.CD_o) - self.Cm_alpha * (self.R_rhox - self.CL_qbar)
        self.sigma_sp_approx   = self.V_o * self.B_sp / (self.cw * self.A_sp)
        self.omega_d_sp_approx = (self.V_o / self.cw) * np.abs((self.B_sp**2 - 4 * self.A_sp * self.C_sp)**0.5 / (self.A_sp))
        self.omega_n_sp_approx = (self.eigvals_lo[self.sort_index[-1]] * self.eigvals_lo[self.sort_index[-2]])**0.5 * 2 * self.V_o / self.cw
        self.sigma_D           = g * self.CD_o / (self.V_o * self.CL_o)
        self.sigma_q           = (g / self.V_o) * ((self.CL_o - self.CD_alpha) * self.Cm_qbar / (self.R_rhox * self.Cm_alpha + (self.CD_o + self.CL_alpha) * self.Cm_qbar))
        self.R_ps              = self.R_rhox * self.Cm_alpha / (self.R_rhox * self.Cm_alpha + (self.CD_o + self.CL_alpha) * self.Cm_qbar)
        self.sigma_phi         = - (g / self.V_o) * self.R_gx * self.R_ps *((self.R_rhox * self.Cm_qbar - self.R_yy * (self.CD_o + self.CL_alpha)) / (self.R_rhox * self.Cm_alpha + (self.CD_o + self.CL_alpha) * self.Cm_qbar))
        self.sigma_ph_approx   = self.sigma_D + self.sigma_q + self.sigma_phi
        self.omega_d_ph_approx = (2 * (g / self.V_o)**2 * self.R_ps - (self.sigma_D + self.sigma_q)**2)**0.5
        self.llambda_sp        = np.empty((2), dtype=complex)
        self.llambda_sp[0]     = self.cw * complex(- self.sigma_sp_approx, self.omega_d_sp_approx)  / (2 * self.V_o)
        self.llambda_sp[1]     = self.cw * complex(- self.sigma_sp_approx, -self.omega_d_sp_approx) / (2 * self.V_o)
        self.llambda_p         = np.empty((2), dtype=complex)
        self.llambda_p[0]      = self.cw * complex(- self.sigma_ph_approx, self.omega_d_ph_approx)  / (2 * self.V_o)
        self.llambda_p[1]      = self.cw * complex(- self.sigma_ph_approx, -self.omega_d_ph_approx) / (2 * self.V_o)

    def lateral(self):
        # self.N = len(eigvals_la)
        self.amps_la  = (self.eigvecs_la.real**2 + self.eigvecs_la.imag**2)**0.5
        self.phase_la = np.arctan2(self.eigvecs_la.imag, self.eigvecs_la.real)

        # ====== find the location of each mode ================================
        self.dump_max      = np.max(np.abs(self.eigvals_la.real))
        self.non_zero_real = np.where(np.abs(self.eigvals_la.real) != 0)
        self.dump_min      = np.min(np.abs(self.eigvals_la[self.non_zero_real]))

        self.index_roll   = np.where(np.abs(self.eigvals_la.real) == self.dump_max)[0]
        self.index_spiral = np.where(np.abs(self.eigvals_la) == self.dump_min)[0]
        self.index_Dutch  = np.where(self.eigvals_la.imag != 0)[0]

        # ============= roll mode ==========================================
        self.eig_roll     = self.eigvals_la[self.index_roll]
        self.sigma_roll   = - self.eig_roll[0].real * 2 * self.V_o / self.bw
        self.sigma99_roll = np.log(0.01) / (- self.sigma_roll)

        # ============= spiral mode ========================================
        self.eig_spiral    = self.eigvals_la[self.index_spiral]
        self.sigma_spiral  = - self.eig_spiral[0].real * 2 * self.V_o / self.bw
        self.doubling_time = - np.log(2) / self.sigma_spiral

        # ============= Dutch mode =========================================
        self.eig_Dutch     = self.eigvals_la[self.index_Dutch]
        self.sigma_Dutch   = - self.eig_Dutch[0].real * 2 * self.V_o / self.bw
        self.sigma99_Dutch = np.log(0.01) / (- self.sigma_Dutch)
        self.omega_d_Dutch = np.abs(self.eig_Dutch[0].imag) * 2 * self.V_o / self.bw
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        self.omega_n_Dutch = (self.eig_Dutch[0] * self.eig_Dutch[1]) *  2 * self.V_o / self.bw
        self.zeta_Dutch    = - (self.eig_Dutch[0] + self.eig_Dutch[1]) / (2 * (self.eig_Dutch[0] * self.eig_Dutch[1])**0.5)

        # =============== lateral approximation =================================
        self.llambda_r              = self.Cl_pbar / self.R_xx
        self.sigma_roll_appox       = - self.rho * self.Sw * self.bw**2 * self.V_o * self.Cl_pbar / (4 * self.Ixx)
        self.llambda_s              = - (g * self.bw / (2 * self.V_o**2)) * ((self.Cl_beta * self.Cn_rbar - self.Cl_rbar * self.Cn_beta) / (self.Cl_beta * self.Cn_pbar - self.Cl_pbar * self.Cn_beta))
        self.sigma_spiral_approx    =   (g / self.V_o) * ((self.Cl_beta * self.Cn_rbar - self.Cl_rbar * self.Cn_beta) / (self.Cl_beta * self.Cn_pbar - self.Cl_pbar * self.Cn_beta))
        self.R_Ds                   = (self.Cl_beta * (self.R_gy * self.R_rhoy * self.R_zz - (self.R_rhoy - self.CY_rbar) * self.Cn_pbar) - self.CY_beta * self.Cl_rbar * self.Cn_pbar) / (self.R_rhoy * self.R_zz * self.Cl_pbar)
        self.omega_d_Dutch_approx   = (2 * self.V_o / self.bw) * ((1 - (self.CY_rbar / self.R_rhoy)) * (self.Cn_beta / self.R_zz)
                                     + ((self.CY_beta * self.Cn_rbar) / (self.R_rhoy * self.R_zz))
                                     + self.R_Ds
                                     - 0.25 * ((self.CY_beta / self.R_rhoy) + (self.Cn_rbar / self.R_zz))**2)**0.5
        self.sigma_Dutch_approx     = - (self.V_o / self.bw) * (
                                     + self.CY_beta / self.R_rhoy
                                     + self.Cn_rbar / self.R_zz
                                     - (self.Cl_rbar * self.Cn_pbar) / (self.Cl_pbar * self.R_zz)
                                     + (self.R_gy * (self.Cl_rbar * self.Cn_beta - self.Cl_beta * self.Cn_rbar)) / (self.Cl_pbar * (self.Cn_beta + self.CY_beta * self.Cn_rbar / self.R_rhoy))
                                     - self.R_xx * self.R_Ds / self.Cl_pbar
                                 )
        self.llambda_DR             = np.empty((2), dtype=complex)
        self.llambda_DR[0]          = self.cw * complex(- self.sigma_Dutch_approx,  self.omega_d_Dutch_approx) / (2 * self.V_o)
        self.llambda_DR[1]          = self.cw * complex(- self.sigma_Dutch_approx, -self.omega_d_Dutch_approx) / (2 * self.V_o)

    def printtotxt(self):
        # ============== print to file ========================================
        filename = "longitudinal.txt"
        with open(filename, "w", encoding="utf-8") as f:
            f.write(" ")

        printMatToFile(self.A_mat_lo, filename, "A Matrix")
        printMatToFile(self.B_mat_lo, filename, "B Matrix")
        printMatToFile(self.C_lo, filename, "C Matrix")


        with open(filename, "a", encoding="utf-8") as f: # print longitydinal result
            for i in range(6):
                print("============================================================", file=f)
                print("eigenvalue = ", "{:.12f}".format(self.eigvals_lo[i]), file=f)
                print("                 real", "             image", "             phase", " Amp", file=f)
                for j in range(6):
                    print(symbol_lo[j],
                        "{:>17.12f}".format(self.eigvecs_lo[j, i].real),
                        "{:>17.12f}".format(self.eigvecs_lo[j, i].imag),
                        "{:>17.12f}".format(np.rad2deg(self.phase_lo[j, i])),
                        "{:>17.12f}".format(self.amps_lo[j, i]), file=f)

                dampingRate   = -self.eigvals_lo[i].real
                dampingRate99 = np.log(0.01) / -dampingRate
                self.doubling_time = -np.log(2) / dampingRate
                freq          = abs(-self.eigvals_lo[i].imag)
                period        = 2 * np.pi / freq

                print("damping rate             =", "{:.12f}".format(dampingRate), file=f)
                if dampingRate > 0:
```

```
                print("99% damping rate         =", "{:.12f}".format(dampingRate99), file=f)
            else:
                print("Doubling time            =", "{:.12f}".format(self.doubling_time), file=f)
            print("damped natural frequency =", "{:.12f}".format(freq), file=f)
            print("damped natural period    =", "{:.12f}".format(period), file=f)

        roots = np.unique(abs(self.eigvals_lo.imag))
        pairs = [i for i in roots if i != 0]
        print("\n==================== ", "complex Pairs", " ========================", end="\n", file=f)
        for i in range(len(pairs)):
            index_pair = np.where(abs(self.eigvals_lo.imag) == pairs[i])[0]
            print("complex pairs:", "{:.12f}".format(self.eigvals_lo[index_pair[0]]), file=f)
            print("              ", "{:.12f}".format(self.eigvals_lo[index_pair[1]]), file=f)

            omega_n = (self.eigvals_lo[index_pair[0]] * self.eigvals_lo[index_pair[1]])**0.5
            zeta = - (self.eigvals_lo[index_pair[0]] + self.eigvals_lo[index_pair[1]]) / \
                (2 * (self.eigvals_lo[index_pair[0]] * self.eigvals_lo[index_pair[1]])**0.5)
            print("damping ratio             =", "{:.12f}".format(zeta.real), file=f)
            print("undamped natural frequecy =", "{:.12f}".format(omega_n.real), file=f)


        print("=============================================================\n\n\n", file=f)

        print("#################### short period ##########################", file=f)
        print("eigenvalues       =", "{:.6f}".format(-self.eigvals_lo[self.sort_index[-1]]), file=f)
        print("                  =", "{:.6f}".format(-self.eigvals_lo[self.sort_index[-2]]), file=f)
        print("========================================", file=f)
        print("damping rate      =",      "{:.6f}".format(self.sigma_sp), file=f)
        print("99\% damping rate =", "{:.6f}".format(self.sigma_sp99), file=f)
        print("======== or ========", file=f)
        print("damping rate      =",      "{:.6f}".format(self.sigma_sp_1), file=f)
        print("99% damping rate =", "{:.6f}".format(self.sigma_sp99_1), file=f)
        print("====================", file=f)
        print("damped frequency  = none" , file=f)
        print("period            = none", file=f)
        print("====================\n", file=f)

        print("#################### phugoid period ########################", file=f)
        print("eigenvalues       =", "{:.6f}".format(-self.eigvals_lo[self.sort_index[-3]]), file=f)
        print("                  =", "{:.6f}".format(-self.eigvals_lo[self.sort_index[-4]]), file=f)
        print("damping rate      =", "{:.6f}".format(self.sigma_ph), file=f)
        print("99% damping rate  =", "{:.6f}".format(self.sigma_ph99), file=f)
        print("======== or ========", file=f)
        print("damping rate      =", "{:.6f}".format(self.sigma_ph_1), file=f)
        print("99% damping rate  =", "{:.6f}".format(self.sigma_ph99_1), file=f)
        print("====================", file=f)
        print("damped frequency  =", "{:.6f}".format(self.omega_d_ph), file=f)
        print("period            =", "{:.6f}".format(self.period_ph), file=f)
        print("====================\n", file=f)

        print("#################### short period approximation ############", file=f)
        print("eigenvalues       =", "{:.6f}".format(self.llambda_sp[0]), file=f)
        print("                  =", "{:.6f}".format(self.llambda_sp[1]), file=f)
        print("damping rate      =", "{:.6f}".format(self.sigma_sp_approx),   file=f)
        print("99% damping rate  =", "{:.6f}".format(np.log(0.01) / (- self.sigma_sp_approx)), file=f)
        print("damped frequency  =", "{:.6f}".format(self.omega_d_sp_approx), file=f)
        print("period            =", "{:.6f}".format(2 * np.pi / self.omega_d_sp_approx), file=f)
        print("============== error ==================", file=f)
        print("damping rate      =", "{:.6f}".format(error(self.sigma_sp, self.sigma_sp_approx)),   file=f)
```

```python
            print("99% damping rate  =", "{:.6f}".format(error(self.sigma_sp99, np.log(0.01) / (- self.sigma_sp_approx))), file=f)
            print("========================================\n", file=f)

            print("#################### phugoid period approximation ##########", file=f)
            print("eigenvalues       =", "{:.6f}".format(-self.eigvals_lo[self.sort_index[-3]]), file=f)
            print("                  =", "{:.6f}".format(-self.eigvals_lo[self.sort_index[-4]]), file=f)
            print("damping rate      =", "{:.6f}".format(self.sigma_ph_approx),   file=f)
            print("99% damping rate  =", "{:.6f}".format(np.log(0.01) / (- self.sigma_ph_approx)), file=f)
            print("damped frequency  =", "{:.6f}".format(self.omega_d_ph_approx), file=f)
            print("period            =", "{:.6f}".format(2 * np.pi / self.omega_d_ph_approx), file=f)
            print("============== error ===================", file=f)
            print("damping rate      =", "{:.6f}".format(error(self.sigma_ph ,self.sigma_ph_approx)),   file=f)
            print("99% damping rate  =", "{:.6f}".format(error(self.sigma_ph99, np.log(0.01) / (- self.sigma_ph_approx))), file=f)
            print("damped frequency  =", "{:.6f}".format(error(self.omega_d_ph, self.omega_d_ph_approx)), file=f)
            print("period            =", "{:.6f}".format(error(2 * np.pi / self.omega_d_ph, 2 * np.pi / self.omega_d_ph_approx)), file=f)
            print("========================================\n", file=f)

        # ========= write into a file ==========================================
        filename = "Lateral.txt"
        with open(filename, "w", encoding="utf-8") as f:
            f.write(" ")

        printMatToFile(self.A_mat_la, filename, "A Matrix")
        printMatToFile(self.B_mat_la, filename, "B Matrix")
        printMatToFile(self.C_la,     filename, "C Matrix")


        with open(filename, "a", encoding="utf-8") as f: # print lateral result
            for i in range(6):
                print("=============================================================", file=f)
                print("eigenvalue = ", "{:.12f}".format(self.eigvals_la[i]), file=f)
                print("                 real", "            image", "            phase", "            Amp", file=f)
                for j in range(6):
                    print(symbol_lo[j],
                            "{:>17.12f}".format(self.eigvecs_la[j, i].real),
                            "{:>17.12f}".format(self.eigvecs_la[j, i].imag),
                            "{:>17.12f}".format(np.rad2deg(self.phase_lo[j, i])),
                            "{:>17.12f}".format(self.amps_lo[j, i]), file=f)

                dampingRate   = -self.eigvals_la[i].real
                dampingRate99 = np.log(0.01) / -dampingRate
                self.doubling_time = -np.log(2) / dampingRate
                freq          = abs(-self.eigvals_la[i].imag)
                period        = 2 * np.pi / freq

                print("damping rate             =", "{:.12f}".format(dampingRate), file=f)
                if dampingRate > 0:
                    print("99% damping rate         =", "{:.12f}".format(dampingRate99), file=f)
                else:
                    print("Doubling time            =", "{:.12f}".format(self.doubling_time), file=f)
                print("damped natural frequency =", "{:.12f}".format(freq), file=f)
                print("damped natural period    =", "{:.12f}".format(period), file=f)

            roots = np.unique(abs(self.eigvals_la.imag))
            pairs = [i for i in roots if i != 0]
            print("\n==================== ", "complex Pairs", " =========================", end="\n", file=f)
```

```python
        for i in range(len(pairs)):
            index_pair = np.where(abs(self.eigvals_la.imag) == pairs[i])[0]
            print("complex pairs:", "{:.12f}".format(self.eigvals_la[index_pair[0]]), file=f)
            print("              ", "{:.12f}".format(self.eigvals_la[index_pair[1]]), file=f)

            omega_n = (self.eigvals_la[index_pair[0]] * self.eigvals_la[index_pair[1]])**0.5
            zeta = - (self.eigvals_la[index_pair[0]] + self.eigvals_la[index_pair[1]]) / \
                (2 * (self.eigvals_la[index_pair[0]] * self.eigvals_la[index_pair[1]])**0.5)
            print("damping ratio             =", "{:.12f}".format(zeta.real), file=f)
            print("undamped natural frequecy =", "{:.12f}".format(omega_n.real), file=f)

        print("==============================================================\n\n\n", file=f)

        print("\n================ roll mode =================", file=f)
        # print("eigenvalues = ",eig_roll , file=f)
        print("damping rate     = ", "{:.6f}".format(self.sigma_roll), file=f)
        print("99% damping rate = ", "{:.6f}".format(self.sigma99_roll), file=f)

        print("\n================ spiral mode ================", file=f)
        # print("eigenvalues = ",eig_spiral , file=f)
        print("damping rate  = ", "{:.6f}".format(self.sigma_spiral), file=f)
        print("doubling time = ", "{:.6f}".format(self.doubling_time), file=f)

        print("\n================ Dutch mode =================", file=f)
        # print("eigenvalues = ",eig_Dutch , file=f)
        print("damping rate              = ", "{:.6f}".format(self.sigma_Dutch), file=f)
        print("99% damping rate          = ", "{:.6f}".format(self.sigma99_Dutch), file=f)
        print("damped frequency          = ", "{:.6f}".format(self.omega_d_Dutch), file=f)
        print("Period                    = ", "{:.6f}".format(2 * np.pi / self.omega_d_Dutch), file=f)

        print("\n================ roll mode approximation =================", file=f)
        print("damping rate     = ", "{:.6f}".format(self.sigma_roll_appox), file=f)
        print("99% damping rate = ", "{:.6f}".format(np.log(0.01) / -self.sigma_roll_appox), file=f)
        print("============== error ==================", file=f)
        print("damping rate     = ", "{:.6f}".format(error(self.sigma_roll, self.sigma_roll_appox)), file=f)
        print("99% damping rate = ", "{:.6f}".format(error(self.sigma99_roll, np.log(0.01) / -self.sigma_roll_appox)), file=f)

        print("\n================ spiral mode approximation ================", file=f)
        print("damping rate  = ", "{:.6f}".format(self.sigma_spiral_approx), file=f)
        print("doubling time = ", "{:.6f}".format(np.log(2) / -self.sigma_spiral_approx), file=f)
        print("============== error ==================", file=f)
        print("damping rate  = ", "{:.6f}".format(error(self.sigma_spiral, self.sigma_spiral_approx)), file=f)
        print("doubling time = ", "{:.6f}".format(error(self.doubling_time, np.log(2) / -self.sigma_spiral_approx)), file=f)


        print("\n================ Dutch mode approximation =================", file=f)
        print("damping rate              = ", "{:.6f}".format(self.sigma_Dutch), file=f)
        print("99% damping rate          = ", "{:.6f}".format(np.log(0.01) / -self.sigma_Dutch), file=f)
        print("damped frequency          = ", "{:.6f}".format(self.omega_d_Dutch_approx), file=f)
        print("Period                    = ", "{:.6f}".format(2 * np.pi / self.omega_d_Dutch_approx), file=f)
        print("============== error ==================", file=f)
        print("damping rate              = ", "{:.6f}".format(error(self.sigma_Dutch, self.sigma_Dutch)), file=f)
        print("99% damping rate          = ", "{:.6f}".format(error(self.sigma99_Dutch, np.log(0.01) / -self.sigma_Dutch)), file=f)
        print("damped frequency          = ", "{:.6f}".format(error(self.omega_d_Dutch, self.omega_d_Dutch_approx)), file=f)
        print("Period                    = ", "{:.6f}".format(error(2 * np.pi / self.
```

```python
omega_d_Dutch ,2 * np.pi / self.omega_d_Dutch_approx)), file=f)

    def handle(self):
        self.CW = self.W / (0.5 * self.rho * self.V_o**2 * self.Sw)
        self.CAP = (self.omega_n_sp_approx)**2 * self.CW / self.CL_alpha

        print(New.omega_n_sp_approx)
        print("CAP = ", self.CAP)


        # short mode
        if  0.085 <= self.CAP <= 3.6 and 0.3 <= self.zeta_sp  <= 2:
            print("Level 1")
        elif 0.038 <= self.CAP <= 10  and 0.2 <= self.zeta_sp <= 2:
            print("Level 2")
        elif 0.15 <= self.zeta_sp:
            print("Level 3")
        else:
            print("Level 4")

        # Phugoid mode
        if self.zeta_ph > 0.04:
            print("Level 1")
        elif self.zeta_ph > 0:
            print("Level 2")
        else:
            print("Level 3")

        # Roll mode
        if 1 / self.sigma_roll < 1.4:
            print("Level 1")
        elif 1 / self.sigma_roll < 3:
            print("Level 2")
        elif 1 / self.sigma_roll < 10:
            print("Level 3")
        else:
            print("Level 4")

        # spiral mode
        if self.sigma_spiral > 0:
            print("Level 1")
        elif - np.log(2) / self.sigma_spiral > 20:
            print("Level 1")
        elif -np.log(2) / self.sigma_spiral > 12:
            print("Level 2")
        elif - np.log(2) / self.sigma_spiral > 4:
            print("Level 3")
        else:
            print("Level 4")

        # Dutch mode
        if self.zeta_Dutch > 0.08 and self.zeta_Dutch * self.omega_n_Dutch > 0.15 and self.omega_n_Dutch > 0.4:
            print("Level 1")
        elif self.zeta_Dutch > 0.02 and self.zeta_Dutch * self.omega_n_Dutch > 0.05 and self.omega_n_Dutch > 0.4:
            print("Level 2")
        elif self.zeta_Dutch  > 0 and self.omega_n_Dutch > 0.4:
            print("Level 3")
        else:
            print("Level 4")

    def run(self):
        self.open()
        self.getMatrix()
        self.geteig()
        self.longitudinal()
        self.lateral()
```

```python
        self.printtotxt()
        self.handle()


if __name__ == "__main__":
    ### INPUT JSON FILE HEREEEEEEEEEE!!!!!!!! ######################
    New = glider("final.json")
    New.run()

    ### INPUT JSON FILE HEREEEEEEEEEE!!!!!!!! ######################
    base = glider("baseline.json")
    base.run()
# ===================== eigenvalue plot ================================

fig, ax = plt.subplots(figsize=(10, 8))

ax.scatter(New.eigvals_lo[New.sort_index[-2:]].real    , New.eigvals_lo[New.sort_index[-2:]].imag
        ,s=100, marker="o", edgecolors="r", facecolors='None', label="New short")
ax.scatter(New.eigvals_lo[New.sort_index[-4:-2]].real , New.eigvals_lo[New.sort_index[-4:-2]].
    imag   ,s=100, marker="v", edgecolors="g", facecolors='None', label="New Phugoid")
ax.scatter(New.eig_roll.real                           , New.eig_roll.imag
        ,s=100, marker="s", edgecolors="b", facecolors='None', label="New Roll")
ax.scatter(New.eig_spiral.real                         , New.eig_spiral.imag
        ,s=100, marker="D", edgecolors="c", facecolors='None', label="New Spiral")
ax.scatter(New.eig_Dutch.real                          , New.eig_Dutch.imag
        ,s=100, marker="*", edgecolors="m", facecolors='None', label="New Dutch")
ax.scatter(base.eigvals_lo[New.sort_index[-2:]].real    , base.eigvals_lo[New.sort_index[-2:]].
    imag   ,s=100, marker="o", c="r", label="baseline short")
ax.scatter(base.eigvals_lo[New.sort_index[-4:-2]].real , base.eigvals_lo[New.sort_index[-4:-2]].
    imag ,s=100, marker="v", c="g", label="baseline Phugoid")
ax.scatter(base.eig_roll.real                           , base.eig_roll.imag
        ,s=100, marker="s", c="b", label="baseline Roll")
ax.scatter(base.eig_spiral.real                         , base.eig_spiral.imag
        ,s=100, marker="D", c="c", label="baseline Spiral")
ax.scatter(base.eig_Dutch.real                          , base.eig_Dutch.imag
        ,s=100, marker="*", c="m", label="baseline Dutch")
# ax.hlines(0, 1, -10, color='black')
ax.vlines(0, 1, -1, color='black', ls=":")
ax.hlines(0, 0, -30, colors="black", ls=":")
ax.set_ylabel("image")
ax.set_xlabel("real")
plt.grid()
plt.legend()
plt.savefig('complex_plot.pdf', bbox_inches="tight")
plt.show()

print("\n")
print("####################################################################")
print("#              Longitudinal                                        #")
print("####################################################################\n")
with open("longitudinal.txt", "r", encoding="utf-8") as f:
    content = f.read()
    print(content)

print("\n")
print("####################################################################")
print("#              Lateral                                             #")
print("####################################################################\n")
with open("lateral.txt", "r", encoding="utf-8") as f:
    content = f.read()
    print(content)
```

Listing 2   dynamic stability analysis program python code