MAE6490 Turbulent Boundary Layer Analysis Assignment

Answers to be submitted as a single pdf, letter size pages, 11pt text. Explain how you got your answers using text, equations and/or Matlab code. Figures need to be appropriately labeled including labels on the x- and y-axes, labels on any colorbars, and legends in figures with multiple lines. All labels and other answers need units. Figures must be numbered and have appropriate captions and legends.

See appendices for data set description, notation used throughout the assignment, and information on specific topics (for questions 4–6). Question 6 is a bonus question. It is a difficult question and I will not be helping with any code for the bonus question. The maximum points for the assignment (including bonus) is capped at 100.

1. **Velocity fields and outer units** (15 points)

   (a) Provide plots of: $\overline{u}(x,y)$, $u(x,y,t_0)$, and $u'(x,y,t_0)$ (use the same fixed t_0 for both; meaning that the latter two are instantaneous fields).

   (b) Find the free-stream velocity U_∞, boundary-layer thickness δ (for which $\overline{u}(\delta) = 0.99U_\infty$), displacement thickness δ^*, momentum thickness θ, and turbulence intensity (in %) in the free stream. Indicate how you determined all these.

   (c) Plot the wall-normal profiles (i.e. averaged in time and in x-direction) of mean velocity and fluctuating velocity, normalized by outer units (U_∞ and δ) i.e. $\overline{u}/U_\infty(y/\delta)$ and $\overline{u'^2}/U_\infty^2(y/\delta)$. Discuss both profiles. What is happening near the wall? What is happening near the edge of the boundary layer?

   (d) The boundary layer thickness is often approximated as

   $$\delta/x \approx 0.375 Re_x^{-0.2}, \tag{1}$$

   where $Re_x = xU_\infty/\nu$. Use this to approximate the downstream distance x at which our data is recorded.

   (e) Based on equation 1, do you expect the boundary layer thickness to vary over the streamwise extent of our data (calculate this)? In a single graph, show two boundary layer profiles $\overline{u}(y)$: averaged over the first 100 and last 100 points in streamwise direction respectively. Discuss the difference between them.

2. **Friction velocity** (25 points)
   In this question, all velocities are averaged in time and in streamwise direction to obtain a 230-point velocity profile.

   (a) The recorded data is not super close to the surface, but we might have one point in the viscous sub-layer. Try estimating the friction velocity u_τ by setting $u^+ = y^+$ based on the data point nearest to the wall. Discuss when this relationship is valid, give the upper limit for when this is still valid, and show whether this applies to this first data point.

   (b) Use u_τ from 2a and δ from 1b to calculate Re_τ.

(c) A more accurate way to determine u_τ is by fitting the log-law:

$$u^+ = \kappa^{-1}\ln(y^+) + A. \tag{2}$$

Which is generally valid for $30 < y^+ < 0.15Re_\tau$. We use $\kappa \approx 0.4$ and $A \approx 5$. Use your u_τ estimate from 2a to determine which of the points on the velocity profile fall in the above range. Then fit equation 2 to the data (on the specified range) to determine u_τ. Show a graph of the fit, and provide the fitted value for u_τ. Re-calculate Re_τ with the new (more accurate) value of u_τ.

(d) For a turbulent boundary layer the friction coefficient, defined as

$$c_f \equiv 2\tau_w/(\rho U_\infty^2), \tag{3}$$

where $\tau_w = \rho u_\tau^2$, can be approximated by

$$c_f/2 \approx 0.01 Re_x^{-0.133}. \tag{4}$$

You can combine equations 3 and 4 to describe the variation of u_τ with downstream distance. Using equations 3 and 4, and the approximation for x from question 1d, what is u_τ expected to be at our downstream location? Does this match your calculations from 2a and 2b? Discuss any difference.

From equations 3 and 4, how is u_τ expected to vary over our streamwise extent?

(e) Use equation 3 to calculate the friction coefficient. For a laminar boundary layer, $c_f \approx 0.664Re_x^{-1/2}$. What is the ratio between the friction in this turbulent boundary layer compared to the laminar case (assuming the same free-stream velocity and downstream location).

3. **Wall-normal profiles** (15 points)
In this question, all velocities are averaged in time and in streamwise direction (over all 1194 points) to obtain a 230-point velocity profile.

(a) Beyond the logarithmic region (typically for $y^+ > 0.15Re_\tau$), the log-law in equation 2 does not describe the velocity profile very well. In the outer layer, or 'wake' region, the velocity profile is described by

$$u^+ = \frac{1}{\kappa}\ln(y^+) + A + 2\frac{\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right), \tag{5}$$

which is an extension of equation 2 and generally valid for $30 < y^+ < Re_\tau$, which is up to the edge of the boundary layer. Use a fit of this equation in the stated range of y^+ to find the wake strength parameter Π (use your u_τ value found in 2c).

(b) Plot the velocity profile $u^+(y^+)$ with the y^+ axis in logarithmic scale. Add the approximations from equation 2 and equation 5 as well as the $u^+ = y^+$ approximation. Discuss whether they match your velocity profile. In your graph, indicate the viscous sublayer, the buffer layer, the log layer, wake region and freestream.

(c) Plot the difference between equation 2 and your data, i.e. $u^+ - \left(\kappa^{-1}\ln(y^+) + A\right)$ as function of y^+; discuss the difference (or lack thereof) in the range $30 < y^+ < 0.15Re_\tau$.

4. **Uniform momentum zones (UMZs)** (25 points)
A brief description on UMZs and their detection is provided in Appendix C

   (a) Show the following histograms, all with bin-sizes of 0.2 m/s:
   (i) Histogram of $u(x, y, t)$ over all data; (ii) histogram of u for only the data in frame 102; and (iii) histogram of u of frame 102 but only for a streamwise range of $0 \leq x/\delta \leq 0.25$. Discuss the difference between the three histograms; why are the peaks more/less pronounced in the different histograms?
   For the latter histogram, indicate the peaks. Use the locations halfway between the peaks to define the boundaries of uniform momentum zones. How many UMZs are indicated by this histogram?
   Plot $u(x, y, 102)$ for $x/\delta \leq 0.25$. Overlay this with contours at the levels of the boundaries. Discuss the result.

   (b) Write a script to automate peak detection in these histograms on a frame-by-frame basis, and in streamwise steps of $\Delta x/\delta = 0.25$. I suggest using matlab's `findpeaks` function with the `'MinPeakProminence'` option to eliminate noise peaks (requires the Signal Processing toolbox). Use your script to count the UMZs in each frame (the same as you did manually in 4a, but looping over all frames and all streamwise extents).
   Show a histogram of number of UMZs and calculate the average number of UMZs. Compare both to the data by de Silva et al. (2016) as in App. C (Figs. 2a **and** b).

5. **Quadrant analysis** (20 points)
A brief description of quadrant analysis is provided in Appendix D

   (a) Perform a quadrant analysis on your data, plotting the bivariate histogram of u' and v' (i.e. a 2D histogram of each combination of u' and v' rather than a regular 1D histogram). In matlab, you can use the function `hist3`. Plot the result using the `contour` function (equivalent to Fig. 3a in Appendix D, but no need to color by quadrant). Only use data that falls in your logarithmic region, $30 < y^+ < 0.15Re_\tau$.

   (b) Also plot the pre-multiplied version, $u'v'P(u', v')$ where $P(u', v')$ indicates your bivariate histogram (equivalent to Fig. 3b in Appendix D, but no need to color by quadrant). Use a colormap for the contour lines from which we can clearly distinguish positive from negative values. Show the colorbar.

   (c) Discuss the shape of the graphs from both 5a and 5b. Discuss what this means for the total Reynolds stresses and what this means for the mean momentum equation and the TKE equation.

6. **BONUS QUESTION: Energy spectra** (10 points)
In Hutchins and Marusic (2007), the authors describe large-scale structures in the logarithmic layer, in addition to small-scale structures closer to the wall. These two parameters (size and wall-normal location) can be captured together using pre-multiplied energy spectra, which show the energy content of the flow divided up into both wave-number and wall-normal location. See their Figure 10 which has these energy spectra in the four panels labeled (i). For the full bonus points in this question, you have to
(1) plot the pre-multiplied energy spectrum, equivalent to their figure 10(i) panels and
(2) discuss your pre-multiplied energy spectra and how it compares to their four versions (i.e. the four panels (i) in Figure 10). Explain any differences/similarities.

Appendices

A Dataset description

The dataset for this assignment is available for download on Box. This dataset comes from a open-return suction wind tunnel, which is a relatively standard facility. Data is recorded in a 2D plane in x (streamwise) and y (wall-normal) directions near the bottom floor of the tunnel. In this plane, only the u (streamwise) and v (wall-normal) components are measured. PIV (particle image velocimetry) is used to record snapshots of velocity at a low framerate of 1/3 Hz, leading to velocity snapshots that are uncorrelated in time. A total of 500 velocity fields is recorded.

The dataset contains the following variables:

- `X`, a 1×1194 vector, which are the coordinates in the x (streamwise) direction in meters;
- `Y`, a 230×1 vector, which are the coordinates in the y (wall-normal) direction in meters;
- `T`, a $1 \times 1 \times 500$ vector, which are the timestamps in seconds;
- `U`, a $230 \times 1194 \times 500$ matrix, which are the u (streamwise) components of velocity in meter/second;
- `V`, a $230 \times 1194 \times 500$ matrix, which are the v (wall-normal) components of velocity in meter/second.
- `nu`, the value of kinematic viscosity in m^2/s.

B Notation

Throughout the assignment, the following notation is used:

- x and u denote streamwise coordinates and velocities;
- y and v denote wall-normal coordinates and velocities;
- Superscript '+' indicates normalization with viscous scales (ν and u_τ), e.g. $u^+ = u/u_\tau$;
- $\overline{u}(x, y) = \overline{u(x, y, t)}$ denotes temporal average;
- $\langle u \rangle = \langle u(x, y) \rangle$ denotes spatial average;
- $u = \overline{u} + u'$ denotes a decomposition into a time-averaged part and a fluctuating part.

C Uniform momentum zones

Uniform momentum zones (UMZs) are regions of relatively constant momentum in the turbulent boundary layer. These zones only show up instantaneously and not in the time-averaged or streamwise-averaged velocity fields. These uniform momentum zones have a limited streamwise extent, so they are usually evaluated over regions of $\Delta x/\delta = 0.25$. A good explanation for the detection of uniform momentum zones is provided in de Silva et al. (2016).

In short, uniform momentum zones are observed in histograms of streamwise velocity (see Fig. 1 a & b) where distinct peaks correspond to regions with unified velocity. To define the boundaries of these zones, the mid-point between peaks is typically used (Fig. 1 c & d). The histograms in Fig. 1b, c & d indicate 2, 3, and 4 UMZs respectively. Note that in this paper they use the turbulent/non-turbulent interface (TNTI) as a boundary, which you do not have to do in this assignment. They also normalize histograms into pdfs, which is not required for this assignment.

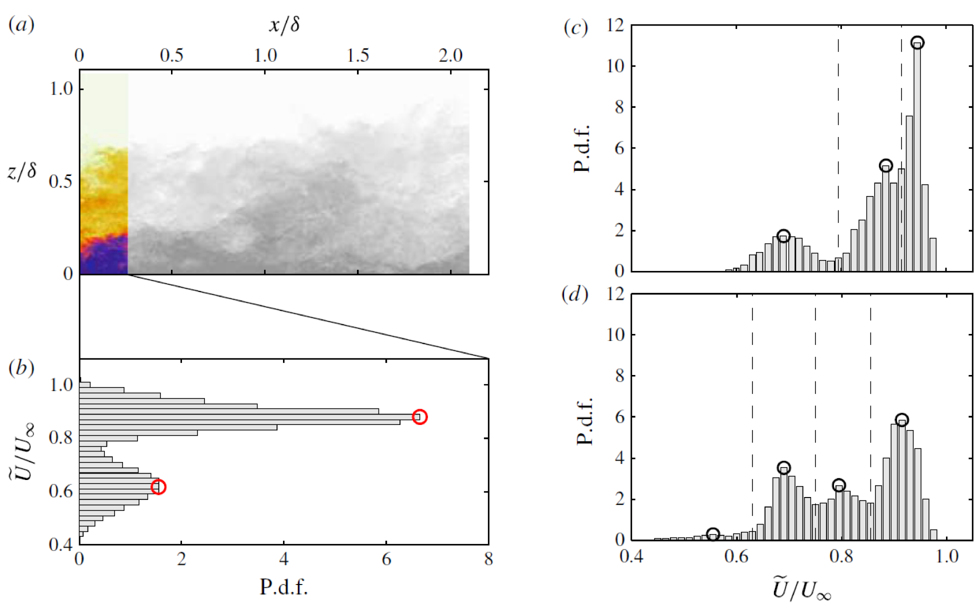

Figure 1: UMZs (de Silva et al., 2016). *Note that z in the figure is what we call y.*

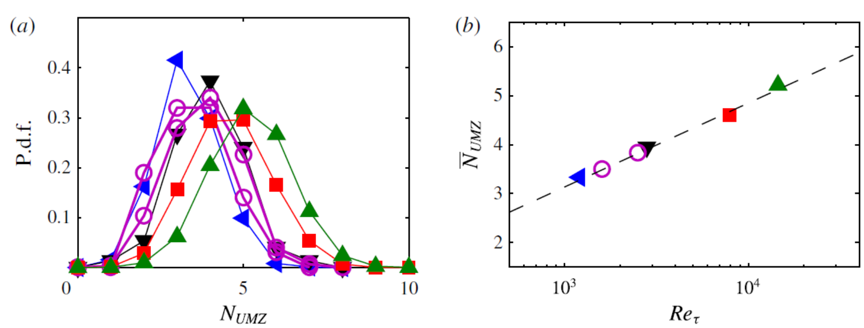

Figure 2: Distribution (a) and total number (b) of UMZs as function of Re_τ (de Silva et al., 2016)

D Quadrant Analysis

Quadrant analysis considers the vectors consisting of the fluctuating velocity components $u'(x, y, t)$ and $v'(x, y, t)$, and groups each velocity vector into one of four quadrants (see Fig. 3a):

Q1 if $u' > 0$ and $v' > 0$;
Q2 if $u' < 0$ but $v' > 0$;
Q3 if $u' < 0$ and $v' < 0$;
Q4 if $u' > 0$ but $v' < 0$.

In terms of contribution to the Reynolds stresses $-\overline{\langle u'v' \rangle}$, each vector in Q1 and Q3 (for which the product $u'v' > 0$) contributes negatively to the Reynolds stresses, whereas each vector in Q2 and Q4 (for which the product $u'v' < 0$) contributes positively to the Reynolds stresses.

For reference, quadrant analysis is further discussed in the Annual Review of Fluids paper Wallace (2016). The below Fig. 3a shows a joint probability distribution function (jpdf) for u' and v'. A jpdf is essentially a normalized histogram such that the integral over the entire jpdf equals one. For this assignment, you can use a bivariate histogram without normalization into a jpdf. Fig. 3b shows the same jpdf, but now multiplied at each location by $u'v'$. This "pre-multiplied" version is a measure for how much each point contributes to the total Reynolds stresses. For example, most data points might have a very small u' and v', both close to zero. This causes the peak in Fig. 3a to be around zero. However, even though there are a lot of these vectors, all of them together don't contribute a lot to the Reynolds stress because their individual contribution is close to zero.

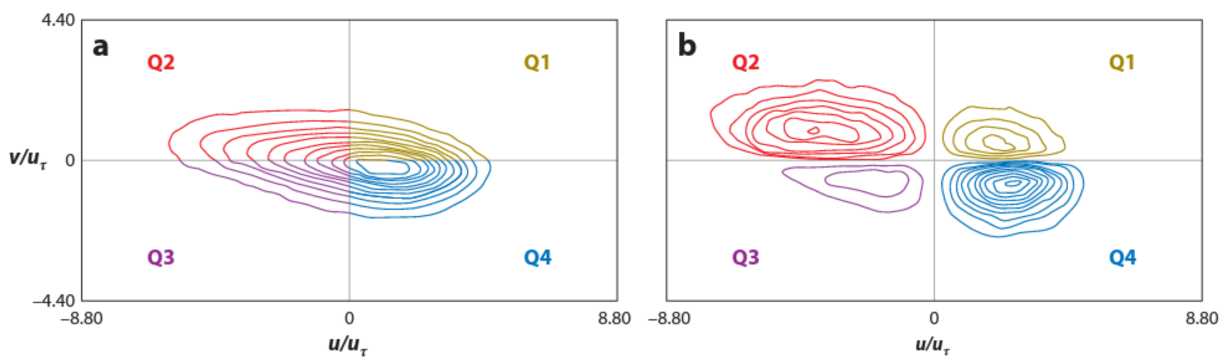

Figure 3: Example of quadrant analysis showing (9)a) the joint probability distribution function, $P(u', v')$ and (b) the pre-multiplied version, $u'v'P(u', v')$ (Wallace 2016). *Note that* **u** *and* **v** *in the figure is what we call* u' *and* v'.

E Useful Matlab commands (same as in previous assignment)

Note: while the dataset is provided in .mat format and Matlab commands are given below, you are free to use other software (e.g. Python).

E.1 Element-wise matrix operations

The '.' indicates applying the operation, such as multiplying or power, to each element in the matrix individually, as opposed to for example a matrix cross product:

```
UV = U .* V;
velocity_magnitude = sqrt( U.^2 + 2*V.^2 );
```

E.2 Averaging

The 3D matrix U is in directions U(y,x,t). To average in specific directions:

```
Uy = mean(U , 1); % average over all y values
Ux = mean(U , 2); % average over all x values
Ut = mean(U , 3); % average over all t values
Uxyt = mean( U(:) ); % transform U into a vector (U(:)), then
    average the vector to get a single value
```

Such that the time average $\overline{u}(x,y)$ is obtained by:

```
U_TA = mean( U, 3); % mean in the third dimension (time)

% can plot this using (see more plotting commands below)
figure
imagesc(X,Y,U_TA)
set(gca,'ydir','normal')
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb=colorbar;
ylabel(cb,'$\overline{u}$ (m/s)','Interpreter','latex')
```

Or a spatial average $\langle u\rangle(t)$:

```
U_xy = mean(U, [1 2]); % average in both x2 and x1 directions.
V_xy = mean(V, [1 2]); % average in both x2 and x1 directions.

figure
plot(T(:),U_xy)
hold on
plot(T(:),V_xy)
xlabel('t (s)')
ylabel('<U_i>(t) (m/s)')
legend('<U>','<V>')
grid on
```

E.3 Plotting

Instantaneous snapshot of scalar:

```
figure
image_number = 100;
imagesc(X,Y, U(:,:,image_number) )
set(gca,'ydir','normal') % sets 0 on the bottom left
xlabel('x (m)') % always use labels (with units!) on axes and
   colorbars
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb,'U (m/s)')
```

Instantaneous snapshot of vector field:

```
figure
image_number = 100;
quiver(X,Y, U(:,:,image_number), V(:,:,image_number), 2 )
xlabel('x (m)')
ylabel('y (m)')
axis tight
```

Combined background scalar and vector field:

```
vorticity_z = NaN(size(U));
vorticity_z(2:end-1,2:end-1,:) = (U(3:end,2:end-1,:) - U(1:end
   -2,2:end-1,:))/(Y(3)-Y(1)) - (V(2:end-1,3:end,:)-V(2:end-1,1:
   end-2,:))/(X(3)-X(1));

figure
image_number = 100;
imagesc(X,Y, vorticity_z(:,:,image_number))
set(gca,'ydir','normal') % sets 0 on the bottom left
xlabel('x (m)')
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb,'\omega_z (1/s)')
hold on % 'hold on' lets you add multiple plots on top of each
   other
quiver(X,Y, U(:,:,image_number), V(:,:,image_number), 2 ,'k')
```

Multi-panel figures:

```
figure
tiledlayout(1,3)
nexttile(1)
imagesc(X,Y, U(:,:,image_number) )
```

```
set(gca,'ydir','normal') % sets 0 on the bottom left
xlabel('x (m)') % always use labels (with units!) on axes and colorbars
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb,'U (m/s)')

nexttile(2)
imagesc(X,Y, V(:,:,image_number) )
set(gca,'ydir','normal') % sets 0 on the bottom left
xlabel('x (m)') % always use labels (with units!) on axes and colorbars
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb,'V (m/s)')

nexttile(3)
imagesc(X,Y, sqrt(U(:,:,image_number).^2 + V(:,:,image_number).^2) )
set(gca,'ydir','normal') % sets 0 on the bottom left
xlabel('x (m)') % always use labels (with units!) on axes and colorbars
ylabel('y (m)')
colormap(jet)
cb = colorbar;
ylabel(cb,'(U.^2 + V.^2)^{0.5} (m/s)')
```