$$ma_{cm} = f - mg\sin\theta$$
$$= m(R-r)\ddot\theta$$

$$\tau = \frac{2}{5}mr^2\ddot\phi$$
$$= rf$$

$$R\theta = -r(\phi+\theta') \qquad \theta' = -\theta$$
$$r\phi = -(R-r)\theta$$
$$r\ddot\phi = -(R-r)\ddot\theta$$
$$\ddot\phi = -\frac{(R-r)}{r}\ddot\theta$$
$$\frac{2}{5}m\cancel{r^2}\frac{-(R-r)}{\cancel{r}}\ddot\theta = \cancel{r}f$$
$$f = -\frac{2}{5}m(R-r)\ddot\theta$$

$$\cancel{m}(R-r)\ddot\theta = -\frac{2}{5}\cancel{m}(R-r)\ddot\theta - \cancel{m}g\sin\theta$$
$$\frac{7}{5}(R-r)\ddot\theta + g\sin\theta = 0$$
$$\ddot\theta + \frac{5g}{7(R-r)}\sin\theta = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) - \frac{\partial L}{\partial\theta} = 0$$

$$L = T - V$$
$$T = \frac12 mv^2 + \frac12 I_0\dot\phi^2$$
$$v = (R-r)\dot\theta$$
$$\dot\phi = \frac{-(R-r)}{r}\dot\theta$$
$$T = \frac12 m(R-r)^2\dot\theta^2 + \frac12\frac25 mr^2\frac{(R-r)^2}{r^2}\dot\theta^2$$
$$= \frac{7}{10}m(R-r)^2\dot\theta^2$$
$$V = mg\left[(R-r) - (R-r)\cos\theta\right]$$
$$= mg(R-r)(1-\cos\theta)$$
$$L = m(R-r)\left[\frac{7}{10}(R-r)\dot\theta^2 - g(1-\cos\theta)\right]$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) = m(R-r)^2\frac{7}{5}\ddot\theta$$
$$\frac{\partial L}{\partial\theta} = -m(R-r)g\sin\theta$$
$$\frac75\cancel{m}(R-r)^{\cancel{2}}\ddot\theta + \cancel{m(R-r)}g\sin\theta = 0$$
$$\ddot\theta + \frac{5g}{7(R-r)}\sin\theta = 0$$

$$I = I_0 + mr^2$$
$$= \frac25 mr^2 + m(R-r)^2$$

$$\tau' = I\ddot\theta$$
$$= Rf - (R-r)mg\sin\theta$$

$$\tau = \frac25 mr^2\ddot\phi$$
$$= rf - mr^2\ddot\theta$$
$$\ddot\phi = \frac{-(R-r)}{r}\ddot\theta$$
$$-\frac25 m\cancel{r^2}\frac{(R-r)}{\cancel{r}}\ddot\theta = \cancel{r}f - mr^{\cancel{2}}\ddot\theta$$
$$f = -\frac25 m(R-2r)\ddot\theta$$

$$\left[\frac25 mr^2 + m(R-r)^2\right]\ddot\theta = -\frac25 mR(R-2r)\ddot\theta - (R-r)mg\sin\theta$$
$$\left[\frac25 r^2 + (R-r)^2 + \frac25R^2 - \frac45Rr\right]\ddot\theta + (R-r)g\sin\theta = 0$$
$$\left[\frac25 r^2 + R^2 + r^2 - 2Rr + \frac25R^2 - \frac45Rr\right]\ddot\theta + (R-r)g\sin\theta = 0$$
$$\left[\frac75 r^2 + \frac75R^2 - \frac{14}{5}Rr\right]\ddot\theta + (R-r)g\sin\theta = 0$$
$$\frac75(R-r)^{\cancel{2}}\ddot\theta + \cancel{(R-r)}g\sin\theta = 0$$
$$\ddot\theta + \frac{5g}{7(R-r)}\sin\theta = 0$$