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CMPT 317 Assignment 5 Question 2

Question 2.

(a) What are the Conditional Probability Distributions implied by the network diagram? List them using the notation P (…). You do not need to indicate any probabilities. Just provide the notation.

**Solution:** From the graph, we know that $X_2, X_3, X_4, X_5$ are both causes for $X_1$, and $X_1$ is dependent on $X_2, X_3, X_4, X_5$, so the notations are:

- P ($X_2$)
- P ($X_3$)
- P ($X_4$)
- P ($X_5$)
- P ($X_1 \mid X_2$)
- P ($X_1 \mid X_3$)
- P ($X_1 \mid X_4$)
- P ($X_1 \mid X_5$)

(b) Assume that each variable Xi has 10 domain values. How many entries in each Conditional Probability table that you listed? In other word, how many numbers would be required if you were to fill in each table (which you thankfully don't have to do). What's the total number of entries, when you add up all the entries for all the CPDs?

**Solution:** Since $X_2, X_3, X_4, X_5$ are both causes to $X_1$, and each Xi has 10 domain values. Then the CPDs for each domain values can be calculated as:

- P ($X_2$): 10
- P ($X_3$): 10
- P ($X_4$): 10
- P ($X_5$): 10
- P ($X_2 \mid X_1$): 10 * 10 = 100
- P ($X_3 \mid X_1$): 10 * 10 = 100
- P ($X_4 \mid X_1$): 10 * 10 = 100
- P ($X_5 \mid X_1$): 10 * 10 = 100

The entire is 10 + 10 + 10 + 10 + 100 + 100 + 100 + 100 = 440

(c) Express the Joint Probability Distribution in terms of the Conditional Probability Distributions you outlined above.

**Solution:** JPD is calculate by multiply each CPD:

JPD: $P(X_2) \bullet P(X_3) \bullet P(X_4) \bullet P(X_5) \bullet P(X_1 \mid X_2) \bullet P(X_1 \mid X_3) \bullet P(X_1 \mid X_4) \bullet P(X_1 \mid X_5)$

(d) Derive a formula for the query $P(X_1|X_2, X_3, X_4)$.

**Solution:** Since $X_1$ is independent on $X_2$, $X_3$, $X_4$, then we can see that:

$P(X_1|X_2, X_3, X_4) = P(X_1|X_2) \bullet P(X_1|X_3) \bullet P(X_1|X_4)$