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Lecture Section: CMPT 260

1. Given a proof by cases to prove that for all integers x, $x^2 - x$ is an even integer.

   By case 1: x is even, $x = 2k$

   Then, $x^2 - x = 2k^2 - 2k = 2k (2k – 1) = 2 (k (2k - 1))$

   And the result holds.

   By case 2: when x is odd, $x = 2k + 1$

   Then $x^2 - x = (2k – 1)^2 - (2k + 1) = 4k^2 + 4k + 1 – 2k – 1 = 4k^2 - 2k = 2k (2k -1) = 2(k (2k -1))$.

   By these 2 cases, we can see that whatever x is even integer or odd, the result is even.

2. Give a proof by contraposition that for $a \in Z$, if $a^2 + 3$ is odd, then a is even.

   Let $P(x) = (a^2 + 3$ is odd, $a \in Z)$, $Q(x) = (a$ is even$)$;

   $P(x) \rightarrow Q(x)$.

   We know that $a^2 + 3$ is odd. Assume a is not even ($\sim Q(x)$), thus a is odd and there exists an integer k, such that $a = 2k + 1$.

   Then $a^2 + 3 = (2k + 1) (2k + 1) + 3 = 4k^2 + 4 k + 1 + 3 = 4k^2 + 4k + 4 = 4 (k^2 + k + 1)$, let $(k^2 + k + 1) = p$, then $a^2 + 3 =2p$, then and thus $a^2 + 3$ is even $\sim(P(x))$, but the premise that $a^2 + 3$ is odd.

   So, we get that $\sim Q(x) \rightarrow \sim P(x)$

   We can thus conclude that if $a^2 + 3$ is odd, then a is even.

3. Give a proof by contradiction for real numbers a and b that $(a+b)/2 < b \Rightarrow a < b$.

   Let $P(x) = (a+b)/2 < b$, $Q(x) = a<b$; $P(x) \rightarrow Q(x)$

   We know that $(a + b) /2 < b$. Assume to the contrary that $(a+b)/2 >b$ ($\sim P(x)$). Thus $a + b > 2b \rightarrow a > b$ ($\sim Q(x)$). But this contradict the premise that $(a + b) /2 < b$.

   We can then conclude that $(a+b)/2 <b \Rightarrow a<b$.

4. Give a proof by contradiction that given integers j and k where $j \geq 2$ that then $j \nmid k \vee j \nmid (k+1)$.

   We know that P: j and k are both integer and $j \geq 2$, Q: $j \nmid k \vee j \nmid (k+1)$.

   So, we assume to $\sim$P: j and k are both integer and $j<2$, $j =1$, $k =1$

   We are going to prove $\sim$Q: $j \mid k \vee j \mid (k+1)$ is true if $\sim$P

   From $j \mid k$:

   We get that $j * n = k$, and n must be a positive integer, assume $j = 1$, $k = 1$, so that $n = 1$ that is not an integer, this assumption is true.

   From $j \mid (k+1)$:

   We get that $j * m = k + 1$, and m must be a positive integer, assume $j = 1$, $k = 1$, so that $n = 1/2$ that is not an integer, this assumption is false.

   So, we get that $j \mid k \vee j \mid (k+1)$ is true by integers j and k where $j<2$, so we conclude that $j \nmid k \vee j \nmid (k+1)$ by integers j and k where $j \geq 2$.