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Lecture Section: CMPT 260

1. Consider the following expression

   $\forall xP(x) \land Q(x) \leftrightarrow (\exists xR(x) \rightarrow \forall x((S(x) \land Y(y)) \lor \exists y(U(y) \lor \sim T(x))))$

   (a) For each occurrence of each variable, indicate whether the variable is free or bound. If the variable is bound, indicate whether it is bound to a $\forall$ or to a $\exists$.

   P(x) bounded by $\forall$

   Q(x) is a free variable

   R(x) bounded by $\exists$

   S(x) bounded by $\forall$

   Y(y) is a free variable

   U(y) is bounded by $\exists$

   ~T(x) is bounded by $\forall$

   (b) Rename the variables so that distinct names are used for each distinct variable.

   $\forall xP(x) = P$

   $\forall xS(x) = S$

   $\forall xT(x) = T$

   $\exists xR(x) = R$

   $\exists xU(y) = U$

   So, we get that $\forall xP \wedge Q(x) \leftrightarrow (\exists xR \rightarrow (\forall xS \wedge Y(y) \lor (\exists yU \lor \sim\forall xT)$.

2. Show formally that $\sim \exists y(\forall x\exists zP(x, y, z) \lor \exists x\forall zQ(x, y, z))$ is logically equivalent to $\forall y(\exists x\forall z \sim P(x,y,z) \land \forall x\exists z \sim Q(x,y,z))$

   $\sim \exists y(\forall x\exists zP(x, y, z) \lor \exists x\forall zQ(x, y, z))$ Premise

   $\forall y \sim(\forall x\exists zP(x, y, z) \lor \exists x\forall zQ(x, y, z))$ Negation

   $\forall y(\sim\forall x\exists z P(x,y,z) \land \sim\exists x\forall z Q(x,y,z))$ D.M law

$\forall y(\exists x \forall z \sim P(x,y,z) \land \forall x \exists z \sim Q(x,y,z))$ Negation

3. Find an interpretation to show that the following argument form is not valid. $(\forall x(P(x) \rightarrow Q(x)) \land (\forall x(P(x) \rightarrow R(x)))) \rightarrow \forall x((Q(x) \rightarrow R(x)))$.

   From the variables that questions gave, the argument has a domain $\forall x$ that is unique individuals for each variable, each predicate of the expression defined, and all variables of the expression are bounded by $\forall x$.
   If it is valid, then the expression should be true for all interpretations. So we get a chart.

x	P(x)	Q(x)	R(x)	P(x)→Q(x)	P(x)→R(x)	(P(x)→Q(x)) ^(P(x)→R(x))	Q(x)→R(x)
a	T	T	T	T	T	T	T
d	T	T	F	F	F	F	T

The last column shows that $\forall x((Q(x) \rightarrow R(x)$ is true, but from the 5th column $\forall x(P(x) \rightarrow Q(x))$ is false, the 6th column $\forall x(P(x) \rightarrow R(x))$ is false, so the 7th column $\forall x\ ((P(x) \rightarrow Q(x)) \wedge (P(x) \rightarrow R(x)))$ is false. Thus, the $\forall x((Q(x) \rightarrow R(x)))$ expression is not valid.