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Lecture Section: CMPT 260

Assignment 2

1. Give a formal proof that $((p \rightarrow q) \rightarrow p) \rightarrow ((p \rightarrow q) \rightarrow q)$. Use the rules of equivalence from Thm 2.1.1 of the text. Or the rules of inference in table 2.3.1 of the text, or the deduction theorem. For each of the proof, give the reason for the stop and the numbers of any previous steps referred to.

   There are 4 conditions that define the questions:

   1. p=true, q=true;
   2. p=true, q=false;
   3. p=false, q=true;
   4. p=false, q=true;

   For 1st condition:

   p=true, q =true;

   $p \rightarrow q$=true;

   $(p \rightarrow q) \rightarrow p$=true;

   $(p \rightarrow q) \rightarrow q$=true;

   $((p \rightarrow q) \rightarrow p) \rightarrow ((p \rightarrow q) \rightarrow q)$=true.

   For 2nd condition:

   p=true, q=false;

   $p \rightarrow q$=false;

   $(p \rightarrow q) \rightarrow p$=true;

   $(p \rightarrow q) \rightarrow q$=true;

   $((p \rightarrow q) \rightarrow p) \rightarrow ((p \rightarrow q) \rightarrow q)$=true;

For 3rd condition

p=false, q=true;

$p \rightarrow q$=true;

$(p \rightarrow q) \rightarrow p$=false;

$(p \rightarrow q) \rightarrow q$=true;

$((p \rightarrow q) \rightarrow p) \rightarrow ((p \rightarrow q) \rightarrow q)$=true;

For 4th condition

p=false, q=false;

$p \rightarrow q$=true;

$(p \rightarrow q) \rightarrow p$=false;

$(p \rightarrow q) \rightarrow q$=false;

$((p \rightarrow q) \rightarrow p) \rightarrow ((p \rightarrow q) \rightarrow q)$=true;

To conclude, in all 4 conditions, it shows that the $((p \rightarrow q) \rightarrow p) \rightarrow ((p \rightarrow q) \rightarrow q)$ is always true.

2. A strange island is inhabited only by knights and knave. Knights always tell the truth, and knaves always lie.

   You meet 2 inhabitants: A and B. A claims, ‘both I am a knight and B is a knave’, and B says, ‘I tell you A is a knight’.

   Determine who is a knight and who is a knave, if possible.

   Let A is a knight = a.
   Let b is a knight = b.
   So, if A is a knave, we get ~a, and if B is a knave, we get ~b.
   A said: both I am a knight and B is a knave = a ^ ~b
   B said: I tell you A is a knight = a
   Assume that a is true, so we get that
   1. $a \leftrightarrow a \wedge \sim b$. Premise.
   2. $b \leftrightarrow a$ Premise.

3. a assumption for DT
4. a→ a ^ ~b 1.remove biconditional
5. b→a 2 remove biconditonal
6. a→b 2 remove biconditional
7. a ^ ~b 3, 4 Modus Ponens
8. ~b 7 Specialization
9. ~a 5, 8 Modeus Tollens
10. ~b 6, 9 Modeus Tollens

From line 9 and line 10, they show that both A and B are not knight, they are both knaves.