Computer Science 260
Assignment 3

Due Oct 5, 2016

1. Consider the following expression

$\forall x P(x) \wedge Q(x) \leftrightarrow (\exists x R(x) \rightarrow \forall x((S(x) \wedge Y(y)) \vee \exists y(U(y) \vee \sim T(x))))$

(a) For each occurrence of each variable, indicate whether the variable is free or bound. If the variable is bound, indicate whether it is bound to a $\forall$ or to a $\exists$.

(b) Rename the variables so that distinct names are used for each distinct variable.

2. Show formally that $\sim \exists y(\forall x \exists z P(x, y, z) \vee \exists x \forall z Q(x, y, z))$ is logically equivalent to $\forall y(\exists x \forall z \sim P(x, y, z) \wedge \forall x \exists z \sim Q(x, y, z))$.

State the reason for each step.

3. Find an interpretation to show that the following argument form is not valid.
$(\forall x(P(x) \rightarrow Q(x)) \wedge (\forall x(P(x) \rightarrow R(x)))) \rightarrow \forall x((Q(x) \rightarrow R(x)))$.