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Lecture: CMPT 260.

Assignment 6.

① $\sum_{i=0}^{n} i\cdot 2^i = (n-1)2^{n+1}+2$.

Base: Let $n=0$; $\sum_{i=0}^{0} i\cdot 2^i = 0\cdot 2^0 = 0$.

$(0-1)2^{0+1}+2=0$

Since these are equal, the formula holds for $n=0$

Inductive Hypothesis: Assume that $n=k$. and $k\in Z^+$

Inductive Step: need show $\sum_{i=0}^{k+1} i2^i = (k+1-1)2^{k+1+1}+2 = k\cdot 2^{k+2}+2$

$$\sum_{i=0}^{k+1} i2^i = \sum_{i=0}^{k} i2^i + (k+1)\cdot 2^{k+1} \text{ (summation)}$$

$$\sum_{i=0}^{k} i2^i = (k-1)\cdot 2^{k+1}+2$$

$$\sum_{i=0}^{k+1} i\cdot 2^i = \sum_{i=0}^{k} i\cdot 2^i + (k+1)\cdot 2^{k+1}$$

$$= (k-1)\cdot 2^{k+1}+2+k\cdot 2^{k+1}+2^{k+1}$$

$$= k\cdot 2^{k+1}-2^{k+1}+2+k\cdot 2^{k+1}+2^{k+1}$$

$$= 2\cdot k\cdot 2^{k+1}+2$$

$$= k\cdot 2^{k+2}+2$$

Thus by the equations above, we have $\sum_{i=0}^{n} i\cdot 2^i = (n-1)^{2n+1}+2$

12). f_k be the k^{th} fibonacci number (strong).

prove $n \geq 3$, $f_n \geq (\frac{3}{2})^{n-2}$.

Basis Step:

Assume $n=3$, $f(n)=2$.

$(\frac{3}{2})^{3-2} = \frac{3}{2}$; $f(n) \geq \frac{3}{2}$

Since it is true, so the formula holds for $n=3$

Assume $n=3+1=4$. $f(n)=3$.

$(\frac{3}{2})^{4-2} = \frac{9}{4}$ $f(n) \geq \frac{9}{4}$

Since it is true, so the formula holds for $n=4$.

Assume $n=3+2=5$ $f(n)=5$.

$(\frac{3}{2})^{5-2} = \frac{27}{8}$ $f(n) \geq \frac{27}{8}$

Since it is true, so the formula holds for $n=5$.

Inductive Hypothesis:

Assume $f(i)$ holds for all i from n to k where $k \geq n+3$

Inductive Step:

need to show $f(k+1)$ also hold.

$f(k+1) = f(k) + f(k-1)$

$f(k) \geq (\frac{3}{2})^{k-2}$.

$f(k-1) \geq (\frac{3}{2})^{(k-1)-2}$.

so, $f(k)+f(k-1) \geq (\frac{3}{3})^{k-2} + (\frac{3}{2})^{(k-1)-2}$

Thus, by the equations above, $n \geq 3$, $f_n \geq (\frac{3}{2})^{n-2}$

for f_k be the k^{th} fibonacci number

if $k = n+4 = 7$.

$f(k) = 13 \geq (\frac{3}{2})^5$.

if $k-1 = 6$.

$f(k) = 8 \geq (\frac{3}{2})^4$.

3. Given the loop: while $i \neq n$.
$p := 3 \times f$.
$f := p + f + 1$.
$i = i + 1$.
end while.

pre-condition: $\{ \text{int } n, n \geq 0, i = 0, f = 1 \}$.
post-condition $\{ f = \frac{4^{n+1} - 1}{3} \}$.

loop invariant: $\{ I(k) : i = k \text{ and } f = \frac{4^{k+1} - 1}{3} \}$.

(1). Basis Property: show $I(0)$.

while $i \neq n$,
$p := 3 \times f = 3 \times 1 = 3$.
$f := p + f + 1 = 3 + 1 + 1 = 5$.
$i := 1$.

post: $5 = \frac{4^{n+1} - 1}{3}$. $\therefore n = i = \log 4(3f+1) - 1 = 1$;

$n = i$, so, the loop end, so, $I(0)$ holds.

(2) Inductive Property: If before an iteration G and $I(k)$ then
$(i = k) \neq m$, and $f = \frac{4^{k+1} - 1}{3}$
then after the iteration since $i := i + 1$ exected $i = k + 1$
$f = \frac{4^{k+2} - 1}{3}$ $k = \log_4(3f+1) - 1$ end loop.

(3). when $i = n$, and the loop teminates.

(4). Since all faction above, this loop 's correctness is proved.