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Lecture Section: CMPT 260

1. Prove that a graph G = (V,E) is connected if and only if for every partition of V into two nonempty sets V1 and V2 (V1 $\cap$V2 = $\varnothing$ and V1 $\cup$V2 = V), there is an edge with one end in V1 and one end in V2.

Suppose G is connected, and V1 and V2 are 2 non-empty parts. Let a $\in$ V1, b $\in$ V2. Let P be a path from a to b in G. Ordering the vertices from a to b, if vertex x equals the last vertex of P in V1, the next vertex y is in V2, so ex is the desired edge.

By contradiction, there are no an edge with one end in V1 and one end in V2, suppose V1 only has one vertex x, and V2 only has one vertex y, assume that V has two vertices those are x and y. If there is not an edge with one end in V1 and one end in V2, it must not be a connected graph.

2. The question provides that the simple connected planar graph with at least 3 vertices, every face is bounded by at least 3 edges, every edge connects at most 2 faces.

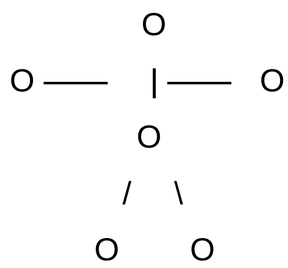
   From the question, substituting into Euler's formula:

   $$e <= 3v-6$$

   From the Handshake Theorem: average deg(v) = 2e/v.

   Because of the average degree in a simple planar graph is less 6, so in any simple planar graph has 5 vertices at most.

   Because of a simple planar graph has 5 vertices, a simple planar graph should have 6 color.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