Computer Science 260

Assignment 7

Due November 2, 2016 at 17:00 [For this assignment no late submissions accepted]

1. Prove that a graph $G = (V, E)$ is connected if and only if for every partition of V into two nonempty sets V_1 and V_2 ($V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = V$), there is an edge with one end in V_1 and one end in V_2. (10 marks)

2. In a plane drawing of a simple connected planar graph with at least three vertices, every face is bounded by at least 3 edges and every edge separates at most 2 faces. Therefore $2e \geq 3f$. Substituting into euler's formula $v - e + \frac{2e}{3} \geq 2$, or $e \leq 3v - 6$. By the handshake theorem the average degree in a graph is $\frac{2e}{v}$. Thus in a simple planar graph the average degree is at most $\frac{2(3v-6)}{v} = 6 - \frac{12}{v}$. Since the average degree in a simple planar graph is less than 6, and since every vertex cannot have above average degree, in any simple planar graph there exists a vertex of degree at most 5.

A legal coloring of a graph is an assignment of colors to vertices so that adjacent vertices receive different colors. A graph is k-colorable iff it is possible to legally color the vertices using at most k colors.

Prove using mathematical induction that every simple planar graph is 6-colorable. (8 marks)