REALTIME7 math work

Evan Cribbie

August 2023

1 Triangulation

$$k_l = \text{left cam intrinsic matrix} \tag{1}$$
$$k_r = \text{right cam intrinsic matrix} \tag{2}$$

Matrix k_l:

$$k_l = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix k_r:

$$k_r = \begin{bmatrix} f'_x & 0 & o'_x \\ 0 & f'_y & o'_y \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix M_l:

$$M_l = \begin{bmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Matrix M_r:

$$M_r = \begin{bmatrix} f'_x & 0 & c'_x & 0 \\ 0 & f'_y & c'_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Equation for U_l:

$$U_l = \begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix}$$

Equation for X_l:

$$X_l = \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix}$$

Equation for $M_l \cdot X_l$:

$$U_l = M_l \cdot X_l = \begin{bmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix}$$

Equation for U_r:

$$U_r = \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$

Equation for X_r:

$$X_r = \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Equation for $M_r \cdot X_r$:

$$U_r = M_r \cdot X_r = \begin{bmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Equation for R:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Equation for T:

$$T = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Equation for RT:

$$RT = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation for X_l:

$$X_l = RT \cdot X_r = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Equation for P_l:

$$P_l = M_l{\cdot}RT = \begin{bmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$$

Equation for U_l:

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} = U_l = M_l \cdot X_l = M_l \cdot (RT \cdot X_r) = P_l \cdot X_r = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Equation for U_r:

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = U_r = M_r \cdot X_r = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Equations:

$$u_r = m_{11}x_r + m_{12}y_r + m_{13}z_r + m_{14} \tag{3}$$
$$v_r = m_{21}x_r + m_{22}y_r + m_{23}z_r + m_{24} \tag{4}$$
$$1 = m_{31}x_r + m_{32}y_r + m_{23}z_r + m_{24} \tag{5}$$

So by rearranging, we get:

$$\begin{bmatrix} u_r(m_{31} - m_{11}) & u_r(m_{32} - m_{12}) & u_r(m_{33} - m_{13}) \\ v_r(m_{31} - m_{21}) & v_r(m_{32} - m_{22}) & v_r(m_{33} - m_{23}) \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} m_{14} - m_{34} \\ m_{24} - m_{34} \end{bmatrix}$$

Equations:

$$u_l = p_{11}x_r + p_{12}y_r + p_{13}z_r + p_{14} \tag{6}$$
$$v_l = p_{21}x_r + p_{22}y_r + p_{23}z_r + p_{24} \tag{7}$$
$$1 = p_{31}x_r + p_{32}y_r + p_{33}z_r + p_{34} \tag{8}$$

So by rearranging, we get:

$$\begin{bmatrix} u_l(p_{31} - p_{11}) & u_l(p_{32} - p_{12}) & u_l(p_{33} - p_{13}) \\ v_l(p_{31} - p_{21}) & v_l(p_{32} - p_{22}) & v_l(p_{33} - p_{23}) \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} p_{14} - p_{34} \\ p_{24} - p_{34} \end{bmatrix}$$

Combining gives us:

$$\begin{bmatrix} u_r(m_{31}-m_{11}) & u_r(m_{32}-m_{12}) & u_r(m_{33}-m_{13}) \\ v_r(m_{31}-m_{21}) & v_r(m_{32}-m_{22}) & v_r(m_{33}-m_{23}) \\ u_l(p_{31}-p_{11}) & u_l(p_{32}-p_{12}) & u_l(p_{33}-p_{13}) \\ v_l(p_{31}-p_{21}) & v_l(p_{32}-p_{22}) & v_l(p_{33}-p_{23}) \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} m_{14}-m_{34} \\ m_{24}-m_{34} \\ p_{14}-p_{34} \\ p_{24}-p_{34} \end{bmatrix}$$

Matrix A:

$$A = \begin{bmatrix} u_r(m_{31}-m_{11}) & u_r(m_{32}-m_{12}) & u_r(m_{33}-m_{13}) \\ v_r(m_{31}-m_{21}) & v_r(m_{32}-m_{22}) & v_r(m_{33}-m_{23}) \\ u_l(p_{31}-p_{11}) & u_l(p_{32}-p_{12}) & u_l(p_{33}-p_{13}) \\ v_l(p_{31}-p_{21}) & v_l(p_{32}-p_{22}) & v_l(p_{33}-p_{23}) \end{bmatrix}$$

Matrix b:

$$b = \begin{bmatrix} m_{14}-m_{34} \\ m_{24}-m_{34} \\ p_{14}-p_{34} \\ p_{24}-p_{34} \end{bmatrix}$$

So...

$$A \cdot X_r = b$$

Gives:

3D Coordinate X_r:

$$X_r = (A^T A)^{-1} A^T b$$