

EE232 Homework 4

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Question 1

$R = [2.28125 \ 2.35484 \ 2.57560 \ 2.20767];$

```
cvx_begin
    variables Ic Vc;
    maximize Vc;
    subject to
        Vc == Ic*sum(R);
        Ic*R(1) <= 3.65;
        Ic*R(2) <= 3.65;
        Ic*R(3) <= 3.65;
        Ic*R(4) <= 3.65;
        Ic >= 0; Ic <= 1.6;
cvx_end
```

By solving the above convex problem, we can obtain V_c and I_c :

$$V_c = 13.3486 \text{ V}$$

$$I_c = 1.4171 \text{ A}$$

Question 2

Part a

We can obtain the Y matrix according to the transmission network:

$$Y = \begin{bmatrix} -20 & 0 & 10 & 0 \\ 0 & -30 & 10 & 10 \\ 10 & 10 & -40 & 10 \\ 0 & 10 & 10 & -20 \end{bmatrix}$$

```
cvx_begin
    variables P1^g P2^g P5^g L2^ctrl L3^ctrl L4^ctrl theta2 theta3 theta4 theta5
    minimize sum_{h=1}^{10} (C_{1,h}(P1^g) + C_{2,h}(P2^g) + C_{5,h}(P5^g))
    subject to
        1.5 <= P1^g <= 5; 1.5 <= P2^g <= 5; 1.5 <= P5^g <= 5;

        P1^g + P2^g + P5^g == L2^ctrl + L3^ctrl + L4^ctrl + L2^fixed + L3^fixed + L4^fixed

        L2^ctrl >= 0; L3^ctrl >= 0; L4^ctrl >= 0

        sum_{h=1}^{10} L2^ctrl == 10; sum_{h=3}^9 L3^ctrl == 12; sum_{h=2}^6 L4^ctrl == 7;

        L2,h^ctrl == 0;
        L3,h^ctrl == 0 (h < 3, h > 9);
```

$$L_{4,h}^{ctrl} == 0 \ (h < 2, h > 6);$$

$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y] \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

cvx_end

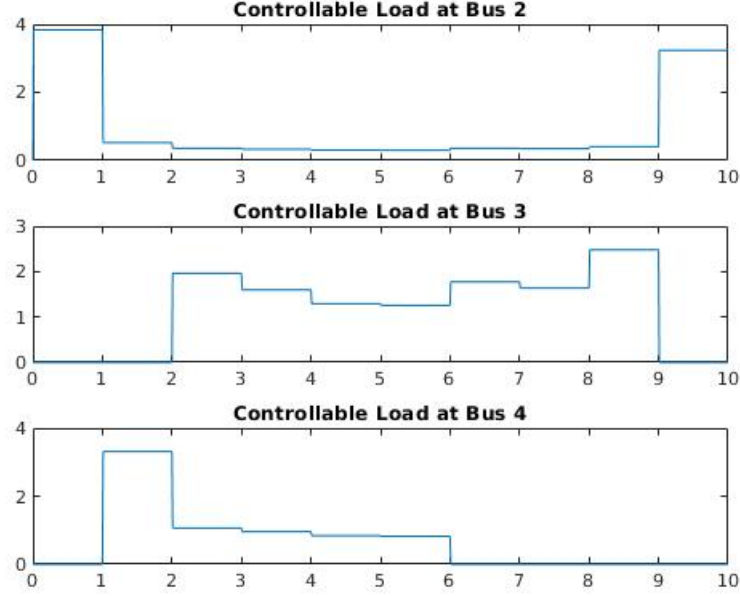


Figure 1: Schedule of controllable load with infinite transmission line capacity

Part b

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cvx_begin
variables P1^g P2^g P5^g L2^ctrl L3^ctrl L4^ctrl theta2 theta3 theta4 theta5
minimize sum_{h=1}^{10} (C_{1,h}(P1^g) + C_{2,h}(P2^g) + C_{5,h}(P5^g))
subject to
    1.5 ≤ P1^g ≤ 5; 1.5 ≤ P2^g ≤ 5; 1.5 ≤ P5^g ≤ 5;

    P1^g + P2^g + P5^g == L2^ctrl + L3^ctrl + L4^ctrl + L2^fixed + L3^fixed + L4^fixed

    L2^ctrl ≥ 0; L3^ctrl ≥ 0; L4^ctrl ≥ 0

    sum_{h=1}^{10} L2^ctrl == 10; sum_{h=3}^9 L3^ctrl == 12; sum_{h=2}^6 L4^ctrl == 7;

    L2^ctrl == 0;
    L3^ctrl == 0 (h < 3, h > 9);

```

$$L_{4,h}^{ctrl} == 0 \ (h < 2, h > 6);$$

$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y] \cdot \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

$$|10 \cdot (-\theta_3)| \leq 1.5$$

$$|10 \cdot (\theta_2 - \theta_4)| \leq 1.5$$

cvx_end

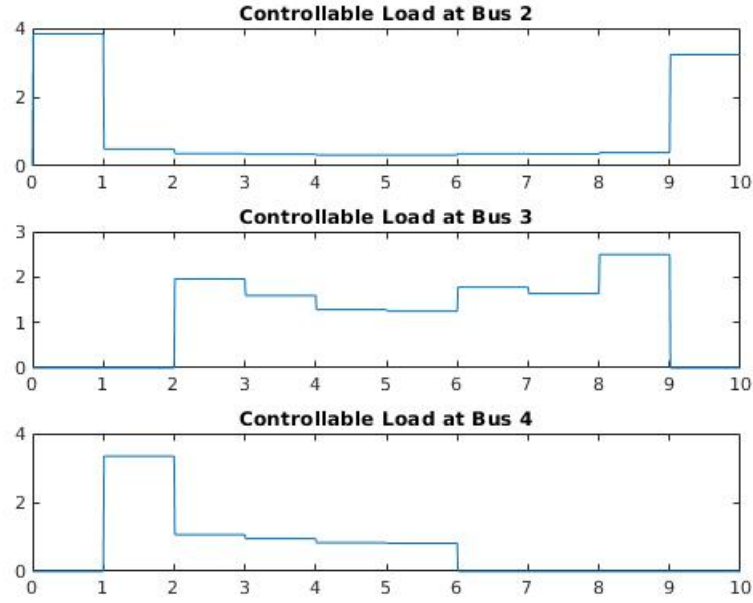


Figure 2: Schedule of controllable load with limited transmission line capacity

Part c

Optimal generation cost in (a): \$49377

Optimal generation cost in (b): \$49377

PAR (controllable) in (a): 1.3172

PAR (controllable) in (b): 1.3172

PAR (total) in (a): 1

PAR (total) in (b): 1

Question 3

```

cvx_begin
variables  $P_1^g$   $P_2^g$   $P_5^g$   $L_2^{ctrl}$   $L_3^{ctrl}$   $L_4^{ctrl}$   $\theta_2$   $\theta_3$   $\theta_4$   $\theta_5$ 
minimize  $\sum_{h=1}^{10} (L_{2,h}^{fixed} + L_{3,h}^{fixed} + L_{4,h}^{fixed} + L_{2,h}^{ctrl} + L_{3,h}^{ctrl} + L_{4,h}^{ctrl}) \times Price_h$ 
subject to
     $1.5 \leq P_1^g \leq 5; 1.5 \leq P_2^g \leq 5; 1.5 \leq P_5^g \leq 5;$ 

     $P_1^g + P_2^g + P_5^g == L_2^{ctrl} + L_3^{ctrl} + L_4^{ctrl} + L_2^{fixed} + L_3^{fixed} + L_4^{fixed}$ 

     $L_2^{ctrl} \geq 0; L_3^{ctrl} \geq 0; L_4^{ctrl} \geq 0$ 

     $\sum_{h=1}^{10} L_{2,h}^{ctrl} == 10; \sum_{h=3}^9 L_{3,h}^{ctrl} == 12; \sum_{h=2}^6 L_{4,h}^{ctrl} == 7;$ 

     $L_{2,h}^{ctrl} == 0;$ 
     $L_{3,h}^{ctrl} == 0 \ (h < 3, h > 9);$ 
     $L_{4,h}^{ctrl} == 0 \ (h < 2, h > 6);$ 

    
$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y] \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

cvx_end

```

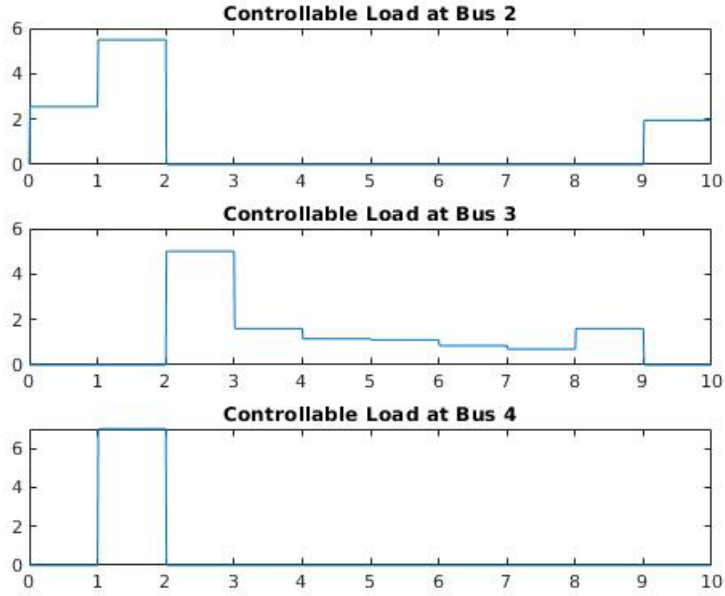


Figure 3: Schedule of controllable load with different hourly price

Optimal price: \$2187
 PAR (controllable) in this case: 1.3172
 PAR (total) in this case: 2.4978

Question 4

```

cvx_begin
  variables  $P_1^g$   $P_2^g$   $P_5^g$   $L_2^{ctrl}$   $L_3^{ctrl}$   $L_4^{ctrl}$   $\theta_2$   $\theta_3$   $\theta_4$   $\theta_5$ 
  minimize  $\sum_{h=1}^{10} (L_{2,h}^{fixed} + L_{3,h}^{fixed} + L_{4,h}^{fixed} + L_{2,h}^{ctrl} + L_{3,h}^{ctrl} + L_{4,h}^{ctrl}) \times 40$ 
            $+ \max (L_{2,h}^{fixed} + L_{3,h}^{fixed} + L_{4,h}^{fixed} + L_{2,h}^{ctrl} + L_{3,h}^{ctrl} + L_{4,h}^{ctrl}) \times 80$ 
  subject to
     $1.5 \leq P_1^g \leq 5; 1.5 \leq P_2^g \leq 5; 1.5 \leq P_5^g \leq 5;$ 

     $P_1^g + P_2^g + P_5^g == L_2^{ctrl} + L_3^{ctrl} + L_4^{ctrl} + L_2^{fixed} + L_3^{fixed} + L_4^{fixed}$ 

     $L_2^{ctrl} \geq 0; L_3^{ctrl} \geq 0; L_4^{ctrl} \geq 0$ 

     $\sum_{h=1}^{10} L_{2,h}^{ctrl} == 10; \sum_{h=3}^9 L_{3,h}^{ctrl} == 12; \sum_{h=2}^6 L_{4,h}^{ctrl} == 7;$ 

     $L_{2,h}^{ctrl} == 0;$ 
     $L_{3,h}^{ctrl} == 0 \ (h < 3, h > 9);$ 
     $L_{4,h}^{ctrl} == 0 \ (h < 2, h > 6);$ 

    
$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y] \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

cvx_end
  
```

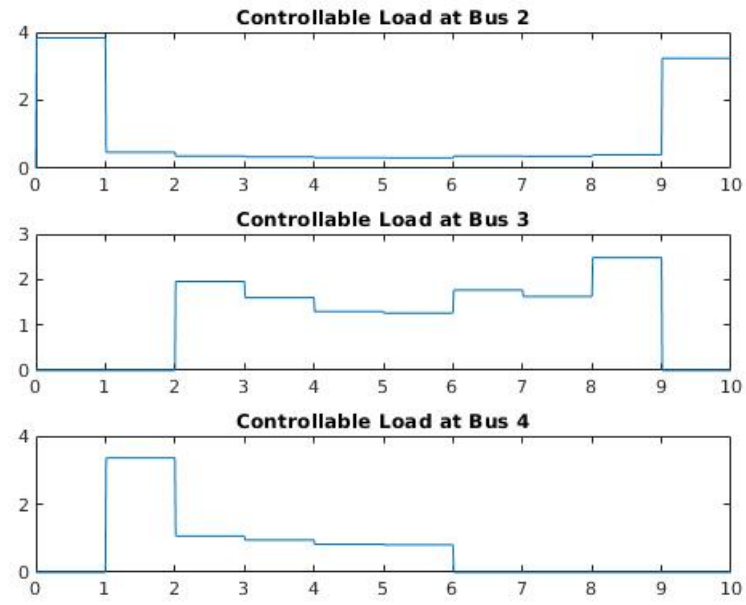


Figure 4: Schedule of controllable load with energy and power charge

Optimal price: \$2776.8

PAR (controllable) in this case: 1.3172

PAR (total) in this case: 1

Question 5
Part a, b

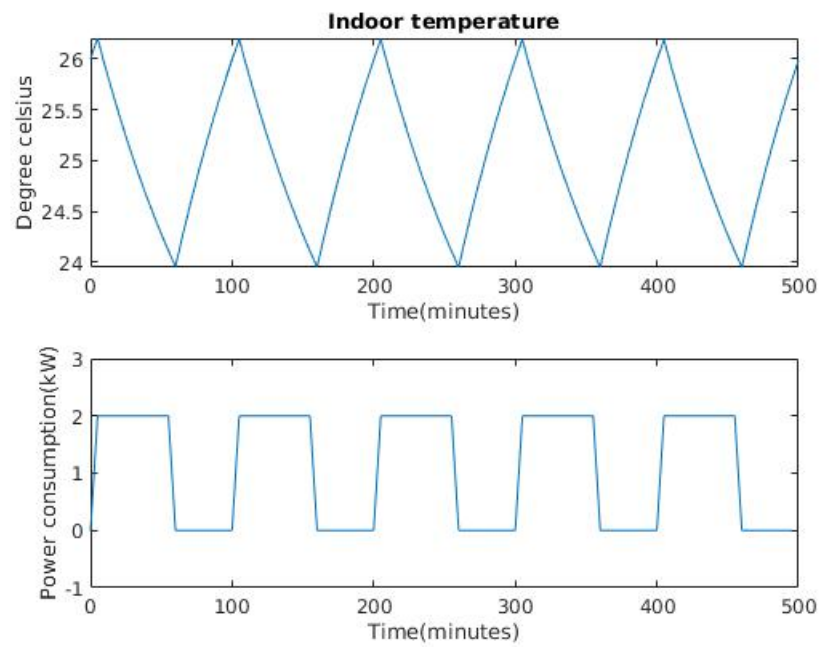


Figure 5: Indoor temperature and power consumption

Total energy usage: 9.1667 kWh

Part c

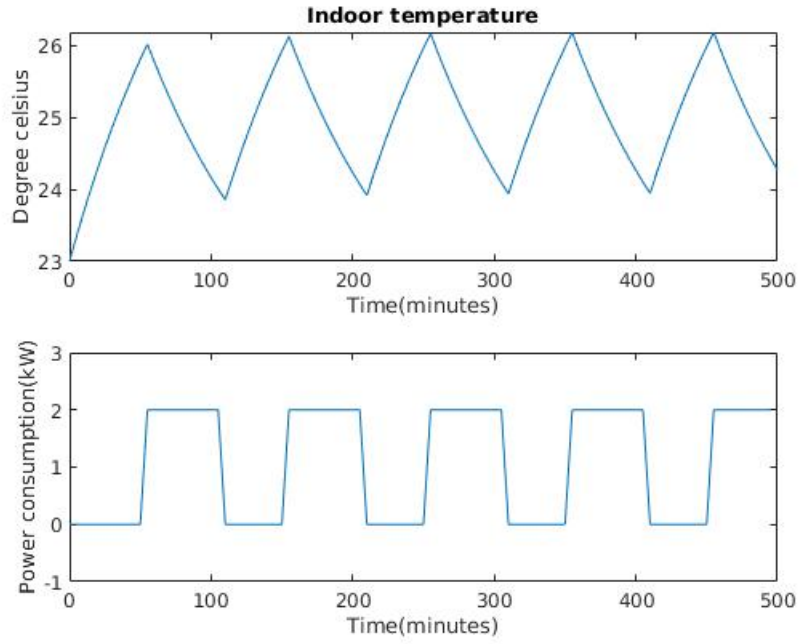


Figure 6: Indoor temperature and power consumption at $x_0 = 23$

Total energy usage: 8.8333 kWh

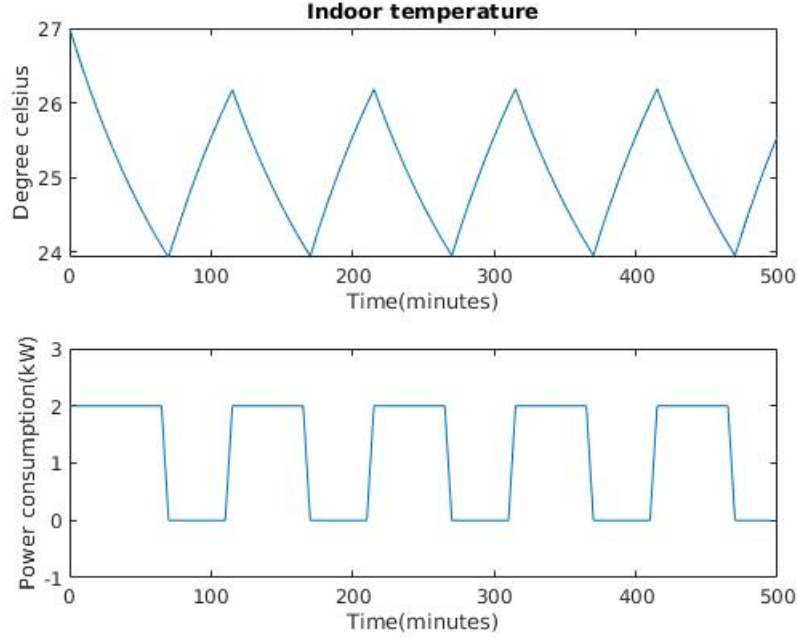


Figure 7: Indoor temperature and power consumption at $x_0 = 27$

Total energy usage: 9.6667 kWh