EE232 Homework 4

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Question 1

```
R = [2.28125 2.35484 2.57560 2.20767];

cvx_begin
    variables Ic Vc;
    maximize Vc;
    subject to
        Vc == Ic*sum(R);
        Ic*R(1) <= 3.65;
        Ic*R(2) <= 3.65;
        Ic*R(3) <= 3.65;
        Ic*R(4) <= 3.65;
        Ic >= 0; Ic <= 1.6;
cvx_end
```

By solving the above convex problem, we can obtain V_c and I_c:

$$V_c = 13.3486\,\mathrm{V}$$
$$I_c = 1.4171\,\mathrm{A}$$

Question 2

Part a

We can obtain the Y matrix according to the transmission network:

$$Y = \begin{bmatrix} -20 & 0 & 10 & 0 \\ 0 & -30 & 10 & 10 \\ 10 & 10 & -40 & 10 \\ 0 & 10 & 10 & -20 \end{bmatrix}$$

`cvx_begin`

`variables` $P_1^g \;\; P_2^g \;\; P_5^g \;\; L_2^{ctrl} \;\; L_3^{ctrl} \;\; L_4^{ctrl} \;\; \theta_2 \;\; \theta_3 \;\; \theta_4 \;\; \theta_5$

`minimize` $\sum_{h=1}^{10} \left(C_{1,h}(P_1^g) + C_{2,h}(P_2^g) + C_{5,h}(P_5^g) \right)$

`subject to`

$1.5 \leq P_1^g \leq 5;\; 1.5 \leq P_2^g \leq 5;\; 1.5 \leq P_5^g \leq 5;$

$P_1^g + P_2^g + P_5^g == L_2^{ctrl} + L_3^{ctrl} + L_4^{ctrl} + L_2^{fixed} + L_3^{fixed} + L_4^{fixed}$

$L_2^{ctrl} \geq 0;\; L_3^{ctrl} \geq 0;\; L_4^{ctrl} \geq 0$

$\sum_{h=1}^{10} L_{2,h}^{ctrl} == 10;\; \sum_{h=3}^{9} L_{3,h}^{ctrl} == 12;\; \sum_{h=2}^{6} L_{4,h}^{ctrl} == 7;$

$L_{2,h}^{ctrl} == 0;$
$L_{3,h}^{ctrl} == 0 \; (h < 3, h > 9);$

$L_{4,h}^{ctrl} == 0 \; (h < 2, h > 6);$

$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y] \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

```
cvx_end
```

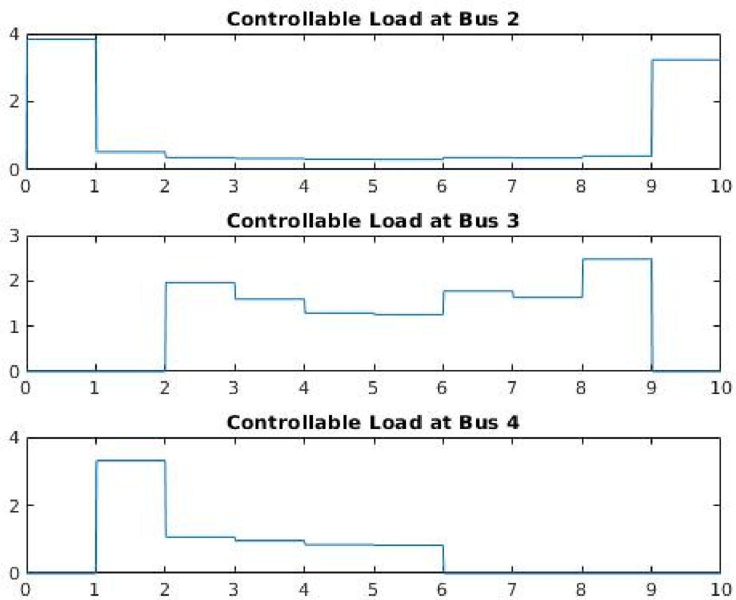

Figure 1: Schedule of controllable load with infinite transmission line capacity

Part b

```
cvx_begin
    variables
```
variables $P_1^g \; P_2^g \; P_5^g \; L_2^{ctrl} \; L_3^{ctrl} \; L_4^{ctrl} \; \theta_2 \; \theta_3 \; \theta_4 \; \theta_5$

minimize $\sum_{h=1}^{10} \left(C_{1,h}(P_1^g) + C_{2,h}(P_2^g) + C_{5,h}(P_5^g)\right)$

subject to

$1.5 \le P_1^g \le 5; \; 1.5 \le P_2^g \le 5; \; 1.5 \le P_5^g \le 5;$

$P_1^g + P_2^g + P_5^g == L_2^{ctrl} + L_3^{ctrl} + L_4^{ctrl} + L_2^{fixed} + L_3^{fixed} + L_4^{fixed}$

$L_2^{ctrl} \ge 0; \; L_3^{ctrl} \ge 0; \; L_4^{ctrl} \ge 0$

$\sum_{h=1}^{10} L_{2,h}^{ctrl} == 10; \; \sum_{h=3}^{9} L_{3,h}^{ctrl} == 12; \; \sum_{h=2}^{6} L_{4,h}^{ctrl} == 7;$

$L_{2,h}^{ctrl} == 0;$
$L_{3,h}^{ctrl} == 0 \; (h < 3, h > 9);$

$L_{4,h}^{ctrl} == 0 \ (h < 2, h > 6);$

$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y] \cdot \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

$|10 \cdot (-\theta_3)| \leq 1.5$
$|10 \cdot (\theta_2 - \theta_4)| \leq 1.5$

```
cvx_end
```

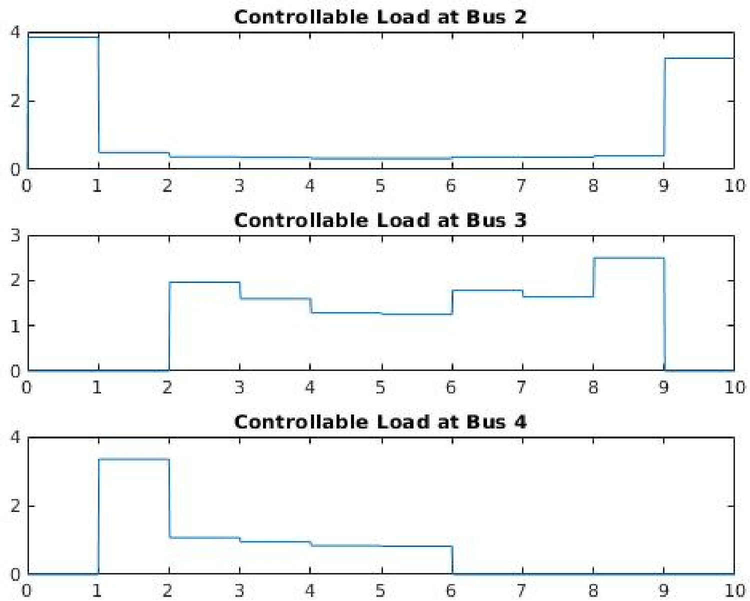

Figure 2: Schedule of controllable load with limited transmission line capacity

Part c
Optimal generation cost in (a): $49377
Optimal generation cost in (b): $49377
PAR (controllable) in (a): 1.3172
PAR (controllable) in (b): 1.3172
PAR (total) in (a): 1
PAR (total) in (b): 1

Question 3

`cvx_begin`

    `variables` $P_1^g\ \ P_2^g\ \ P_5^g\ \ L_2^{ctrl}\ \ L_3^{ctrl}\ \ L_4^{ctrl}\ \ \theta_2\ \ \theta_3\ \ \theta_4\ \ \theta_5$

    `minimize` $\sum_{h=1}^{10}\left(L_{2,h}^{fixed}+L_{3,h}^{fixed}+L_{4,h}^{fixed}+L_{2,h}^{ctrl}+L_{3,h}^{ctrl}+L_{4,h}^{ctrl}\right)\times Price_h$

    `subject to`

$$1.5 \le P_1^g \le 5;\ 1.5 \le P_2^g \le 5;\ 1.5 \le P_5^g \le 5;$$

$$P_1^g + P_2^g + P_5^g == L_2^{ctrl} + L_3^{ctrl} + L_4^{ctrl} + L_2^{fixed} + L_3^{fixed} + L_4^{fixed}$$

$$L_2^{ctrl} \ge 0;\ L_3^{ctrl} \ge 0;\ L_4^{ctrl} \ge 0$$

$$\sum_{h=1}^{10} L_{2,h}^{ctrl} == 10;\ \sum_{h=3}^{9} L_{3,h}^{ctrl} == 12;\ \sum_{h=2}^{6} L_{4,h}^{ctrl} == 7;$$

$$L_{2,h}^{ctrl} == 0;$$
$$L_{3,h}^{ctrl} == 0\ (h<3, h>9);$$
$$L_{4,h}^{ctrl} == 0\ (h<2, h>6);$$

$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y]\begin{bmatrix}\theta_2\\ \theta_3\\ \theta_4\\ \theta_5\end{bmatrix}$$

`cvx_end`

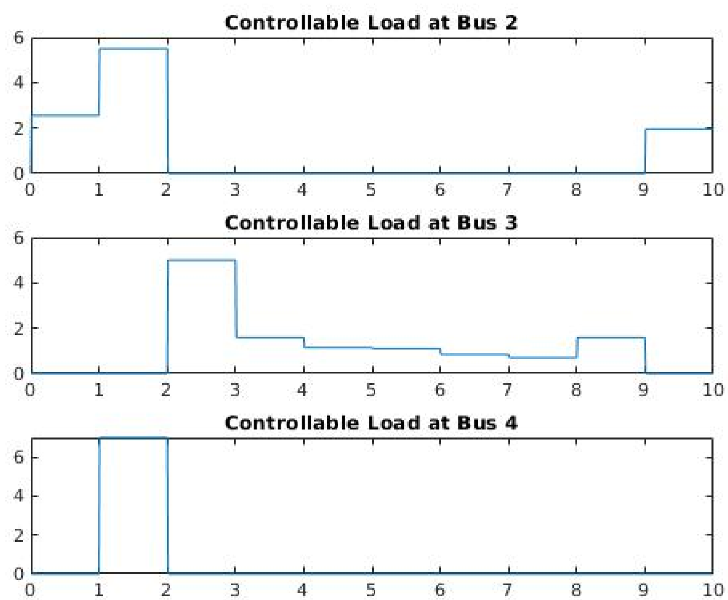

Figure 3: Schedule of controllable load with different hourly price

Optimal price: $2187
PAR (controllable) in this case: 1.3172
PAR (total) in this case: 2.4978
This is not desirable. PAR is far from 1, which means that the load is not stable.

Question 4

`cvx_begin`

`variables` $P_1^g\ \ P_2^g\ \ P_5^g\ \ L_2^{ctrl}\ \ L_3^{ctrl}\ \ L_4^{ctrl}\ \ \theta_2\ \ \theta_3\ \ \theta_4\ \ \theta_5$

`minimize` $\sum_{h=1}^{10}\left(L_{2,h}^{fixed}+L_{3,h}^{fixed}+L_{4,h}^{fixed}+L_{2,h}^{ctrl}+L_{3,h}^{ctrl}+L_{4,h}^{ctrl}\right)\times 40$
$+\max\left(L_{2,h}^{fixed}+L_{3,h}^{fixed}+L_{4,h}^{fixed}+L_{2,h}^{ctrl}+L_{3,h}^{ctrl}+L_{4,h}^{ctrl}\right)\times 80$

`subject to`

$1.5 \le P_1^g \le 5;\ 1.5 \le P_2^g \le 5;\ 1.5 \le P_5^g \le 5;$

$P_1^g + P_2^g + P_5^g == L_2^{ctrl} + L_3^{ctrl} + L_4^{ctrl} + L_2^{fixed} + L_3^{fixed} + L_4^{fixed}$

$L_2^{ctrl} \ge 0;\ L_3^{ctrl} \ge 0;\ L_4^{ctrl} \ge 0$

$\sum_{h=1}^{10} L_{2,h}^{ctrl} == 10;\ \sum_{h=3}^{9} L_{3,h}^{ctrl} == 12;\ \sum_{h=2}^{6} L_{4,h}^{ctrl} == 7;$

$L_{2,h}^{ctrl} == 0;$
$L_{3,h}^{ctrl} == 0\ (h<3, h>9);$
$L_{4,h}^{ctrl} == 0\ (h<2, h>6);$

$$\begin{bmatrix} P_2^g - L_2^{ctrl} - L_2^{fixed} \\ -L_3^{ctrl} - L_3^{fixed} \\ -L_4^{ctrl} - L_4^{fixed} \\ P_5^g \end{bmatrix} = [Y] \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

`cvx_end`

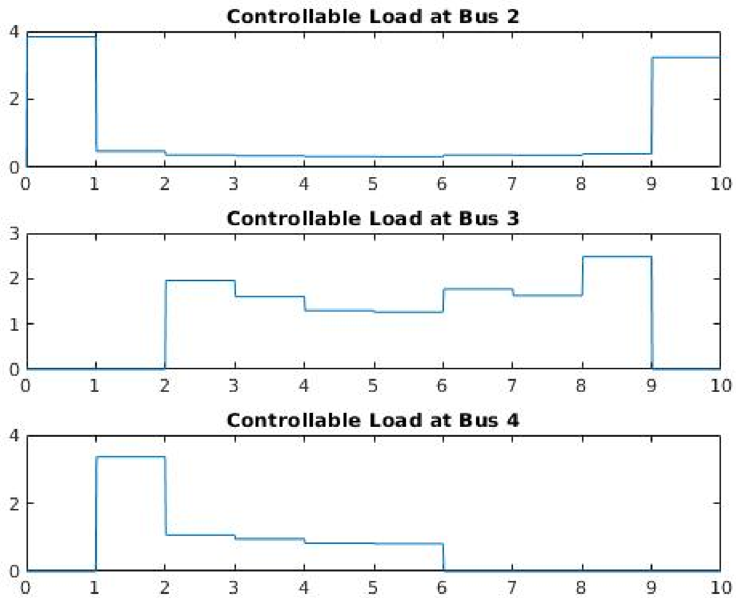

Figure 4: Schedule of controllable load with energy and power charge

Optimal price: $2776.8
PAR (controllable) in this case: 1.3172
PAR (total) in this case: 1

Question 5
Part a, b

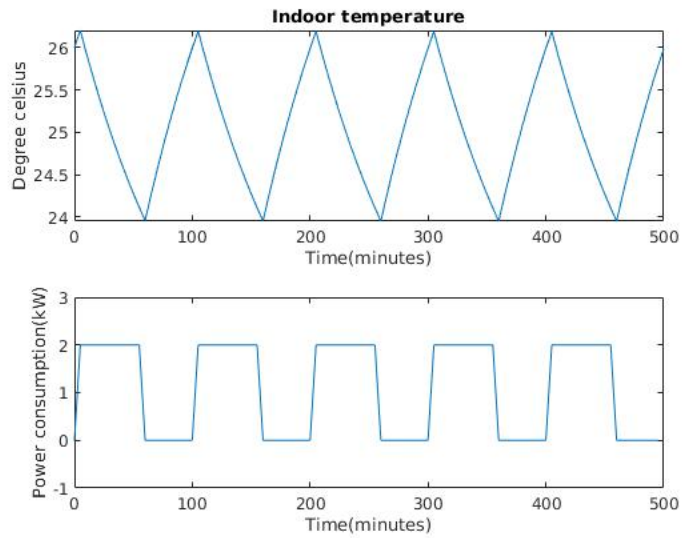

Figure 5: Indoor temperature and power consumption

Total energy usage: 9.1667 kWh

Part c

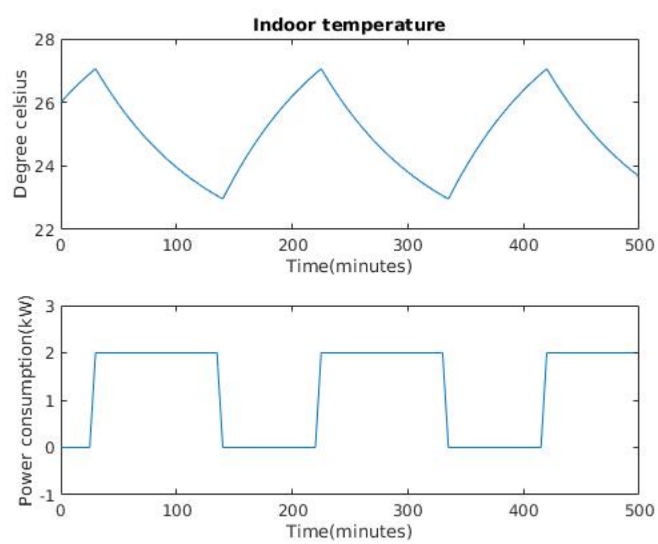

Figure 6: Indoor temperature and power consumption

Total energy usage: 10 kWh