# STA 100 Problem Set

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1. 10 student drivers' speeds were observed for an hour, with their parents in the car and without their parents. Their average speed was recorded in each case (speed varied greatly depending on what kind of driving situation they were in). The data are as follows:

| | Speed |
|---|---|
| With parents | 19, 25, 31, 52, 49, 34, 59, 47, 17, 51 |
| Without parents | 24, 27, 36, 53, 55, 34, 66, 51, 20, 55 |

It is of interest to test whether there is a significant difference in the mean speeds with ($\mu_1$) and without ($\mu_2$) parents in the car. Assume the speed is normally distributed. Use level of significance $\alpha = 0.05$.

(a) State the null and alternative hypotheses.

(b) Calculate the test statistic and determine the critical value for this problem.

(c) Find the range of $p$-value.

(d) Do you reject the null, or fail to reject?

(e) State the conclusion in terms of the problem.

(f) Construct a 95% confidence interval for $\mu_1 - \mu_2$. Is the confidence interval consistent with the conclusion in (d)?

(g) Interpret your confidence interval in terms of the problem.

**Solution:**

(a) $H_0 : \mu_1 - \mu_2 = 0$ v.s. $H_A : \mu_1 - \mu_2 \neq 0$.

(b) The differences are

$$-5, -2, -5, -1, -6, 0, -7, -4, -3, -4.$$

The mean and standard deviation of the differences are thus $\bar{D} = -3.7, s = 2.2136$. Therefore, the test statistic is

$$T = \frac{\bar{D} - 0}{\text{SE}_{\bar{D}}} = \frac{-3.7}{2.2136/\sqrt{10}} = -5.286.$$

The critical value is $t_{10-1}(0.05/2) = t_9(0.025) = 2.262$.

(c) The range of $p$-value is (0, 0.001).

(d) We reject $H_0$ at the $\alpha = 0.05$ level of significance.

(e) There is a significant difference in the mean speeds with and without parents in the car.

(f) The 95% confidence interval for $\mu_1 - \mu_2$ is

$$-3.7 \pm t_9(0.025) \times \frac{2.2136}{\sqrt{10}}$$

or $(-5.2834, -2.1166)$. Since zero is not included in the confidence interval, it is consistent with the conclusion in (d).

(g) We are 95% confident that the difference in the mean speeds with and without parents in the car is between $-5.2834$ and $-2.1166$.

2. In an experiment to test the effectiveness of a new sleeping aid, one group of patients took a new drug (Drug A) and the other group took a commonly used drug (Drug B). Information was recorded on time to fall asleep (in minutes), and the following summary statistics were complied:

| | Drug A | Drug B |
|---|---|---|
| Mean | 27.3 | 32.7 |
| Standard deviation | 5.2 | 4.1 |
| Sample size | 18 | 16 |

The researcher aims to use this data to examine whether Drug A ($\mu_1$) is more effective than Drug B ($\mu_2$) in reducing the average time it takes to fall asleep. You may assume the population is normal and the degrees of freedom $\nu = 30$. Use level of significance $\alpha = 0.01$.

(a) State the null and alternative hypotheses.

(b) Calculate the test statistic and determine the critical value for this problem.

(c) Find the range of $p$-value.

(d) Do you reject the null, or fail to reject?

(e) State the conclusion in terms of the problem.

(f) Construct an upper one-sided 99% confidence interval for $\mu_1 - \mu_2$. Is the confidence interval consistent with the conclusion in (d)?

(g) Interpret your confidence interval in terms of the problem.

**Solution:**

(a) $H_0 : \mu_1 - \mu_2 \geq 0$ v.s. $H_A : \mu_1 - \mu_2 < 0$.

(b) The standard error of $\bar{Y}_1 - \bar{Y}_2$ is

$$\text{SE}_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{\frac{5.2^2}{18} + \frac{4.1^2}{16}} = 1.5978$$

The test statistic is thus

$$T = \frac{(\bar{Y}_1 - \bar{Y}_2) - 0}{\text{SE}_{\bar{Y}_1 - \bar{Y}_2}} = \frac{(27.3 - 32.7) - 0}{1.5978} = -3.3797.$$

The critical value is $-t_{30}(0.01) = -2.457$.

(c) The range of $p$-value is (0.0005, 0.005).

(d) We reject $H_0$ at the $\alpha = 0.01$ level of significance.

(e) Drug A is more effective than Drug B in reducing the average time it takes to fall asleep.

(f) The upper one-sided 99% confidence interval for $\mu_1 - \mu_2$ is

$$(-\infty, (27.3 - 32.7) + t_{30}(0.01) \times 1.5978)$$

or $(-\infty, -1.4743)$. Since zero is not included in the confidence interval, it is consistent with the conclusion in (d).

(g) We are 99% confident that the mean time to fall asleep is at least 1.4743 lower when using Drug A compared to Drug B.

3. Suppose that a fair coin is tossed 400 times. Approximate the probability that the number of heads is equal to 201.

**Solution:**

$Y \sim B(400, 0.5)$ can be approximated by $N(200, 100)$. Using continuity correction, one has

$$\begin{aligned} P(Y = 201) &= P(200.5 < Y < 201.5) \\ &= P(\frac{200.5 - 200}{\sqrt{100}} < Z < \frac{201.5 - 200}{\sqrt{100}}) \\ &= P(Z < 0.15) - P(Z \leq 0.05) \\ &= 0.5596 - 0.5199 \\ &= 0.0397. \end{aligned}$$