Midterm II Bonus (Due May 21 at 4pm)

Consider the following regression model with two predictors X_1 and X_2 ($p = 3$ in the general linear regression model)

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i, i = 1, 2, \cdots, n.$$

In matrix form,

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where

$$\mathbf{Y} = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & X_{11} & X_{12} \\ 1 & X_{21} & X_{22} \\ \vdots & \vdots & \vdots \\ 1 & X_{n1} & X_{n2} \end{pmatrix}, \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}, \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}.$$

- Compute $(\mathbf{X}'\mathbf{X})^{-1}$. Show all the steps and formulas. Hint: Equation 5.23.
- Show $\mathbf{HH} = \mathbf{H}$, where $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ is the hat matrix. Show all the steps and formulas.
- Compute the ANOVA table, formulas. Write the model. Hint: the second column of the ANOVA table (Table 6.1 SS) should be expressed using $Y_i, X_{ij}, i = 1, 2, \cdots, n, j = 1, 2, 3$. Equations 6.74 and 6.75 may be useful.