STA 138 Discussion 6

Fall 2020

Likelihood ratio tests for multinomial models

For our discussion this week, we will implement likelihood ratio tests. To do so, we'll pick up where we left off last week.

Suppose that Amelia has sampled 76 newts out of a tank. There are four species in the tank (A, B, C, and D); the observed counts are given in the table below.

A	B	C	D
14	22	25	15

Beatrice, the lab assistant, feeds the newts in the tank regularly. She takes particular notice when she does so of the brightly colored species A and B. From her experience, she claims that 20% of the newts in the tank are from species A, and 30% from species B.

Assuming that she is correct, use numerical methods to obtain the MLE of the proportions of the species in the tank.

In the problems below, we will interpret the test results individually; there's no need to control for multiple testing. We'll carry out each test at $\alpha = 0.01$.

1. Let's test whether we can rule out the possibility that the four species in the tank exist in equal proportions. Carry out a likelihood ratio test of
$$H_0 : \pi_1 = \pi_2 = \pi_3 = \pi_4 = 0.25$$
against
$$H_1 : \pi_i \neq \pi_j \text{ for some } i \neq j$$

2. Let's put Beatrice's claim to the test: using a likelihood ratio test, test
$$H_0 : \pi_1 = 0.2, \pi_2 = 0.3$$
against
$$H_1 : \pi_1 \neq 0.2 \text{ or } \pi_2 \neq 0.3$$

3. *Assuming that Beatrice's claim is true*, let's test whether there is evidence to rule out the possibility that species C and D occur in the tank in equal proportions.
$$H_0 : \pi_1 = 0.2, \pi_2 = 0.3, \pi_3 = \pi_4 = 0.25$$
against
$$H_1 : \pi_1 = 0.2 \text{ and } \pi_2 = 0.3 \text{ but either } \pi_3 \neq 0.25 \text{ or } \pi_4 \neq 0.25$$