STA 220 - Data and Web Technologies for Data Analysis - Lab 1

We could split a matrix A into a product A=QR where Q is a matrix with unit norm orthogonal vectors and R is an upper triangular matrix. Consider the following matrix A (see Lecture 2 c).) and compute the QR decomposition:

```
x <- c("bob" = 39, "carol" = 31, "ted" = 31, "alice" = 32)
X <- cbind(1, x)
colnames(X) <- c("Intercept", "Age")
X <- cbind(X, Sex = c(0,1,0,1))
X
##       Intercept Age Sex
## bob           1  39   0
## carol         1  31   1
## ted           1  31   0
## alice         1  32   1
A = solve(t(X) %*% X)
A
##           Intercept         Age        Sex
## Intercept 38.192308 -1.07692308 -4.2692308
## Age       -1.076923  0.03076923  0.1076923
## Sex       -4.269231  0.10769231  1.3769231
```

Hint

You may use

`qr.X`, `qr.Q`, `qr.R`

Solution

The QR decomposition:

```
QR <- qr(A)
```

Rank of the matrix:

```
QR$rank
## [1] 3
```

The Q factor:

```
Q <- qr.Q(QR)
Q
##             [,1]        [,2]       [,3]
## [1,] -0.99342031  0.11004687 0.03171409
## [2,]  0.02801185 -0.03502717 0.99899371
## [3,]  0.11104698  0.99330901 0.03171409
```

The R factor:

```
R <- qr.R(QR)
R
##           Intercept         Age        Sex
## Intercept -38.44527  1.08265806 4.39706035
## Age         0.00000 -0.01261803 0.89412247
## Sex         0.00000  0.00000000 0.01585704
```

Reconstructing the matrix A from its decomposition as follows:

```
qr.X(QR)
##           Intercept         Age        Sex
## Intercept 38.192308 -1.07692308 -4.2692308
## Age       -1.076923  0.03076923  0.1076923
## Sex       -4.269231  0.10769231  1.3769231
```