1) Naïve Bayes

a. It is not likely to be true because they are most likely related in spam emails. If "viagra" is in the email, then the probability of "free" being in it increases; vice versa too. Mathematically, $P(A \cap B) = P(A|B)P(B)$ (whether A and B are independent or dependent), and $P(A|B) > P(A)$ because of what we said, so $P(A \cap B) > P(A)P(B)$, where A and B are the existence of these two words.

b. $\mathbf{2n + 1}$. For every word j of the n words, we need to calculate two things – $P(\vec{x}_{test,j} = 1|spam)$ and $P(\vec{x}_{test,j} = 1|not\ spam)$, thus the $2n$. We also need to calculate $P(spam)$, thus the $+1$.

c. $\mathbf{2^{n+1} + 1}$. Not assuming independence requires us to keep track of both cases of $\vec{x}_{test,j}$ for each of the n words. This is 2^n possibilities. We need to do this for both spam and not spam. That's $2^n * 2 = 2^{n+1}$We still need to compute $P(spam)$ once.

d. Naïve Bayes has a much smaller runtime and is often good enough.

e. $P(spam|\text{"probability"} = 1, \text{"free"} = 1, \text{"viagra"} = 1) = P(spam) \times P(\text{"probability"=1}|spam) \times P(\text{"free"=1}|spam) \times P(\text{"viagra"=1}|spam) \times P(\text{"hello"} = 0|spam) = 0$ because $P(\text{"probability"}|spam) = 0$.

$P(not\ spam|\text{"probability"} = 1, \text{"free"} = 1, \text{"viagra"} = 1) = P(not\ spam) \times P(\text{"probability"=1}|not\ spam) \times P(\text{"free"=1}|not\ spam) \times P(\text{"viagra"=1}|not\ spam) \geq 0$.

Non-spam

f. $P(spam|\text{"probability"} = 1, \text{"free"} = 1, \text{"viagra"} = 1) = P(spam) \times P(\text{"probability"=1}|spam) \times P(\text{"free"=1}|spam) \times P(\text{"viagra"=1}|spam) \times P(\text{"hello"} = 0|spam) = 2.7 \times 10^{-5}$

$$P(spam) = \frac{1}{2}$$

$$P(\text{"probability"=1}|spam) = \frac{0+1}{1000+2} = \frac{1}{1002}$$

$$P(\text{"free"=1}|spam) = \frac{400+1}{1000+2} = \frac{401}{1002}$$

$$P(\text{"viagra"=1}|spam) = \frac{300+1}{1000+2} = \frac{301}{1002}$$

$$P(\text{"hello"} = 0|spam) = \frac{450+1}{1000+2} = \frac{451}{1002}$$

$P(not\ spam|\text{"probability"} = 0, \text{"free"} = 0, \text{"viagra"} = 0) = P(not\ spam) \times P(\text{"probability"=1}|not\ spam) \times P(\text{"free"=1}|not\ spam) \times P(\text{"viagra"=1}|not\ spam) = 2.33 \times 10^{-6}$

$$P(not\ spam) = \frac{1}{2}$$

$$P(\text{"probability"=1}|not\ spam) = \frac{20+1}{1000+2} = \frac{21}{1002}$$

$$P(\text{"free"=1}|not\ spam) = \frac{200+1}{1000+2} = \frac{201}{1002}$$

$$P(\text{"viagra"=1}|not\ spam) = \frac{10+1}{1000+2} = \frac{11}{1002}$$

$$P(\text{"hello"} = 0|not\ spam) = \frac{100+1}{1000+2} = \frac{101}{1002}$$

$2.33 \times 10^{-6} < 2.7 \times 10^{-5}$. **Spam**.

g. The intuition behind the $+2$ in the denominator is that we start out as if all classes are equally likely ($\frac{1}{|V_j|}$), and there are 2 such classes, spam and not spam.

2) Logic

Predicate	True or False?
Overlap(Open Sky, MidCo)	F
Overlap(SpecCom, FiveCo)	T
Overlap(EastCom, MidCo)	T
Overlap(MidCo, MidCo)	T
Overlap(Central, MidCo)	T

a. ^
b. If two networks overlap, then they cannot use the same channel.
c. $\neg Overlap(x,y) \vee \left(\neg HasChannel(x,c) \vee \neg HasChannel(y,c)\right)$
$= \neg \boldsymbol{Overlap}(\boldsymbol{x},\boldsymbol{y}) \vee \neg \boldsymbol{HasChannel}(\boldsymbol{x},\boldsymbol{c}) \vee \neg \boldsymbol{HasChannel}(\boldsymbol{y},\boldsymbol{c})$
d. $\forall_x\ IsNetwork(x)\ \left(\exists_c\ IsChannel(c)\ \left(HasChannel(x,c) \wedge (\forall_{d \neq c}\ IsChannel(d)\ \left(\neg HasChannel(x,d)\right)\right)\right)$
e. Yes, it's satisfiable:
Open Sky: 1
SpecCom: 1
Central: 3
FiveCo: 2
MidCo: 1
EastCom: 2
f. No, it's not satisfiable. If Open Sky, SpecCom, and EastCom all take channel 1, then that leaves all three of FiveCo, MidCo, and Central to choose between channels 2 and 3, but that's only two channels for three overlapping networks.

3) CNF and Resolution

a. $(a \wedge b) \vee (c \wedge d)$ converted to CNF is $(a \vee c) \wedge (a \vee d) \wedge (b \vee c) \wedge (b \vee d)$

Statement	Value
HasChannel(Open Sky, C_1)	T
HasChannel(Open Sky, C_2)	F
HasChannel(Open Sky, C_3)	F
Overlap(Open Sky, Central)	T

b. ^
c. $\left(\neg O(Open\ Sky, Central) \vee \neg HC(Open\ Sky, C_1) \vee \neg HC(Central, C_1)\right) \wedge \left(O(Open\ Sky, Central)\right) \wedge (HC(Open\ Sky, C_1)) \vDash \neg HC(Central, C_1)$??

$\left(\neg O(Open\ Sky, Central) \vee \neg HC(Open\ Sky, C_1) \vee \neg HC(Central, C_1)\right) \wedge \left(O(Open\ Sky, Central)\right) \wedge \left(HC(Open\ Sky, C_1)\right) \wedge HC(Central, C_1)$

$\left(\neg HC(Open\ Sky, C_1) \vee \neg HC(Central, C_1)\right) \wedge \left(HC(Open\ Sky, C_1)\right) \wedge HC(Central, C_1)$

$\left(\neg HC(Open\ Sky, C_1)\right) \wedge \left(HC(Open\ Sky, C_1)\right)$
()

d. Open Sky cannot both overlap with Central and not overlap with Central.
Central cannot both have channel 1 and not have channel 1.
Open Sky cannot both have channel 1 and not have channel 1.