

1) Naïve Bayes

- a. It is not likely to be true because they are most likely related in spam emails. If "viagra" is in the email, then the probability of "free" being in it increases; vice versa too. Mathematically, $P(A \cap B) = P(A|B)P(B)$ (whether A and B are independent or dependent), and $P(A|B) > P(A)$ because of what we said, so $P(A \cap B) > P(A)P(B)$, where A and B are the existence of these two words.
- b. $2n + 1$. For every word j of the n words, we need to calculate two things – $P(\vec{x}_{test,j} = 1|spam)$ and $P(\vec{x}_{test,j} = 1|not\ spam)$, thus the $2n$. We also need to calculate $P(spam)$, thus the $+1$.
- c. $2^{n+1} - 1$. Not assuming independence requires us to keep track of both cases of $\vec{x}_{test,j}$ for each of the n words. $P(x_{test,1}, \dots, x_{test,n}|c)$ thus has 2^n possibilities for each c . However, the sum of all these 2^n possibilities for each c is 1, so we can just do 1 minus the sum of the previous 2^{n-1} P 's to get the last P . That makes $2^n - 1$ calculations for each c . We need to do this for both spam and not spam. That's $(2^n - 1) * 2 = 2^{n+1} - 2$. We still need to compute $P(spam)$ once. $2^{n+1} - 2 + 1 = 2^{n+1} - 1$.
- d. Naïve Bayes has a much smaller runtime and is often good enough.
- e. $P(spam|"probability" = 1, "free" = 1, "viagra" = 1) = P(spam) \times P("probability"=1|spam) \times P("free"=1|spam) \times P("viagra"=1|spam) \times P("hello" = 0|spam) = 0$ because $P("probability"|spam) = 0$.

$$P(not\ spam|"probability" = 1, "free" = 1, "viagra" = 1) = P(not\ spam) \times P("probability"=1|not\ spam) \times P("free"=1|not\ spam) \times P("viagra"=1|not\ spam) \geq 0.$$

Non-spam

- f. $P(spam|"probability" = 1, "free" = 1, "viagra" = 1) = P(spam) \times P("probability"=1|spam) \times P("free"=1|spam) \times P("viagra"=1|spam) \times P("hello" = 0|spam) = 2.7 \times 10^{-5}$

$$P(spam) = \frac{1}{2}$$

$$P("probability"=1|spam) = \frac{0 + 1}{1000 + 2} = \frac{1}{1002}$$

$$P("free"=1|spam) = \frac{400 + 1}{1000 + 2} = \frac{401}{1002}$$

$$P("viagra"=1|spam) = \frac{300 + 1}{1000 + 2} = \frac{301}{1002}$$

$$P("hello" = 0|spam) = \frac{450 + 1}{1000 + 2} = \frac{451}{1002}$$

$$P(not\ spam|"probability" = 0, "free" = 0, "viagra" = 0) = P(not\ spam) \times P("probability"=1|not\ spam) \times P("free"=1|not\ spam) \times P("viagra"=1|not\ spam) = 2.33 \times 10^{-6}$$

$$P(not\ spam) = \frac{1}{2}$$

$$P("probability"=1|not\ spam) = \frac{20 + 1}{1000 + 2} = \frac{21}{1002}$$

$$P(\text{"free"}=1|\text{not spam}) = \frac{200 + 1}{1000 + 2} = \frac{201}{1002}$$

$$P(\text{"viagra"}=1|\text{not spam}) = \frac{10 + 1}{1000 + 2} = \frac{11}{1002}$$

$$P(\text{"hello"} = 0|\text{not spam}) = \frac{100 + 1}{1000 + 2} = \frac{101}{1002}$$

$2.33 \times 10^{-6} < 2.7 \times 10^{-5}$. **Spam.**

- g. The intuition behind the +2 in the denominator is that we start out as if all classes are equally likely ($\frac{1}{|V_j|}$), and there are 2 such classes, spam and not spam.

2) Logic

Predicate	True or False?
Overlap(Open Sky, MidCo)	F
Overlap(SpecCom, FiveCo)	T
Overlap(EastCom, MidCo)	T
Overlap(MidCo, MidCo)	F
Overlap(Central, MidCo)	T

- \wedge
- If two networks overlap, then they cannot use the same channel.
- $\neg \text{Overlap}(x, y) \vee (\neg \text{HasChannel}(x, c) \vee \neg \text{HasChannel}(y, c))$
 $= \neg \text{Overlap}(x, y) \vee \neg \text{HasChannel}(x, c) \vee \neg \text{HasChannel}(y, c)$
- $\forall_x \text{IsNetwork}(x) \left(\exists_c \text{IsChannel}(c) \left(\text{HasChannel}(x, c) \wedge \right. \right.$
 $\left. \left. (\forall_{d \neq c} \text{IsChannel}(d) (\neg \text{HasChannel}(x, d))) \right) \right)$
- Yes, it's satisfiable:
 Open Sky: 1
 SpecCom: 1
 Central: 3
 FiveCo: 2
 MidCo: 1
 EastCom: 2
- No, it's not satisfiable. If Open Sky, SpecCom, and EastCom all take channel 1, then that leaves all three of FiveCo, MidCo, and Central to choose between channels 2 and 3, but that's only two channels for three overlapping networks.

3) CNF and Resolution

- a. $(a \wedge b) \vee (c \wedge d)$ converted to CNF is $(a \vee c) \wedge (a \vee d) \wedge (b \vee c) \wedge (b \vee d)$

Statement	Value
HasChannel(Open Sky, C_1)	T
HasChannel(Open Sky, C_2)	F
HasChannel(Open Sky, C_3)	F
Overlap(Open Sky, Central)	T

- b. \wedge

c. $(\neg O(\text{Open Sky}, \text{Central}) \vee \neg HC(\text{Open Sky}, C_1) \vee \neg HC(\text{Central}, C_1)) \wedge (O(\text{Open Sky}, \text{Central})) \wedge (HC(\text{Open Sky}, C_1)) \models \neg HC(\text{Central}, C_1) ??$

$$(\neg O(\text{Open Sky}, \text{Central}) \vee \neg HC(\text{Open Sky}, C_1) \vee \neg HC(\text{Central}, C_1)) \wedge (O(\text{Open Sky}, \text{Central})) \wedge (HC(\text{Open Sky}, C_1)) \wedge HC(\text{Central}, C_1)$$

$$(\neg HC(\text{Open Sky}, C_1) \vee \neg HC(\text{Central}, C_1)) \wedge (HC(\text{Open Sky}, C_1)) \wedge HC(\text{Central}, C_1)$$

$$(\neg HC(\text{Open Sky}, C_1)) \wedge (HC(\text{Open Sky}, C_1))$$

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d. Conversion to CNF

Open Sky cannot both overlap with Central and not overlap with Central.

Central cannot both have channel 1 and not have channel 1.

Open Sky cannot both have channel 1 and not have channel 1.