

- The negation of a  $\neq$ -assignment is also an  $\neq$ -assignment because flipping the truth value of each literal inside  $(true \vee false \vee whatever)$  is  $(false \vee true \vee \neg whatever)$ ; the pair of literals that oppose each other is still opposing each other.
- To prove that this is NP-complete, we need to prove that it is both NP and NP-hard. It's obviously in NP because it's easy to verify in polynomial-time whether a potential answer is a  $\neq$ -assignment (see if each clause has a pair of opposing literals).

To prove it's NP-hard, we will reduce from any 3-SAT problem to obtain an answer for 3-SAT. The polynomial-time conversion from a 3-SAT problem  $\phi$  to a  $\neq$ -SAT problem  $\phi_{\neq}$  is as follows:

The  $i$ th clause in  $\phi$  looks like this:

$$(a_i \vee b_i \vee c_i)$$

We will transform it into two clauses to obtain  $\phi_{\neq}$  as such:

$$(a_i \vee b_i \vee new_i) \wedge (c_i \vee \neg new_i \vee false)$$

This is a polynomial time conversion because it's a constant time conversion (it takes 6 steps at most regardless of the clause).

As you can see, there's a new variable called  $new$  for each clause in the transformed formula, and  $false$  is a constant that's always false.

Claim:  $\phi$  is satisfiable if and only if  $\phi_{\neq}$  is  $\neq$ -satisfiable.

Proof:

Claim: if  $\phi$  is satisfiable, then  $\phi_{\neq}$  is  $\neq$ -satisfiable.

Proof: See this table for what  $new_i$  and  $w$  should be given  $\phi$ 's satisfying assignment (0 is  $false$  and 1 is  $true$ ); the last row is if the clause is unsatisfiable:

$a_i$	$b_i$	$new_i$	$c_i$	$\neg new_i$	if $false$ were a variable
1	1	0	1	1	0
1	1	0	0	1	$x$
1	0	0, 1	1	1, 0	0, $x$
1	0	0, 1	0	1, 0	$x$ , 1
0	1	0, 1	1	1, 0	0, $x$
0	1	0, 1	0	1, 0	$x$ , 1
0	0	1	1	0	$x$
0	0	1	0	0	1

Cells with two values correspond in order (e.g. 0, 1 in  $new_i$  would force  $\neg new_i$  to be 1, 0).  $x$  means whatever (could be either 0 or 1).

We set  $new_i$  to be always 0 unless both  $a$  and  $b$  are 0. In any case,  $false$  is always 0 (it's a constant) and it would make for a  $\neq$ -assignment if given a satisfying assignment for  $\phi$ .

Claim: if  $\phi$  is unsatisfiable, then  $\phi_{\neq}$  is also unsatisfiable.

Proof: If  $\phi$  is unsatisfiable, then  $\exists$  at least one clause where all three literals are false (i.e.  $a_i = 0, b_i = 0, c_i = 0$ ). This would force  $new_i$  to be 1, and of course  $\neg new_i$  to be 0. But *false* is a constant and is always 0, so the second clause would be all 0 and thus  $\neq$ -unsatisfiable, causing the entire  $\phi_{neq}$  to be  $\neq$ -unsatisfiable.