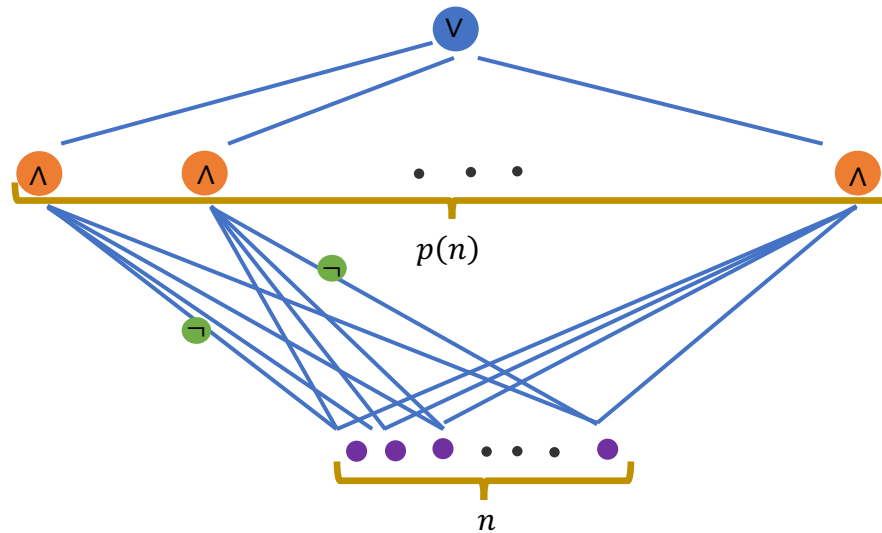


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The circuit for each  $n$  would look like this:



The blue gate at the top is an OR gate, all the orange gates are AND gates, each green gate is a NOT gate. The in-degree of each AND gate is  $n$ , as we can see by the group of  $n$  purple nodes (inputs to the circuit), and there are  $p(n)$  orange AND gates, and the top blue node tells us whether the input belongs to  $L \cap \{0,1\}^n$  – it is the output. The  $n$  purple nodes are the  $n$ -length input.

This circuit is correct because if the input  $x \in L \cap \{0,1\}^n$ , then it equals one of the  $p(n)$  accepted inputs, i.e. it's the first one OR the second one OR ... OR the  $p(n)$ -th one, and for  $x$  to equal  $y \in \{p(n) \text{ accepted inputs}\}$ , it has to be that  $(x_1 = y_1) \text{ AND } (x_2 = y_2) \text{ AND } \dots \text{ AND } (x_n = y_n)$ . There is a NOT gate between the  $x_i$  and the  $z$ -th AND gate if the  $z$ -th accepted input's  $i$ -th bit is 0.

This circuit is polynomial size because the number of edges is  $O(p(n) + 2 * (p(n) \cdot n))$ . There is a wire connecting the blue (output) gate and each of the  $p(n)$  AND gates, and there are at most  $p(n) \cdot n$  NOT gates, making us have at most  $2 \cdot p(n) \cdot n$  wires between the orange AND gates and the purple input nodes. QED.