To prove that TWICE-SAT is NP-complete, we need to prove that it is both NP and NP-hard.

It's easy to see why it is NP; given the two (or more) potential solutions, we can check their correctness in polynomial time by plugging in their values and computing the truth values of ϕ under them.

To see why it's NP-hard, assume for the sake of contradiction that there's a TWICE-SAT solver. We then show that 3-SAT (which is NP-hard) is reducible to TWICE-SAT, producing a contradiction. Given a regular 3-SAT ϕ, add an extra clause with three new literals x, y, and z to produce $\phi_2 = \phi \wedge (x, y, z)$. Solve for the two or more satisfying assignments of ϕ_2 using the TWICE-SAT solver. The solutions fall into either or both of the following categories:

1. The assignments differ in x, y, or z values.
2. The assignments differ in values of variables in ϕ

If the solutions fall *only* into case 1, then we know ϕ has only one satisfying assignment because those three new literals don't exist in ϕ; if the solutions fall into case two (with or without falling into case 1), then we know ϕ has more than one valid assignment (but that doesn't matter because we only need one in regular 3-SAT). Also note that this reduction is in polynomial time because we're just adding three new literals.

Claim: 3-SAT ϕ is satisfiable if and only if the corresponding TWICE-SAT ϕ_2 is satisfiable.
Proof:

Claim: If TWICE-SAT ϕ_2 is satisfiable with two or more assignments, then 3-SAT ϕ is satisfiable.
Proof: Take any of TWICE-SAT's solutions, discard the three new literals x, y, and z, and assign all truth values from that solution to variables in ϕ. This is a valid assignment to ϕ because the assignment to ϕ_2 makes all of ϕ_2's clauses true, and ϕ's clauses are a subset of ϕ_2's, so it makes ϕ true also.

Claim: If 3-SAT ϕ is satisfiable, then TWICE-SAT ϕ_2 is satisfiable.
Proof: Take 3-SAT's valid assignment and assign the values of its variables to their corresponding variables in ϕ_2. Then assign any of x, y, or z in ϕ_2 to true. It's obvious that ϕ_2 is true.

We have solved 3-SAT (which we know to be NP-hard) via a TWICE-SAT solver, a contradiction. Thus, our initial assumption that a TWICE-SAT solver exists is false. TWICE-SAT is thus NP-complete.