

Hand in your solutions electronically using CMS. Each solution should be submitted as a separate file. Collaboration is encouraged while solving the problems, but:

1. list the names of those with whom you collaborated;
2. you must write up the solutions in your own words;

Remember that when a problem asks you to design an algorithm, you must also prove the algorithm's correctness and analyze its running time. The running time must be bounded by a polynomial function of the input size.

The problem numbers below are from the **Exercise section in Chapter 1** from the online version of the Arora-Barak textbook which can be found at <https://theory.cs.princeton.edu/complexity/book.pdf>.

(1) (10 points) Problem 12

(2) (10 points) Problem 13.

(3) (10 points) Define a two dimensional Turing machine to be a TM where each of its tapes is an infinite grid (and the machine can move not only Left and Right but also Up and Down). Show that for every (time-constructible)  $T : \mathbb{N} \rightarrow \mathbb{N}$  and every Boolean function  $f$ , if  $f$  can be computed in time  $T(n)$  using a two-dimensional TM then  $f \in \mathbf{DTIME}(T(n)^2)$ .