Introduction to Computational Complexity	**Problem Set 3**
CS 4814, Spring 2020	**Due 11:59pm Tuesday, 18th February**

Hand in your solutions electronically using CMS. Each solution should be submitted as a separate file. Collaboration is encouraged while solving the problems, but:

1. **list the names of those with whom you collaborated;**
2. **you must write up the solutions in your own words;**

Remember that when a problem asks you to design an algorithm, you must also prove the algorithm's correctness and analyze its running time. The running time must be bounded by a polynomial function of the input size.

(1) *(10 points)* Define the language TWICE-SAT $= \{\phi$ is a 3CNF: ϕ has at least 2 satisfying assignments$\}$. Prove that TWICE-SAT is NP-complete.

(2) *(10 points)* Let ϕ be a 3CNF formula on n variables. An $\neq$-assignment $z \in \{0,1\}^n$ to the variables of ϕ is such that each clause contains two literals that evaluate to unequal truth values (under the assignment z).

1. Show that the negation of any $\neq$-assignment to ϕ is also an $\neq$-assignment.
2. Let $\neq$SAT be the collection of 3CNF formulas that have an $\neq$-assignment. Prove that $\neq$SAT is NP-complete.

(3) *(10 points)* A cut in an undirected graph $G = (V, E)$ is a partition of the vertices into two disjoint sets S and $\overline{S} = V \setminus S$. The size of the cut is the number of edges that have one endpoint in S and the other in $\overline{S}$. Define the language MAX-CUT$= \{\langle G, k\rangle : G$ has a cut of size at least $k\}$. Prove that MAX-CUT is NP-complete.

[**Hint**: Reduce $\neq$SAT (defined in Problem 2, HW3) to MAX-CUT. Let ϕ be a 3CNF formula on n variables with ℓ clauses. For each *literal* y, create 3ℓ nodes. Connect all nodes labelled x_i to all nodes labelled $\neg x_i$. For each clause C_i in ϕ, create a triangle by adding edges between literals appearing in the clause C_i. Do not use the same node for more than one clause. Note that the constructed undirected graph is on $6n$ nodes and has $9\ell^2 n + 3\ell$ edges.]