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Let us assume that $PSPACE = E$, then arrive at a violation of the time hierarchy theorem (i.e. a contradiction).

Let us define an arbitrary language L from $DTIME\left(n \cdot 2^{O(n)}\right)$. So, there exists a deterministic Turing machine M that decides L in $O\left(n \cdot 2^{O(n)}\right)$ time (which of course decides it in at most $O\left(n \cdot 2^{O(n)}\right)$ space).

Then let us define a language $L_{pad} = \left\{< x, 1^{2^{O(|x|)}} > s.t.\ x \in L\right\}$. We now create a deterministic Turing machine N to decide whether an input $y \in L_{pad}$:

> Does there exist a string z such that $y = < z, 1^{2^{O(|z|)}} >$? If not, then outright reject. If there is, then run $M(z)$ for $|z| \cdot 2^{O(|z|)}$ steps and output that answer.

Clearly, the space is PSPACE in $|y|$ because the input to L_{pad} is $|z| + 2^{O(|z|)}$ in length and the space of the algorithm is $|z| \cdot 2^{O(|z|)}$, which is $polynomial\left(|z| + 2^{O(|z|)}\right)$. Thus, $L_{pad} \in PSPACE$. Which in turn makes $L_{pad} \in E$ because we assumed $PSPACE = E$.

Now we want to show that $L \in E$ also. We use L_{pad} to do this. Given an input x, pad it with $2^{O(|x|)}$ number of 1's to create y, then run $N(y)$. The padding algorithm is E time because $2^{O(|x|)}$ is $E(|x|)$. And N runs in E time. Hence, L is overall decidable in E time. But L is an arbitrary $DTIME\left(n \cdot 2^{O(n)}\right)$ language, violating the time hierarchy theorem, because $2^{O(n)}$ is $o\left(n \cdot 2^{O(n)}\right)$.