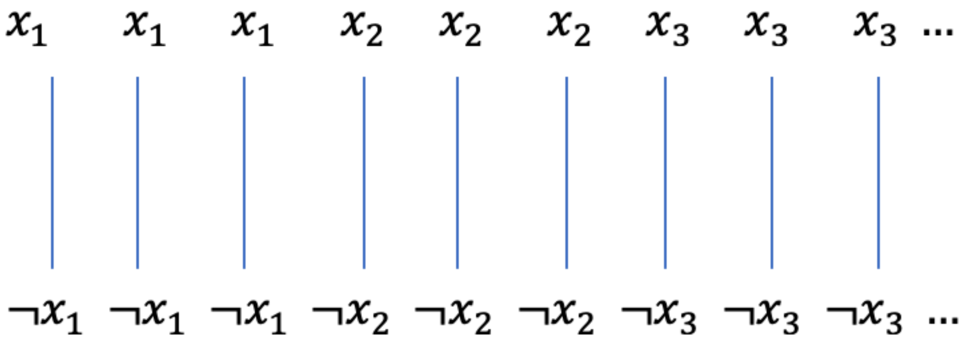
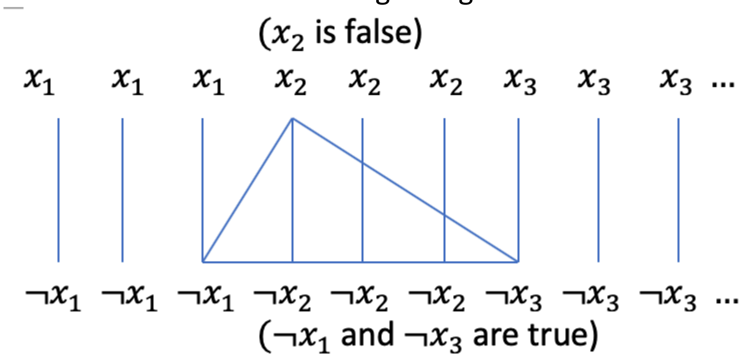
To prove MAX-CUT is NP-complete, we need to prove it's both NP and NP-hard. It's NP because it's easy to verify in polynomial time whether a potential solution is correct because we just need to see whether the cut goes through k edges that have one endpoint in S and the other in $\bar{S}$.
To prove it's NP-hard, we will reduce from a $\neq$-SAT problem ϕ as per the hint. This reduction is polynomial time because we're constructing $O(n)$ nodes and $O(l^2 n)$ edges.

Claim: There is a cut of size $n(3l)^2 + 2l$ on the transformed graph if and only if ϕ is $\neq$-satisfiable.
Proof:

Claim: If ϕ is $\neq$-satisfiable, then there exists a cut of size $n(3l)^2 + 2l$ on the transformed graph.
Proof: Visualize all the x_i nodes to be lined up horizontally on the upper side of the page and all the $\neg x_i$ nodes to be lined up horizontally on the lower side of the page as such:



If a literal a is true, put it in S; otherwise put it in $\bar{S}$. There are thus $n(3l)^2$ edges between all the x_i's and $\neg x_i$'s. The satisfying $\neq$-assignment makes it so that one or more but less than three literals in each clause is true. The resulting triangle thus has nodes on both sides, as such:



A cut for a satisfying $\neq$-assignment thus has the size $n(3l)^2 + 2l$. The $n(3l)^2$ is because of the edges between x_i's and $\neg x_i$'s, and the $2l$ is because each satisfied clause's triangle would get cut twice.

Claim: If there exists a cut of size $n(3l)^2 + 2l$, then ϕ is $\neq$-satisfiable.
Proof: Consider all the triangles getting cut. Try two things. The first thing to try is

Claim: If ϕ is $\neq$-unsatisfiable, then there does not exist a cut of size $n(3l)^2 + 2l$ or greater.
Proof: There would be at least one clause triangle with all three vertices entirely in S or entirely in $\bar{S}$. (because they all have the same truth value). The biggest cut for such a graph is at most $n(3l)^2 + 2(l - 1)$.