

Let $L \in L_1 \oplus L_2$. We want to prove that $L \in \text{NP} \cap \text{coNP}$. First of all, if L_1 and L_2 are in NP and coNP, then that means there exist non-deterministic Turing machines that decide L_1 , $\overline{L_1}$, L_2 , and $\overline{L_2}$ in polynomial time. We name these machines 1M, $\overline{1M}$, 2M, and $\overline{2M}$, respectively. Such machines exist because of the aforementioned definition of NP, and that the definition of a language being coNP is if its complement is in NP (i.e. if its complement can be decided by such a machine). This gives rise to our following algorithm:

To show $L \in \text{NP}$:

If $((L \in L_1) \wedge (L \notin L_2))$, i.e. $(L \in L_1) \wedge (L \in \overline{L_2})$, we output yes if and only if 1M accepts and $\overline{2M}$ accepts. Then if $((L \in L_2) \wedge (L \notin L_1))$, i.e. $(L \in L_2) \wedge (L \in \overline{L_1})$, we output yes if and only if $\overline{1M}$ accepts and 2M accepts.

Similarly, to show that $L \in \text{coNP}$:

If $((L \in L_1) \wedge (L \notin L_2))$, we output yes if and only if $\overline{1M}$ accepts and $\overline{2M}$ accepts. Then if $((L \in L_2) \wedge (L \notin L_1))$, we output yes if and only if 1M accepts and $\overline{2M}(L)$ accepts.