

If a 3CNF is unsatisfiable, then that means every single one of its 2^n assignments makes the 3CNF false. If an assignment is satisfying, that means all $10n$ of its clauses are true. The probability of a clause being false under an assignment is $\frac{1}{8}$, as all three literals have to be false, so the probability of a clause being true under an assignment is $\frac{7}{8}$. The probability that a 3CNF is satisfiable under an assignment is thus

$$\left(\frac{7}{8}\right)^{10n}$$

To get the expectation of N_ϕ , we do

$$\sum_{i=1}^{2^n} i \times S_i$$

Where S_i is the probability that there are exactly i satisfying assignments for the formula. This is equal to

$$\sum_{i=1}^{2^n} i \times \binom{2^n}{i} \times \left(\left(\frac{7}{8}\right)^{10n}\right)^i \times \left(1 - \left(\frac{7}{8}\right)^{10n}\right)^{10n-i}$$

Because when i assignments are satisfying, the remaining $10n - i$ assignments are not. This is upper bounded by

$$N_\phi \leq 2^n \times \left(1 - \left(1 - \left(\frac{7}{8}\right)^{10n}\right)^{10n-1}\right)$$

Which approaches 0 as n approaches infinity. We use Markov's inequality: $P(X \geq 1) \leq \frac{E(X)}{1}$ where X is the number of satisfying assignments. $E(X) = N_\phi$. There exists a $\delta > 0$ that satisfies $N_\phi \leq \frac{1}{2^{-\delta n}}$ when $n \geq 7$. QED.