To prove that TWICE-SAT is NP-complete, we need to prove that it is both NP and NP-hard.

It’s easy to see why it is NP; given the two (or more) potential solutions, we can check their correctness in polynomial time by plugging in their values and computing the truth values of under them.

To see why it’s NP-hard, assume for the sake of contradiction that there’s a TWICE-SAT solver. We then show that 3-SAT (which is NP-hard) is reducible to TWICE-SAT, producing a contradiction. Given a regular 3-SAT , add an extra clause with three new literals , , and to produce . Solve for the two or more satisfying assignments of using the TWICE-SAT solver. The solutions fall into either or both of the following categories:

1. The assignments differ in , , or values.
2. The assignments differ in values of variables in

If the solutions fall *only* into case 1, then we know has only one satisfying assignment because those three new literals don’t exist in ; if the solutions fall into case two (with or without falling into case 1), then we know has more than one valid assignment (but that doesn’t matter because we only need one in regular 3-SAT). Also note that this reduction is in polynomial time because we’re just adding three new literals.

Claim: 3-SAT is satisfiable if and only if the corresponding TWICE-SAT is satisfiable.

Proof:

Claim: If TWICE-SAT is satisfiable with two or more assignments, then 3-SAT is satisfiable.

Proof: Take any of TWICE-SAT’s solutions, discard the three new literals , , and , and assign all truth values from that solution to variables in . This is a valid assignment to because the assignment to makes all of ’s clauses true, and ’s clauses are a subset of ’s, so it makes true also.

Claim: If 3-SAT is satisfiable, then TWICE-SAT is satisfiable.

Proof: Take 3-SAT’s valid assignment and assign the values of its variables to their corresponding variables in . Then assign any of or in to true. It’s obvious that is true.

We have solved 3-SAT (which we know to be NP-hard) via a TWICE-SAT solver, a contradiction. Thus, our initial assumption that a TWICE-SAT solver exists is false. TWICE-SAT is thus NP-complete.