1. The negation of a -assignment is also an -assignment because flipping the truth value of each literal inside is ; the pair of literals that oppose each other is still opposing each other.
2. To prove that this is NP-complete, we need to prove that it is both NP and NP-hard. It’s obviously in NP because it’s easy to verify in polynomial-time whether a potential answer is a -assignment (see if each clause has a pair of opposing literals).

To prove it’s NP-hard, we will reduce from any 3-SAT problem to obtain an answer for 3-SAT. The polynomial-time conversion from a 3-SAT problem to a -SAT problem is as follows:

The th clause in looks like this:

We will transform it into two clauses to obtain as such:

This is a polynomial time conversion because it’s a constant time conversion (it takes 6 steps at most regardless of the clause).

As you can see, there’s a new variable called for each clause in the transformed formula, and is a constant that’s always false.

Claim: is satisfiable if and only if is -satisfiable.

Proof:

Claim: if is satisfiable, then is -satisfiable.

Proof: See this table for what and should be given ’s satisfying assignment ( is and is ); the last row is if the clause is unsatisfiable:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | if were a variable |
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|  |  |  |  |  |  |

Cells with two values correspond in order (e.g. in would force to be ). means whatever (could be either or ).

We set to be always unless both and are . In any case, is always (it’s a constant) and it would make for a -assignment if given a satisfying assignment for .

Claim: if is unsatisfiable, then is also unsatisfiable.

Proof: If is unsatisfiable, then at least one clause where all three literals are false (i.e. ). This would force to be , and of course to be . But is a constant and is always , so the second clause would be all and thus -unsatisfiable, causing the entire to be -unsatisfiable.