1. Let us negate the goal and assume the negated goal is correct to prove the contrapositive. The negated goal is “, ” because is . Since is a polynomial time reduction, we can reduce any SAT problem to its shorter, reduced version . We can keep doing this until the result is of length . Now we can solve this in time, which is time. This makes NPP, and thus P=NP because PNP and we can reduce any NP problem to SAT, and a series of polynomial time reductions is still polynomial time. This proves the contrapositive. QED.
2. Let us negate the goal and assume the negated goal is correct. The negated goal is “SATH is NP-complete.” If SAThhH is NP-complete, then we can reduce SAT to SATH. This is obviously a polynomial time reduction because we’re just padding with a known number of ’s.