1. Let us negate the goal and assume the negated goal is correct. The negated goal is “, ” because is . Since is a polynomial time reduction, then we can reduce any SAT problem to its shorter reduction . We can keep doing this until the result is of length . Now we can solve this in time, which is time. This makes because we can reduce any NP problem to SAT, and a series of polynomial time reductions is still polynomial time. This contradiction we arrive at proves our initial assumption of the negated goal is wrong. Thus, if P=NP, then such that . QED
2. Let us negate the goal and assume the negated goal is correct. The negated goal is “SATH is NP-complete.” If SAThhH is NP-complete, then we can reduce SAT to SATH. This is obviously a polynomial time reduction because we’re just padding with a known number of ’s.