Let us assume that , then arrive at a violation of the time hierarchy theorem (i.e. a contradiction).

Let us define an arbitrary language from . So, there exists a deterministic Turing machine that decides in time (which of course decides it in at most space).

Then let us define a language . We now create a deterministic Turing machine to decide whether an input :

Does there exist a string such that ? If not, then outright reject. If there is, then run for steps and output that answer.

Clearly, the space is PSPACE in because the input to is in length and the space of the algorithm is , which is . Thus,. Which in turn makes because we assumed .

Now we want to show that also. We use to do this. Given an input , pad it with number of ’s to create , then run . The padding algorithm is time because is . And runs in time. Hence, is overall decidable in time. But is an arbitrary language, violating the time hierarchy theorem, because is .