Let us assume that , then arrive at a violation of the space hierarchy theorem (i.e. a contradiction).

Let us define an arbitrary language from . So, there exists a deterministic Turing machine that decides in space (which decides it in at most time).

Then let us define a language . We now create a deterministic Turing machine to decide whether an input :

Does there exists a string such that ? If not, then outright reject. If there is, then run for steps and output that answer.

Clearly, the space is linear in because the input to is in length and the space of the algorithm is , which is . Since runs in , because of our assumption that .

Now we want to show that also. We use and to do this. Given an input , pad it with number of ’s to create , then run . The padding algorithm is obviously deterministic polynomial time because is . And runs in time. Since , is thus decidable in time, which makes it decidable in . But is an arbitrary language from , violating the space hierarchy theorem, because is