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The circuit for each would look like this:



The blue gate at the top is an OR gate, all the orange gates are AND gates, each green gate is a NOT gate. The in-degree of each AND gate is , as we can see by the group of purple nodes (inputs to the circuit), and there are orange AND gates, and the top blue node tells us whether the input belongs to – it is the output. The purple nodes are the -length input.

This circuit is correct because if the input , then it equals one of the accepted inputs, i.e. it’s the first one OR the second one OR … OR the -th one, and for to equal , it has to be that . There is a NOT gate between the and the -th AND gate if the -th accepted input’s i-th bit is 0.

This circuit is polynomial size because the number of edges is . There is a wire connecting the blue (output) gate and each of the AND gates, and there are at most NOT gates, making us have at most wires between the orange AND gates and the purple input nodes. QED.