a. The probability of the output N being actually prime is the "probability of output N is actually prime *given* that Rabin-Miller-k says it's 'probably prime'". Let E denote the event that "Rabin-Miller-k says N is 'probably prime'" and A denote the event that "N *is* prime". The problem is now formally to show $P(A|E) \geq \frac{2^k}{2^k+logP}$.

Baye's Rule: $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

Use it:

$$P(A|E) = \frac{P(E|A)P(A)}{P(E)}$$

The Total Probability Rule: $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$

Use it to rewrite $P(E)$:

$$P(E) = P(E|A)P(A) + P(E|A^c)P(A^c)$$

(A^c means the complement of A, so, "N is composite".)

$$P(E|A) = 1$$
$$P(A) = \frac{1}{\log P}$$
$$P(E|A^c) \leq \frac{1}{2^k}$$
$$P(A^c) = 1 - \frac{1}{\log P}$$

Bringing it all together:

$$P(A|E) \geq \frac{\frac{1}{\log P}}{\frac{1}{\log P} + \frac{1}{2^k}\left(1 - \frac{1}{\log P}\right)} = \frac{\frac{1}{\log P}}{\frac{1}{\log P} + \frac{1}{2^k} - \frac{1}{2^k \log P}}$$

Multiply by $\frac{\log P}{\log P} * \frac{2^k}{2^k}$:

$$P(A|E) \geq \frac{2^k}{2^k + \log P - 1} \geq \frac{2^k}{2^k + \log P}$$

b.

$$\frac{2^k}{2^k + \log P} \geq 0.99$$

But make it an equal sign for now

$$\frac{2^k}{2^k + \log P} = 0.99$$
$$0.99(2^k + \log P) = 2^k$$
$$0.99 \log P = 2^k - 0.99 * 2^k = 2^k(1 - 0.99) = 0.01 * 2^k$$

$$\frac{0.99 \log P}{0.01} = 2^k = 99 \log P$$

$$\log 2^k = \log(99 \log P)$$

$$k \log 2 = \log 99 + \log(\log P)$$

$$k = \frac{\log 99 + \log(\log P)}{\log 2} = 6.63 + \log_2(\log P)$$

$$\boldsymbol{k \geq 6.63 + \log_2(\log P)}$$