The Algorithm

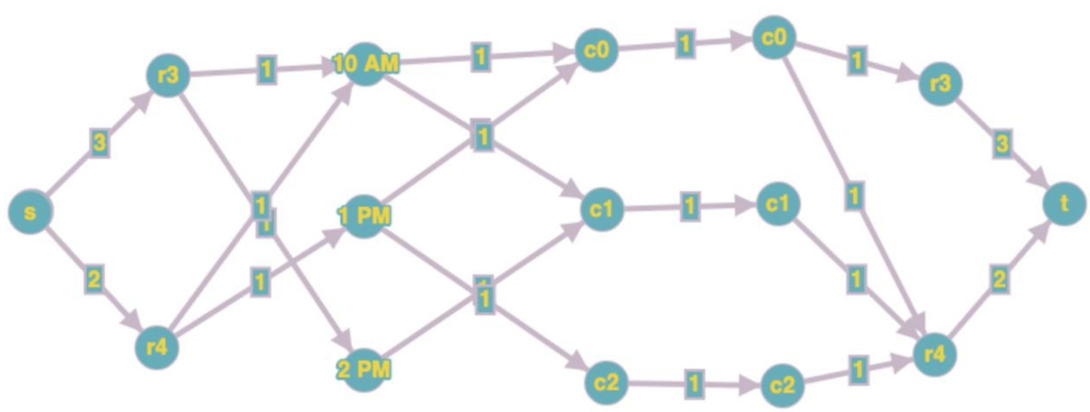
We reduce this to a max flow problem, where one unit of flow means one interview. We construct a graph. There is a source s, sink t, and five layers between.

1. The first layer has k nodes for the k recruiters
2. the second layer has T nodes for the T timeslots
3. the third layer has n nodes for the n candidates
4. the fourth layer has n nodes for the candidates again
5. the fifth layer has k nodes for the k recruiters again

1. There is a directed edge (s, r_p) of capacity c_p from s to every node r_p in the first layer, with c_p as defined in the problem – the interviewing capacity of recruiter p.
2. There is a directed edge (r_p, h_j) of capacity 1 from the first to second layer if recruiter p is available at timeslot j.
3. There is a directed edge (h_j, c_i) of capacity 1 from the second to third layer if candidate i is available at timeslot j.
4. There are n directed edges (c_i, c_i) of capacity 1 from the third to fourth layer. There is a directed edge (c_i, r_p) of capacity 1 from the fourth to fifth layer if recruiter p is qualified to interview candidate i.
5. There is a directed edge (r_p, t) of capacity c_p from every node in the fifth layer to t.

Run Ford-Fulkerson on this graph. If the max flow is less than the number of candidates n, then there is no interview schedule where every candidate gets interviewed; if the max flow is equal to n, then there is.

Example: there are three candidates: c_0, c_1, and c_2. There are two recruiters: r_3 and r_4. c_0 could be interviewed by r_3 and r_4 and is available at 10 AM and 1 PM. c_1 could be interviewed by r_4 and is available at 10 AM and 2 PM. c_2 could be interviewed by r_4 and is available at 1 PM. r_3 has capacity 3 and is available at 10 AM and 2 PM. r_4 has capacity 2 and is available at 10 AM and 1 PM. The constructed graph for this problem would look as such:



Recruiters r3 and r4, candidates c0, c1, and c2, and their time availabilities and eligibilities

Proof of Correctness

This problem is under several constraints:

1. Each candidate gets at most 1 interview
   ⇒This is satisfied by the edges from layer 3 to layer 4, which all have capacity 1.
2. Each candidate can only be interviewed at a timeslot he is available for
   ⇒This is satisfied by the edges from layer 2 to layer 3, which only exist if the candidate is available at the time slot the edge is connecting him to
3. Each candidate can only be interviewed by qualified recruiters
   ⇒This is satisfied by the edges from layer 4 to layer 5, which only exist if the recruiter is qualified to interview the candidate the edge is connecting her to
4. Each recruiter r_p conducts at most c_p interviews
   ⇒This is satisfied by the edges from s to layer 1 and from layer 5 to t, which have capacities c_p
5. Each recruiter can only conduct interviews at timeslots she is available for
   ⇒This is satisfied by the edges from layer 1 to layer 2, which only exist if the recruiter is available at the time slot the edge is connecting her to
6. Each timeslot can have multiple interviews, but they must be of different recruiters *and* different candidates, i.e. a timeslot can't have one recruiter interviewing more than one candidate and a timeslot can't have a candidate interviewed by more than one recruiter.
   ⇒This is satisfied by the edges between layers 1 and 3 all having capacities 1, meaning if a flow of value 1 goes into a node in layer 2, then only 1 can flow out (thus one recruiter only interviewing one candidate), and if a flow of value 1 goes out from layer 2, that means a flow of 1 went in (thus one candidate only have one recruiter)

Runtime Analysis

The runtime of Ford-Fulkerson is $O(mC)$, where m is the number of edges in the graph and C is the sum of the capacities of all edges leaving the source. m here is $k * (kT) * (nT) * n * m * k$. C here is the sum of all recruiter capacities, which is kn in the worst case (all recruiters can interview every candidate). This algorithm is thus $O(k^4 T^2 n^3 m)$, which is polynomial is all the variables involved.