The Algorithm

1. Add a new node s_1
2. Add a new node s_2
3. Add a directed edge (s_1, s) with capacity $\min(C_1, C)$
4. Add a directed edge (s_2, s) with capacity $\min(C_2, C)$
5. Run Ford-Fulkerson on this new graph with source s_1 and sink t_1
6. Let $fMax_1$ equal the maximum flow from that
7. Run Ford-Fulkerson on this new graph with source s_2 and sink t_2
8. Let $fMax_2$ equal the maximum flow from that
9. Remove s_1 and s_2 and their outgoing edges
10. Add new node s_3
11. Add a directed edge (s_3, s) with capacity $C_1 + C_2$
12. add a new node t_{new}
13. direct one edge from each of the two old sinks to t_{new}
14. recall that $C = \sum_{e\ leaving\ s} c_e$
15. let $C_{half} = \left\lceil \frac{C_1 + C_2}{2} \right\rceil$ (e.g. $C_{half} = 3$ when the numerator 6, $C_{half} = 4$ when the numerator is 7)
16. let $C_{smaller} = \min(C_{half}, fMax_1 + 1, fMax_2 + 1)$
17. make the capacities of each of the two new directed edges we made in step 3 $C_{smaller}$
18. run the Ford-Fulkerson algorithm on this new graph with source s_3 and sink t_{new}.
19. Let $vMax_1$ and $vMax_2$ equal the flows on edges (t_1, t_{new}) and (t_2, t_{new}), respectively. They are the fair flows.

Proof of Correctness

Observe that the most flow that could go into a sink i is the minimum between the sum of the capacities of edges that go out from the source (C) and the sum of the capacities of edges that go into said sink (C_i). We could thus restrict C to this minimum to save time, which is implemented in steps 1 through 4. Also observe that $vMax_i \leq fMax_i$ for $i = 1,2$ because $fMax_i$ is computed when there's no constraints put on the problem. So, for the flows to be fair, both flows cannot exceed the smaller of the $fMax_i$'s by more than 1. If both of them are greater than C_{half}, then the fair flows have to be upper bounded by C_{half}. C_{half} is defined like that because when C is even, then obviously each sink cannot exceed $\frac{C}{2}$, but if C is odd, say 7, then the fair flows could be 3 and 4, which are upper bounded by $\left\lceil \frac{C}{2} \right\rceil = 4$. Thus, the fair flows to the two sinks have to be restricted to the three-way minimum in step 16, which is implemented in step 17. The explanation for step 11 is that the total flow that goes into t_3 cannot exceed the $C_1 + C_2$

Runtime Analysis

Recall that the runtime complexity of Ford-Fulkerson is $O(mC)$ with m being the number of edges and C as defined in step 14. Step 5 is $O(mC_1)$ and step 7 is $O(mC_2)$ because we made the new sources' (s_1 and s_2) outgoing capacity to C_1 and C_2. Step 18 is $O(m(C_1 + C_2))$ because we restricted the s_3's outgoing capacity to $C_1 + C_2$. The runtime overall is thus $O(mC_1) + O(mC_2) + O\big(m(C_1 + C_2)\big) = O(m(C_1 + C_2))$.