CS 6817 HW 2

2.38, 2.46, 2.56 (a) and (b)

# 2.38

We use this formula of stability:

The formula for Fourier coefficients of the Tribes function is given in the textbook on page 97:

There are tribes with members each. For each , we write it as , where is the intersection of with the i-th tribe:

Since the we need in the stability formula:

If we define , then the above could be rewritten as

Since when . Notice both instances have in common.

So,

We want to simplify something:

So,

The Piazza hint says to take asymptotics into account. I’m not sure, but so maybe

# 2.46

The Mean Value Theorem for a function continuous in the domain interval is:

We propose that

So, for some :

Which means we have to show

We differentiate with respect to the noise parameter :

Because when , the expression inside the summation is anyway.

Now we set and employ knowledge from the previous exercise, 2.45:

Which makes

Now we need to prove

Well, since and ,

Thus,

QED.

# 2.56

## (a)

Ideally, we want , which means ideally, , since . Maximizing the absolute value is equivalent to maximizing the square, so we try to maximize .

Due to the noise operator formula.

We know that

So

Now we use the Cauchy-Schwarz inequality. We set

So,

Left side is

Because of Parseval’s identity.

So we want to maximize

Because , . is maximized when is as small as possible. And since we can’t have a trivial solution, they have to be greater than , so , **so and must be dictator functions**:

or for some

and

or for some

And we need

To be positive and as large as possible. We need it to be positive because we squared it during the Cauchy-Schwarz process. To make this as likely as possible, we need the two dictator functions to be dictated by the same bit because we have no guarantees about choosing two different bits and since they’re independent of each other; and we need them to be either both positive dictators or both negative dictators because , whereas , and because and are actually more likely to be equal than unequal:

(Because they’re equal when they both get right or both get wrong)

(Because they’re unequal when one of them gets right and the other gets it wrong)

Thus, the functions themselves need to be:

and for some bit

or

and for some bit

i.e. dictated by the same bit, and both multiplied by or .

## (b)

We strive for a similar objective as in part a of this problem, but this time it’s 3 objectives:

Maximize

Maximize

Maximize

We do the steps we did above for each of these, giving two possible results for each:

and for some bit

or

and for some bit

and

and for some bit

or

and for some bit

and

and for some bit

or

and for some bit

And because we have to link the three together, has to equal has to equal . QED.