Analysis of Algorithms	**Problem Set 3**
CS 6820, Fall 2020	**Due Friday, November 6, 6:00pm**

In solving the following problems you are allowed to use, without proof, any facts from the lectures or assigned readings in the Fall 2020 version of CS 6820. Hand in your solutions electronically using CMS. Scanned PDF files are allowed, if you prefer to hand-write your solutions. Each group is responsible for submitting just one solution set.

(1) *(10 points)* The *support* of a vector $x \in \mathbb{R}^n$ is the set of indices $i \in [n]$ such that $x_i \neq 0$. We say that x is s-sparse if its support has at most s elements. In the SPARSE LINEAR SYSTEM SOLUTION problem, one is given a natural number s, a matrix A, and a vector b. (Assume A and b have integer entries, specified in the input as binary numbers.) The problem is to decide whether there exists an s-sparse vector x that satisfies $Ax = b$. Prove that SPARSE LINEAR SYSTEM SOLUTION is NP-Complete.

Extra credit question: Prove that SPARSE LINEAR SYSTEM SOLUTION remains NP-Complete if we restrict the problem by requiring all the entries of A and b to belong to $\{-1, 0, 1\}$.

(2) *(10 points)* Some of your friends are co-founders of UnfortuNet, an Internet service provider that had a promising business plan but keeps running up against NP-Complete problems in their day-to-day operations. Lately they've been experimenting with some new routing software, but unfortunately it's incompatible with the old version of the protocol. Thus, they want to subdivide their network into a "live network" running the old software, and a "test network" running the new version. Here are the constraints they face.

1. There's one particular router, v_1, that **must** belong to the live network.
2. There's another particular router, v_2, that **must** belong to the test network.
3. There's an integer $k > 0$, such that the test network must contain exactly k routers. All others will belong to the live network, i.e. every router belongs to exactly one of L or T.
4. The live network L and the test network T must both be connected subgraphs of G. In other words, every two routers in L must be joined in G by a path consisting only of routers in L, and every two routers in T must be joined in G by a path consisting only of routers in T.

The TEST NETWORK DESIGN problem is the following computational problem: given an undirected graph G, two nodes v_1, v_2, and a desired test network size k, decide whether there exist a live network L and test network T which satisfy (1)-(4). Prove that TEST NETWORK DESIGN is NP-Complete.

(3) *(10 points)* In the SET COVER WITH RECTANGLES problem, one is given:

1. a finite set of points $x_1, x_2, \ldots, x_n \in \mathbb{R}^2$
2. a finite set of axis-parallel rectangles $R_1, R_2, \ldots, R_m \subset \mathbb{R}^2$
3. a natural number k.

Assume that the points x_i and the corners of the rectangles R_j all have integer coordinates, and that the input specifies these integers as binary numbers. The problem is to decide whether there exist k of the given rectangles whose union contains all of the points $x_1, \ldots, x_n$. Prove that SET COVER WITH RECTANGLES is NP-complete.