CS 6820

HW 2

Question 4

We will solve this using dynamic programming.

We defined to mean the amount of garbage that piled up in town from days to , inclusive.

We define to mean the previous time before day that the garbage got disposed in town .

We define to mean true or false, whether there exists an admissible garbage disposable schedule from day to (inclusive) in which the garbage is disposed on day in town .

Before we mathematically define the recurrence scheme, we should explain it in plain English. The recurrence scheme should be true if and only if ALL of the following is true:

* the amount of garbage piled up in town after the previous disposal and up until YESTERDAY (day ) is below (remember we’re disposing town ’s garbage today)
* the garbage piled up in all the other towns after their previous disposal up until TODAY (day ) is below (remember we’re not disposing the other towns’ garbage today)
* every town had an admissible schedule every day up until the days their garbage previously got disposed

We now put this mathematically:

Eventually, to know if there’s an overall admissible schedule over the span of all days, we need this to be true:

i.e. such that is true.

# Algorithm and Runtime Analysis

One can easily imagine that is a 2D array, with each row signifying a town and each column signifying a day. Let’s say there’s towns and days, then the problem is solved after all cells in the 2D array is populated, and you look at and backtrack on to get the garbage truck schedule; this process is . You fill the array in column by column, left to right. Each cell takes time to fill out because you need to iterate over all the other towns. Altogether, the algorithm takes time .

# Proof of Correctness

Want to prove is true.

## Base Case

because everyone together produces unit of garbage, and .

## Induction

Assume , want to prove . From the assumption, we know this is true by plugging in and :

Assume for the sake of contradiction that is false, which means .