1) .

   a. Von Neumann's definition of ordinal: An *ordinal* is a set *y* that is transitive, connected, and well founded.

   b. Claim: the ordinals are well ordered by $\in$.
   Proof: For a set to be well ordered, every non-empty subset has to have a least element, which in our case is an $\in$-minimal element. This is well foundedness. Consider any nonempty set A of ordinals. Let $a \in A$ be an arbitrary element of A. If $a \cap A = \emptyset$, then a is an $\in$-minimal element of A and we're done; if not, we need to show A has an $\in$-minimal element. We consider $B = a^+ \cap A$, where $a^+ = a \cup \{a\}$ is the immediate successor of a. We know that a^+ is also an ordinal (Proposition 9.1). We know that there's an $\in$-minimal b element in B because a^+ is an ordinal, and we claim that b is also the $\in$-minimal element in A. So, we need to prove that $\forall a' \in A$, either $b = a'$ or $b \in a'$. Regarding a^+ and a', we know that it's one of three cases: $a' \in a^+$, $a^+ = a'$, or $a^+ \in a'$. If $a' \in a^+$, then we know $a' \in B$, so either $b = a'$ or $b \in a'$ because b is the $\in$-minimal element of B. For other two cases of if $a^+ = a'$ or $a^+ \in a'$, then $a^+ \subseteq a'$ due to transitivity of an ordinal, and because $b \in a^+$, it is so that $b \in a'$. QED.

   Claim: the ordinals are not a set.
   Proof: Assume for the sake of contradiction that there were a set O of all ordinals, then O would be an ordinal itself (Theorem 8.5), so $O \in O$, violating the well foundedness of ordinals, which is a contradiction.

2) .

   a. Axiom of choice: For every set A of nonempty sets there is a function f with domain A such that $\forall x \in A$, $f(x) \in x$.
   b. The 1-1 mapping $g: \alpha \rightarrow A$, where α is an ordinal and A is any set. Let us set z to some set not in A. By the Axiom of Choice, there is a choice function $f: P(A) \rightarrow \bigcup P(A)$ (where $P(A)$ is the power set of A) such that, for all nonempty $B \subseteq A$, $f(B) \in B$. The functional property will be defined inductively (transfinite induction).
   Base case: $g(0) = f(A)$.
   Inductive case: Suppose $g(\beta)$ has been defined for all $\beta \in \alpha$. If $A \setminus \{g(\beta) | \beta \in a\}$ is empty, then $g(\alpha) = z$; otherwise $g(\alpha) = f(A \setminus \{g(\beta) | \beta \in a\})$. It is clear that $\forall a \in A$, there exists at most one ordinal α such that $g(\alpha) = a$. Thus, g is 1-1.

3) Consider if g's domain was the collection of all ordinals, then g would be a 1-1 mapping from the collection of all ordinals into A. If there were no α such that $g(\alpha) = z$, then g^{-1} (the inverse of g) would be a $1-1$ mapping from A into the collection of all ordinals, making the collection of all ordinals a set by the Replacement Axiom, contradicting what we proved in question 1 part b.

4) Thus, there must exist an α who is the least ordinal such that $g(\alpha) = z$. We already proved in question 2 that $g|_\alpha$ (restriction of g to α) is 1-1. We now claim that g's range is all of A as required to prove the initial claim that every set A is well orderable. If g's range does not include all of A (assuming that for the sake of contradiction), then $A \setminus \{g(\beta) | \beta \in \alpha\}$ is not empty, and according to how we defined g earlier, $g(\alpha) = f(A \setminus \{g(\beta) | \beta \in \alpha\})$, which is an element of A, a contradiction.

   Since there's some ordinal mapped 1-1 onto A, and ordinals are well ordered A can be well ordered. QED.