Exercises 32, 34a, 39, 40, 41 from Boolean algebra notes

Exercise 32)

1. We have to show the Cartesian product of partially ordered sets and is reflexive, transitive, and antisymmetric.
2. Reflexive: want to prove . We know that and are partial orders, so they must be reflexive, so and . And by the definition of partial order on Cartesian products,
3. Transitive: want to prove and if and , then . From the assumption and givens, we know that and and and , which means and . And thus .
4. Antisymmetry: want to prove and , if and , then . Once again, from the assumptions and the givens, this means abd , which means , and and , which means . Thus, .