1. .
   1. Von Neumann’s definition of ordinal: An *ordinal* is a set *y* that is transitive, connected, and well founded.
   2. Claim: the ordinals are well ordered by .

Proof: For a set to be well ordered, every non-empty subset has to have a least element, which in our case is an -minimal element. This is well foundedness. Consider any nonempty set of ordinals. Let be an arbitrary element of . If , then is an -minimal element of and we’re done; if not, we need to show has an -minimal element. We consider , where is the immediate successor of . We know that is also an ordinal (Proposition 9.1). We know that there’s an -minimal element in because is an ordinal, and we claim that is also the -minimal element in . So, we need to prove that , either or . Regarding and , we know that it’s one of three cases: , , or . If , then we know , so either or because is the -minimal element of . For other two cases of if or , then due to transitivity of an ordinal, and because , it is so that . QED.

Claim: the ordinals are not a set.

Proof: Assume for the sake of contradiction that there were a set of all ordinals, then would be an ordinal itself (Theorem 8.5), so , violating the well foundedness of ordinals, which is a contradiction.

1. .
   1. Axiom of choice: For every set of nonempty sets there is a function with domain such that , .
   2. The 1-1 mapping , where is an ordinal and is any set. By the Axiom of Choice, there is a choice function (where is the power set of ) such that, for all nonempty , . The functional property will be defined inductively (transfinite induction).

Base case: .

Inductive case: Suppose has been defined for all If is empty, stop; otherwise . It is clear that , there exists at most one ordinal such that . is thus injective, i.e. 1-1.