1. is **primitive recursion**:

Text

Description automatically generated

In our case, is .

When ,

which is equivalent to

Say there’s a projection function

(which is of course primitive recursive), then

QED.

1. This is a 2-parameter function. Let’s call it We will do induction on .

Base case:

Which is a projection function, which we know is primitive recursive.

Inductive case. Assuming is primitive recursive, prove is also primitive recursive:

Which equals

And we just proved that is primitive recursive. This is a composition schema. QED.

1. We define

Because if is the min, then , and ; similarly, if is the min, then , so it will also return correctly. is thus primitive recursive by composition. QED.

1. can be done via an algorithm similar to mergesort:

As one can see, this is recursive. The base case is projection of one variable onto the variable itself (which is primitive recursive), and the inductive case is primitive recursive by composition. QED.

1. We define

Because if is the max, then , and it returns ; similarly, if is the max, then , and , which is correct. is thus primitive recursive by composition of and .