1. is **primitive recursion**:

Text

Description automatically generated

In our case, is .

When ,

which is equivalent to

Say there’s a projection function

(which is of course primitive recursive), then

QED.

1. This is a 2-parameter function. Let’s call it We will do induction on .

Base case:

Which is a projection function, which we know is primitive recursive.

Inductive case. Assuming is primitive recursive, prove is also primitive recursive:

Which equals

And we just proved that is primitive recursive. This is a composition schema. QED.

1. We define

Because if is the min, then , and ; similarly, if is the min, then , so it will also return correctly. is thus primitive recursive by composition. QED.