Madagascar Project Report

SAVING WITH LCC PROJECTION

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1. INTRODUCTION

Lillybank Tours previously designed a sightseeing flight path in Madagascar using the Mercator projection. The path starts in Antsiranana in the north, passes through Tsingy De Bemaraha National Park, and ends in Fararagana in the southeast. However, the Mercator projection, known for its conformality and ability to preserve angles, suffers from increasing distance distortion especially in areas away from the central line. In this project, such distortions lead to significant discrepancies in the actual route, resulting in higher fuel costs. To address this issue, the Lambert Conformal Conic (LCC) projection with standard parallels at $-10°$ and $-25°$ are used, approximately covering Madagascar's northern and southern boundaries. The LCC projection, known for its ability to minimize distortions over a region of interest, provides a more accurate estimation for fuel usage. This report is structured in the order of introduction, methodology, results, discussion, and conclusion.

Mercator projection is a cylindrical conformal map projection "where lines of constant bearing appear as straight segments [1]." LCC is short for Lambert Conformal Conic, named after Lambert, who first developed it in 1772 (Snyder 1987). Distortion between 2 standard parallels is small, so LCC is often "employed for aeronautical and regional mapping due to its ability to accurately represent shapes and distances in mid-latitudes.[1]"

2. METHODOLOGIES

2.1 Data sources: Pixel, geographic, and Mercator coordinates of four control points. Digitized pixel coordinates of 12 coastal points.

2.2 Methodologies: Used the pixel and Mercator coordinates of control points to build a matrix equation [2]. Applie 2D similarity transformation with 2 points and 4 points to calculate the transformation constants a, b, T_x and T_y. Computed the Mercator coordinates of all control points and coastline points.

Formula [3] is used to convert the Mercator coordinates of all points into geographic coordinates.

$$\lambda = \frac{x}{R} + \lambda_0$$
$$\phi = 2\arctan\left(e^{\frac{y}{R}}\right) - \frac{\pi}{2}$$

The Lambert Conformal Conic (LCC) projection formula from reference [4] is applied to calculate the Easting (X) and Northing (Y) for all points.

$$X = \rho\sin(\theta)$$
$$Y = \rho_0 - \rho\cos(\theta)$$

$$\rho = \frac{RF}{\tan^{n}(\frac{\pi}{4} + \frac{\phi}{2})}$$

$$\rho_0 = \frac{RF}{\tan^{n}(\frac{\pi}{4} + \frac{\phi_0}{2})}$$

$$\theta = n(\lambda - \lambda_0)$$

$$F = \frac{\cos(\phi_1)\cdot\tan^{n}(\frac{\pi}{4} + \frac{\phi_1}{2})}{n}$$

$$n = \frac{ln\left(\frac{\cos(\phi_1)}{\cos(\phi_2)}\right)}{\left(ln\left(\frac{\tan(\frac{\pi}{4}+\frac{\phi_2}{2})}{\tan(\frac{\pi}{4}+\frac{\phi_1}{2})}\right)\right)} = \text{cone constant}$$

The error propagation formula is employed to quantify error accumulation during the transformation process. Initial errors are derived from the deviation between the computed Mercator coordinates of the four control points and the source data, which are treated as the true values. Error matrices [5] are calculated for each transformation step: from pixel to Mercator, Mercator to geographic, and geographic to LCC coordinates.

$$\sigma_y^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{\partial f}{\partial x_i} \cdot \frac{\partial f}{\partial x_j} \cdot \mathrm{Cov}(x_i, x_j)\right)$$

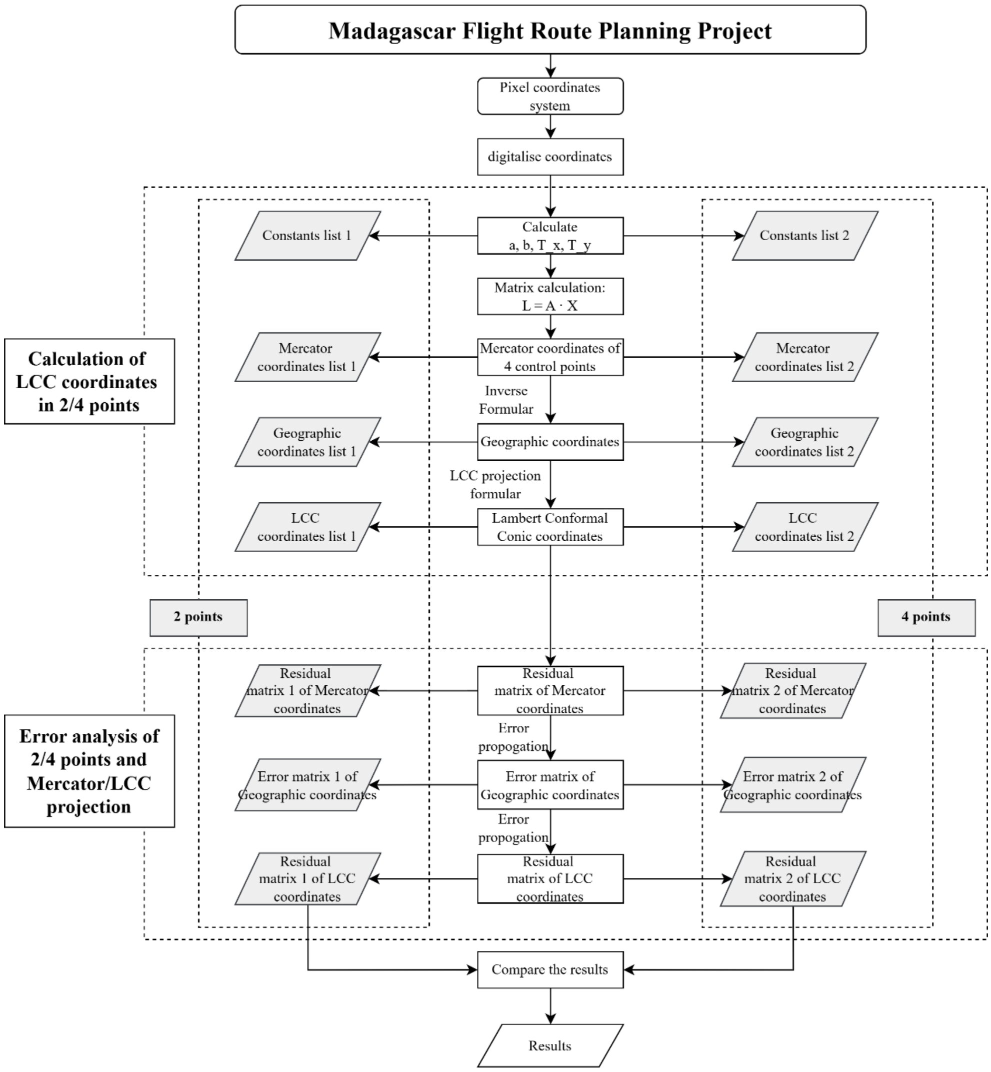
The total errors for Mercator and LCC projections are evaluated. The impact of least squares optimization on the results is analyzed, along with the performance and applicability of each projection. The projection with smaller errors is identified by comparing relative RMSE values, calculated using the formula below:

$$\text{Relative RMSE (\%)} = \frac{\text{RMSE}}{\text{Reference Value}} \times 100$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}}$$

2.3 Workflow: Figure 1:

Figure 1 Workflow of Madagascar project



3. Results and Analysis of Results

3.1 Constants results: Based on the first step of the methodology, the calculated 2D similarity transformation constants using 2 and 4 control points are presented in **Table 1**:

Parameter	2 points	4 points
a	19889.45	19889.45
b	101.5492	-1.46E-11
T_x	-5010721	-4985181
T_y	3199432	3250460

Table 1 Constants Table

The coordinates calculated using 2 and 4 control points are shown in **Table 2**:

crtl_pt _ID	Mer_x_2p	Mer_y_2p	Lam_x_2p	Lam_y_2p	Mer_x_4p	Mer_y_4p	Lam_x_4p	Lam_y_4p
A	3339512.99	4.6566E-10	3453058.86	-273190.327	3348500.1	8479.3618	3463662.27	-265963.786
B	6661051.77	16958.7236	6807915.64	-1063426.23	6670038.87	8479.3618	6814062.04	-1074696.06
C	6679025.98	-3503474.59	5779021.47	-4272542.56	6670038.87	-3511953.95	5769191.6	-4277179.22
D	3357487.21	-3520433.31	2939169.13	-3601067.92	3348500.1	-3511953.95	2932543.2	-3592458.21
C1	5454903.53	-1381497.54	5247102.19	-2069030.25	5456782.2	-1383782.4	5248301.98	-2071645.12
C2	5596059.15	-1758686.32	5284643.44	-2455460.62	5596008.37	-1761682.02	5283848.13	-2458210.4
C3	5516602.88	-1778981.97	5206332.47	-2454471.06	5516450.56	-1781571.47	5205554.59	-2456821.6
C4	5517212.18	-1898318.69	5177596.75	-2564369.23	5516450.56	-1900908.2	5176264.48	-2566557.39
C5	5301982.41	-2595566.61	4818451.66	-3143352.12	5297666.57	-2597039.08	4814276.63	-3143679.28
C6	5223947.83	-2894314.62	4682381.77	-3389871.77	5218108.75	-2895380.88	4677018.91	-3389515.98
C7	4845235.82	-2737128.42	4381573.64	-3169876.59	4840209.13	-2736265.25	4377286.31	-3168068.8
C8	4901959.25	-2160029.62	4554552.02	-2662252.2	4899877.49	-2159471.1	4552772.77	-2661294.35
C9	4940113.37	-1841595.26	4658980.14	-2377877.61	4939656.4	-1841239.84	4658635.33	-2377447.87
C10	5158694.26	-1800699.31	4870515.66	-2389267.77	5158440.39	-1801460.93	4870105.36	-2389914.05
C11	5296701.85	-1561315.02	5055008.79	-2199062.96	5297666.57	-1562787.48	5055558.16	-2200669.27
C12	5454903.53	-1381497.54	5247102.19	-2069030.25	5456782.2	-1383782.4	5248301.98	-2071645.12

Table 2 Coordinates table

In the 2-point process, points A and C were selected as control points for georeferencing. As shown in **Table 2**, for the Mercator projection, the Easting (Mer_x) values computed with the 4-point transformation are consistently slightly larger than those from the 2-point transformation, with differences ranging from approximately 800 m (e.g., point A) to nearly 1000 m (e.g., point C). In contrast, the Northing (Mer_y) values exhibit smaller variations.

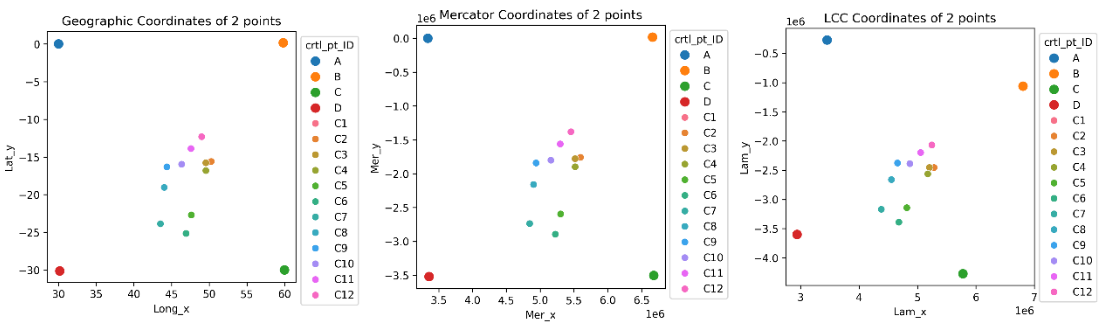
For the LCC projection, a similar trend is observed. The 4-point transformation consistently produces larger Easting (Lam_x) values compared to the 2-point results, with the magnitude of differences varying between approximately 1000 m (e.g., point A) and 1200 m (e.g., point B).

The complete coordinate results are presented in **Appendix A.**

3.3 Plots results:

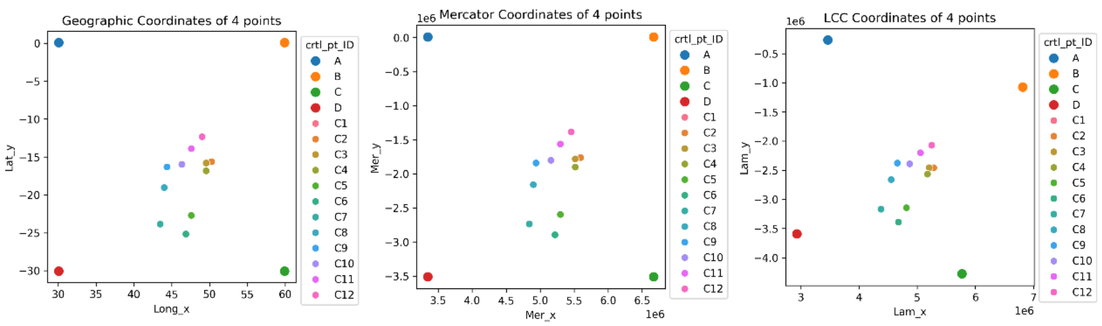
Geographic, Mercator and LCC Coordinates of 2-point method:

Figure 2 Plots of 2-point method



Geographic, Mercator and LCC Coordinates of 4-point method:

Figure 3 Plots of 4-point method



3.3 Error results:

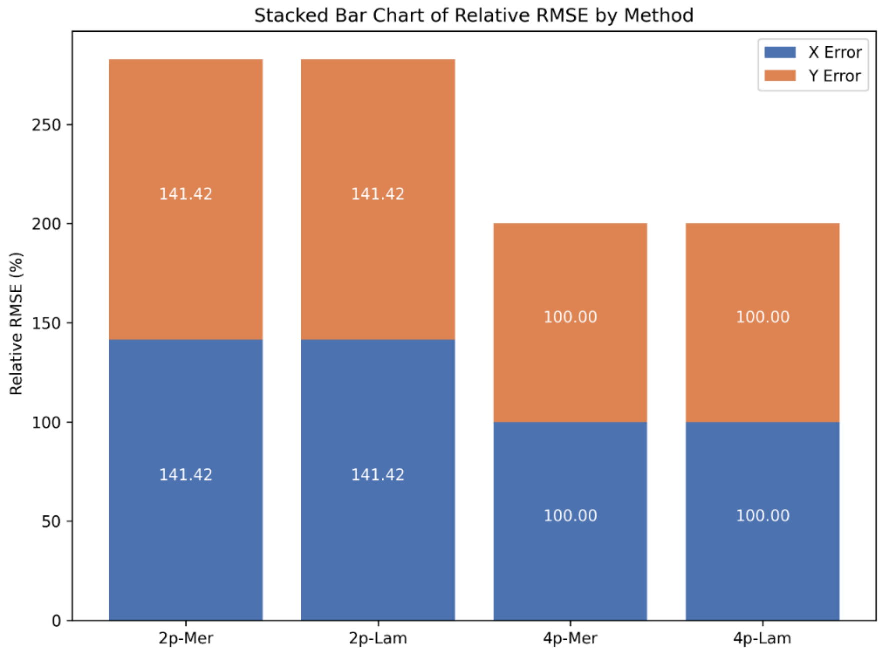
In **Table 3** for the Mercator projection, the 2-point transformation shows highly variable errors, with near-zero values for points A and C (Mer_X=2.33×10^{-9}), but large errors for points B and D (Mer_X=±17974). The 4-point transformation significantly reduces variability, with errors stabilizing around ±8987 for all points. A similar trend is observed in the LCC projection, where the 2-point transformation produces large errors for points B and D (Lam_X≈18600) and near-zero values for points A and C. In contrast, the 4-point transformation reduces errors to approximately 9300(Lam_X) and 8800(LamY) across all points.

Control Point ID	MerX_errors_2p	MerY_errors_2p	LamX_errors_2p	LamY_errors_2p	MerX_errors_4p	MerY_errors_4p	LamX_errors_4p	LamY_errors_4p
A	2.33E-09	7.08E-10	2.42E-09	7.35E-10	8987.108	8479.362	9349.381	8821.168
B	-17974.2	16958.72	18646.95	17593.46	-8987.11	8479.362	9341.234	8813.481
C	1.86E-09	-4.7E-10	1.93E-09	4.84E-10	-8987.11	-8479.36	9341.234	8813.481
D	17974.22	-16958.7	18678.68	17623.39	8987.108	-8479.36	9349.381	8821.168

Table 3 Error table

Figure 4 is a stacked bar chart of relative Root Mean Square Error. It clearly demonstrates that the errors obtained with 2 points are significantly larger than those using the Least Squares method with 4 points, consistent with **Table 3**. For both the Mercator (2p-Mer) and Lambert (2p-Lam) projections, the relative RMSE reaches approximately 141.42%, about 1.41 times greater than the 100% error observed when 4 points are used (4p-Mer and 4p-Lam).

Figure 4 Relative RMSE stacked bar chart



3.4 Distance results:

Table 4 lists the distances between Antsiranana, Tsingy De Bemaraha, and Fararagana, as well as the total distance.

Type	Antsiranana to Tsingy De Bemaraha(m)	Tsingy De Bemaraha to Fararagana(m)	Fararagana to Antsiranana(m)	Total Distance(m)
Geo Distance	918706.0357	552406.2602	1164948.313	2636060.608
Mercator Distance	954900.9389	591355.9733	1223646.007	2769902.919
LCC Distance	911837.2143	548707.1066	1156561.819	2617106.14

Table 4 Distance table

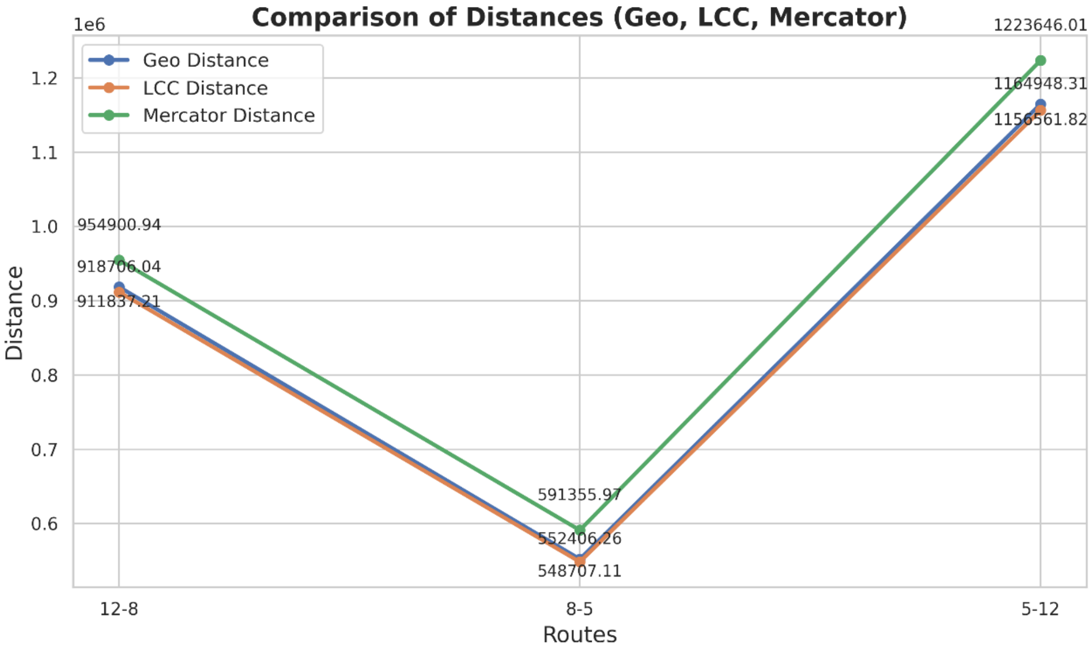
Table 5 presents the relative error values (%) of distances compared to the ground surface distances under the Mercator and LCC projections.

Type	Relative Error - A-to-TDB(%)	Relative Error - TDB-to-F(%)	Relative Error - F-to-A(%)	Relative Error - Total(%)
Mercator Distance	3.939769832	7.050918129	5.038652276	5.077360915
LCC Distance	0.747662592	0.669643679	0.719902633	0.719045248

Table 5 Proportion of distance difference

Figure 5 presents the line chart of the distances.

Figure 5 Distance line graph



Based on the data, the cost saving of a one-way trip using the LCC projection is $44.44, with a saving rate compared to the Mercator projection of 5.52%.

4. Discussion

The Lambert Conformal Conic (LCC) projection preserves angles and shapes but often shortens distances over large areas [1]. In contrast, the Mercator projection exaggerates distances at higher latitudes, making them significantly longer than actual surface distances [1].The absolute distances in LCC are shorter than surface distances, and its equidistance property within 10-25° latitude makes it more suitable for estimating transportation costs.

Although from **Figure 2 & 3,** there is not a big difference between the 2-point and 4-point images. From **Table 3**, it can be seen that using points A and C as control points for georeferencing minimizes the error between them (-4.7E-10), making it negligible. However, the error values for points B and D range from 16,958.7 to 17,974.2, resulting in a large variance in the transformation. In contrast, the least squares 4-point method shows more evenly distributed errors. For example, in the Mercator 4-point transformation, the X and Y errors are similar, at 8,987.108 and 8,479.362, respectively. As stated in Step 6 of the Methodology, its relative RMSE is smaller than that of the 2-point method, as reflected in **Figure 4.** This demonstrates that the 4-point transformation improves accuracy and reduces error variability.

It is worth noting that since both Mercator error and LCC error result from error propagation, their distribution trends are consistent under the same method (2-point or 4-point). This also leads to LCC error values being slightly higher than Mercator's, regardless of the method used—this is determined by the differential formula of LCC.

Table 4 shows that the ground surface distance is approximately 2,636,061 m, the Mercator projection distance is 2,769,903 m, while the LCC projection distance is 2,617,106 m. Compared to the ground surface distance (taken as the true value), LCC is closer with a difference of around 20 km, while Mercator has a larger discrepancy of about 130 km. From **Table 5**, the relative error for Mercator distance compared to the geographic distance is 5.08%, while for LCC, it is 0.72%, showing a significant reduction. **Figure 5** illustrates the distance trends, where all Mercator distances deviate from and are noticeably higher than the ground surface distance. In contrast, LCC distances largely overlap with the ground surface distance at the current precision level.

Potential issues: (a) The four control points are relatively few and form the vertices of a rectangle in the Mercator projection, where their parallel arrangement may introduce computational bias. (b) Scanning precision: Some coastal points are quite close to each other, and their positions may slightly shift due to limited resolution. (c) The exact datum used in the original Mercator projection is not explicitly stated in the dataset provided. However, a commonly used global datum such as WGS84 could be properly used.

5. Conclusions

Compared to the 2D similarity transformation using two points, the Least Squares method achieves a relative error ratio of 1:1.41, indicating a significant improvement in fitting accuracy. The LCC projection shows a 0.72% difference from the geographic distance, while the Mercator projection has a 5.08% difference, making LCC more accurate for flight path design. Using four control points with the Least Squares method and the LCC projection, a one-way trip saves $44.44, with a saving rate of 5.52%.

References

[1] J. P. Snyder, *Map Projections: A Working Manual*, U.S. Geological Survey, Washington, D.C., USA, 1987, pp. 43–45, 107–109.

[2] J. Iliffe and R. Lott, *Datums and Map Projections for Remote Sensing, GIS, and Surveying*. (Second ed.) 2008.

[3] University of Glasgow, “2023 GeoFun Week 4 Linear Algebra Solutions,” *Jupyter Notebook, GeoFun 2024, Week 4*, Dec. 2024.

[4] University of Glasgow, *MadagascarExDay3, Geospatial Fundamentals 24-25, Week 8 Course Materials*, 2024.

[5] C. Ghilani, “Statistics and adjustments explained part 3: Error propagation,” *Surveying and Land Information Science*, vol. 64, pp. 29–33, Mar. 2004.

Appendix A

Results of 2 control points

crtl_pt _ID	LatN deg	LongE deg	pixelx	pixely	Mer_x	Mer_y	Lat_y	Long_x	Lam_x	Lam_y
A	0	30	419	-163	3339512.99	4.6566E-10	0	30	3453058.86	-273190.327
B	0	60	586	-163	6661051.77	16958.7236	0.1523459	59.8385314	6807915.64	-1063426.23
C	-30	60	586	-340	6679025.98	-3503474.59	-30	60	5779021.47	-4272542.56
D	-30	30	419	-340	3357487.21	-3520433.31	-30.1318478	30.1614686	2939169.13	-3601067.92
C1			525	-233	5454903.53	-1381497.54	-12.3145492	49.0032848	5247102.19	-2069030.25
C2			532	-252	5596059.15	-1758686.32	-15.6024022	50.2713344	5284643.44	-2455460.62
C3			528	-253	5516602.88	-1778981.97	-15.7779313	49.5575513	5206332.47	-2454471.06
C4			528	-259	5517212.18	-1898318.69	-16.8069075	49.5630248	5177596.75	-2564369.23
C5			517	-294	5301982.41	-2595566.61	-22.6987036	47.6295414	4818451.66	-3143352.12
C6			513	-309	5223947.83	-2894314.62	-25.1516	46.9285298	4682381.77	-3389871.77
C7			494	-301	4845235.82	-2737128.42	-23.8668142	43.5264288	4381573.64	-3169876.59
C8			497	-272	4901959.25	-2160029.62	-19.0436529	44.035995	4554552.02	-2662252.2
C9			499	-256	4940113.37	-1841595.26	-16.3184858	44.3787467	4658980.14	-2377877.61
C10			510	-254	5158694.26	-1800699.31	-15.9655878	46.3423344	4870515.66	-2389267.77
C11			517	-242	5296701.85	-1561315.02	-13.8878112	47.5821043	5055008.79	-2199062.96
C12			525	-233	5454903.53	-1381497.54	-12.3145492	49.0032848	5247102.19	-2069030.25

Results of 4 control points

crtl_pt_ID	Lat_deg_N	Long_deg_E	pixel_x	pixel_y	Mer_x	Mer_y	Long_x	Lat_y	Lam_x	Lam_y
A	0	30	419	163	3348500.1	8479.3618	30.0807343	0.07617302	3463662.27	-265963.786
B	0	60	586	163	6670038.87	8479.3618	59.9192657	0.07617302	6814062.04	-1074696.06
C	-30	60	586	340	6670038.87	-3511953.95	59.9192657	-30.0659459	5769191.6	-4277179.22
D	-30	30	419	340	3348500.1	-3511953.95	30.0807343	-30.0659459	2932543.2	-3592458.21

C1	525	233	5456782.2	-1383782.4	49.0201614	-12.3346019	5248301.98	-2071645.12
C2	532	252	5596008.37	-1761682.02	50.2708783	-15.6283204	5283848.13	-2458210.4
C3	528	253	5516450.56	-1781571.47	49.5561829	-15.800316	5205554.59	-2456821.6
C4	528	259	5516450.56	-1900908.2	49.5561829	-16.8291749	5176264.48	-2566557.39
C5	517	294	5297666.57	-2597039.08	47.5907707	-22.7109061	4814276.63	-3143679.28
C6	513	309	5218108.75	-2895380.88	46.8760753	-25.1602702	4677018.91	-3389515.98
C7	494	301	4840209.13	-2736265.25	43.4812724	-23.8597229	4377286.31	-3168068.8
C8	497	272	4899877.49	-2159471.1	44.0172939	-19.0389101	4552772.77	-2661294.35
C9	499	256	4939656.4	-1841239.84	44.3746416	-16.3154215	4658635.33	-2377447.87
C10	510	254	5158440.39	-1801460.93	46.3400538	-15.9721657	4870105.36	-2389914.05
C11	517	242	5297666.57	-1562787.48	47.5907707	-13.9006519	5055558.16	-2200669.27
C12	525	233	5456782.2	-1383782.4	49.0201614	-12.3346019	5248301.98	-2071645.12

Appendix B

Detailed Plots:

Figure 6

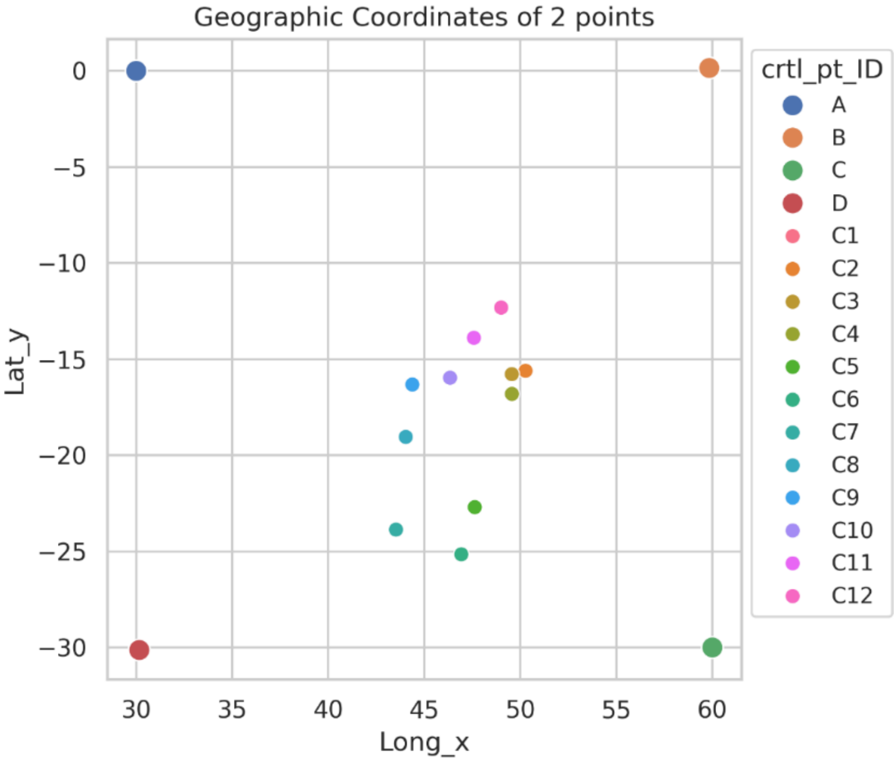


Figure 7

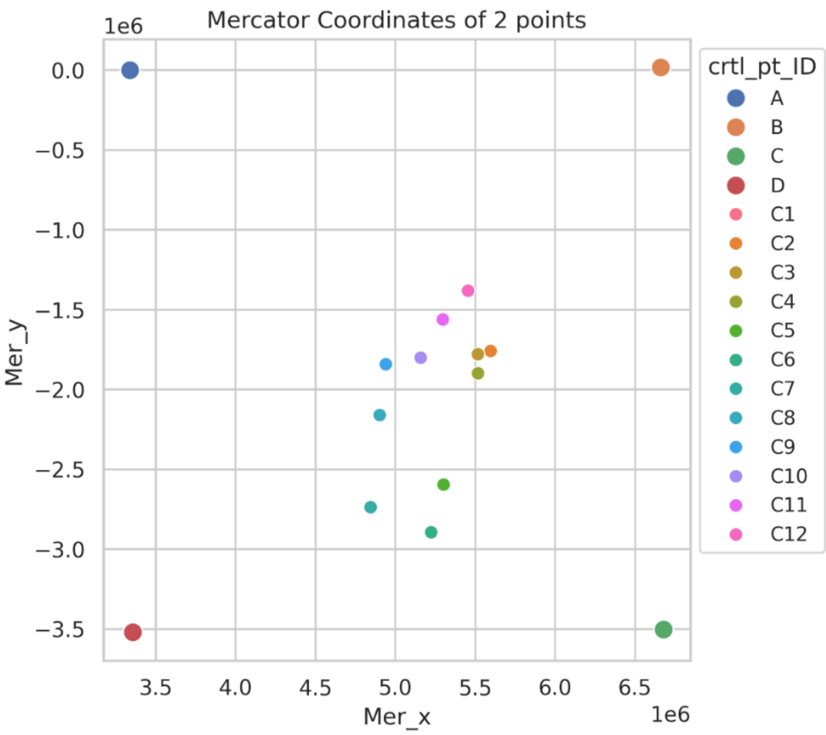


Figure 8

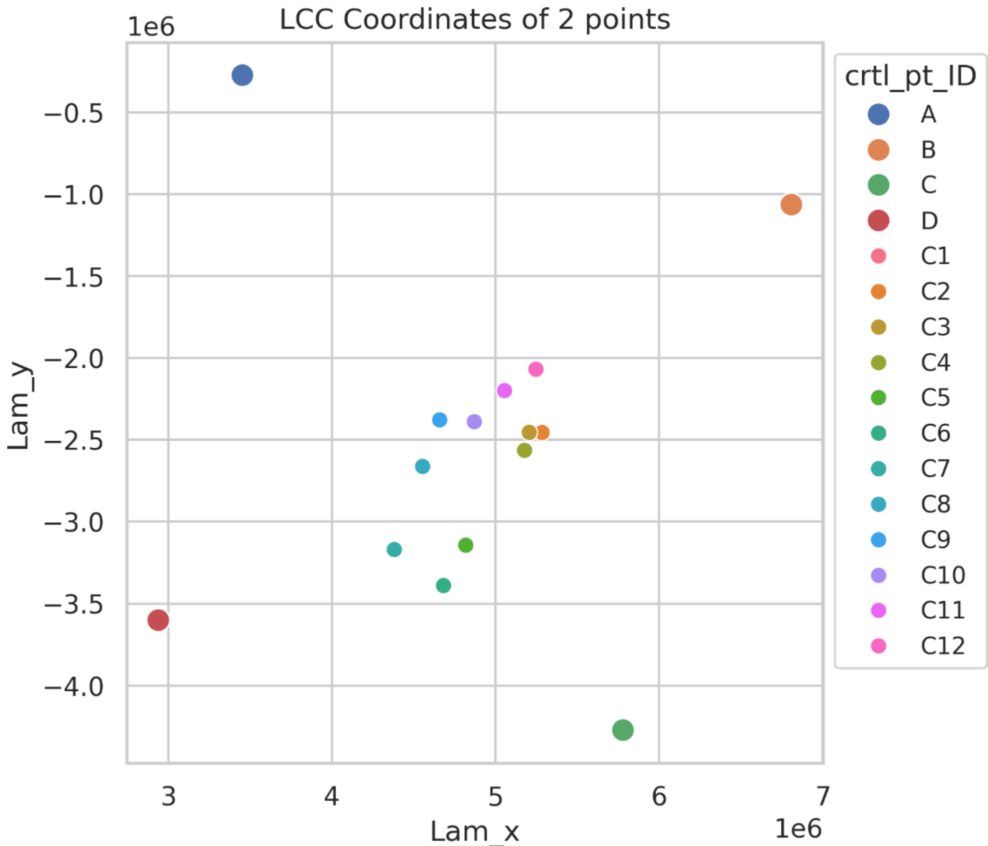


Figure 9

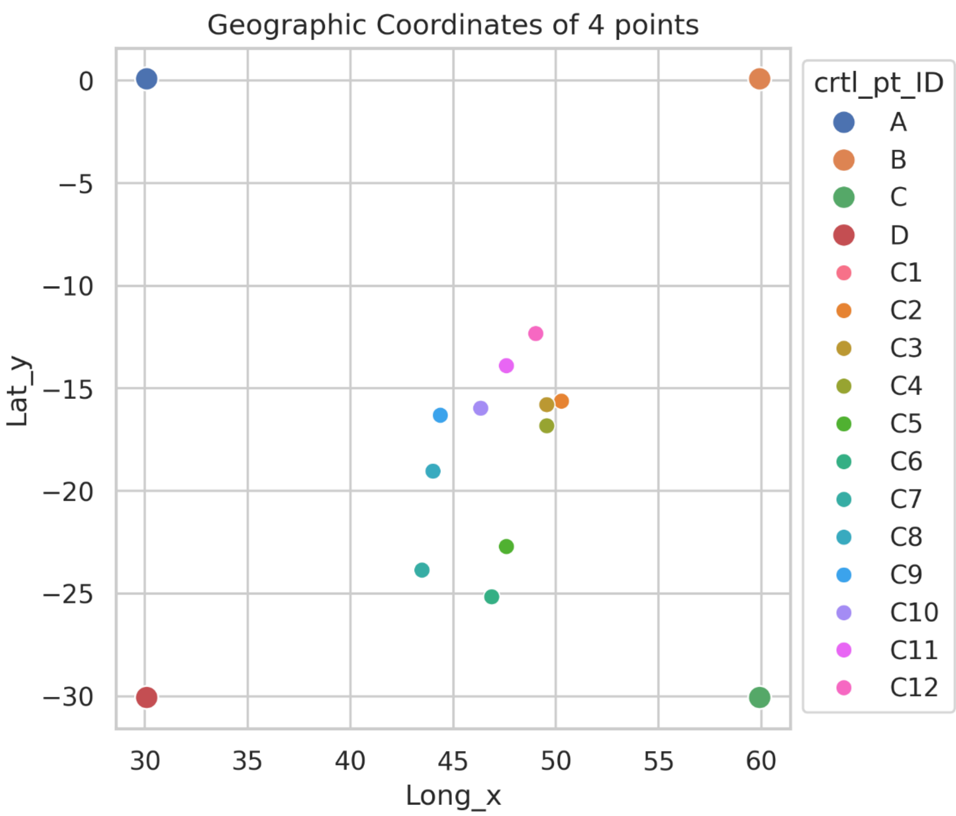


Figure 10

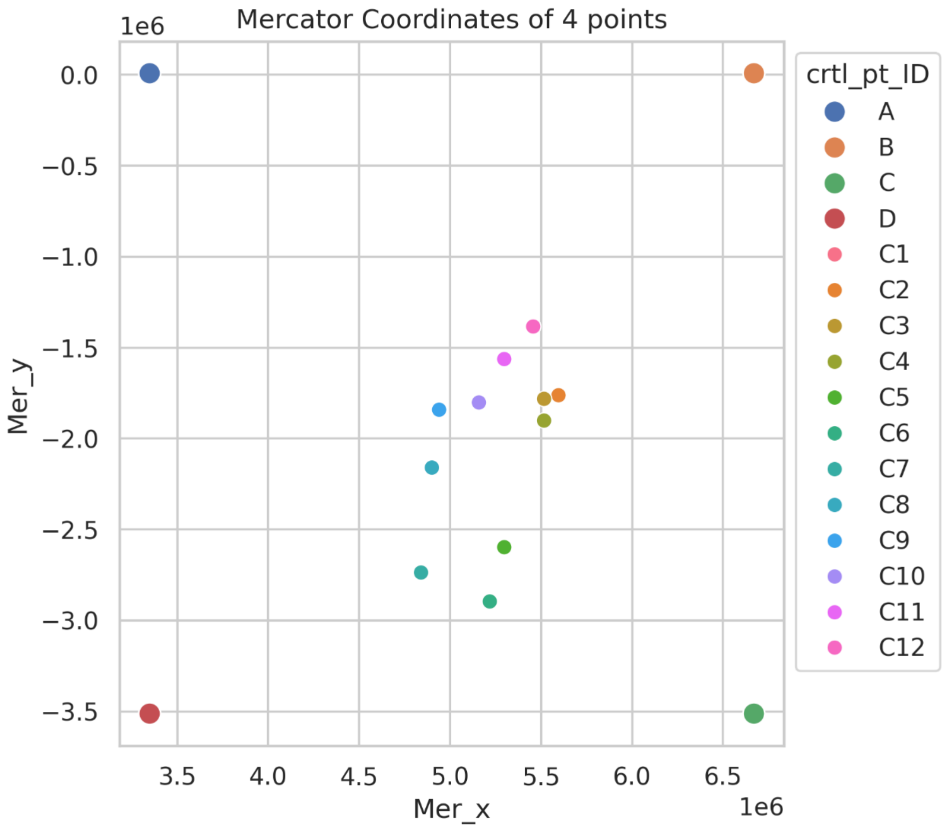
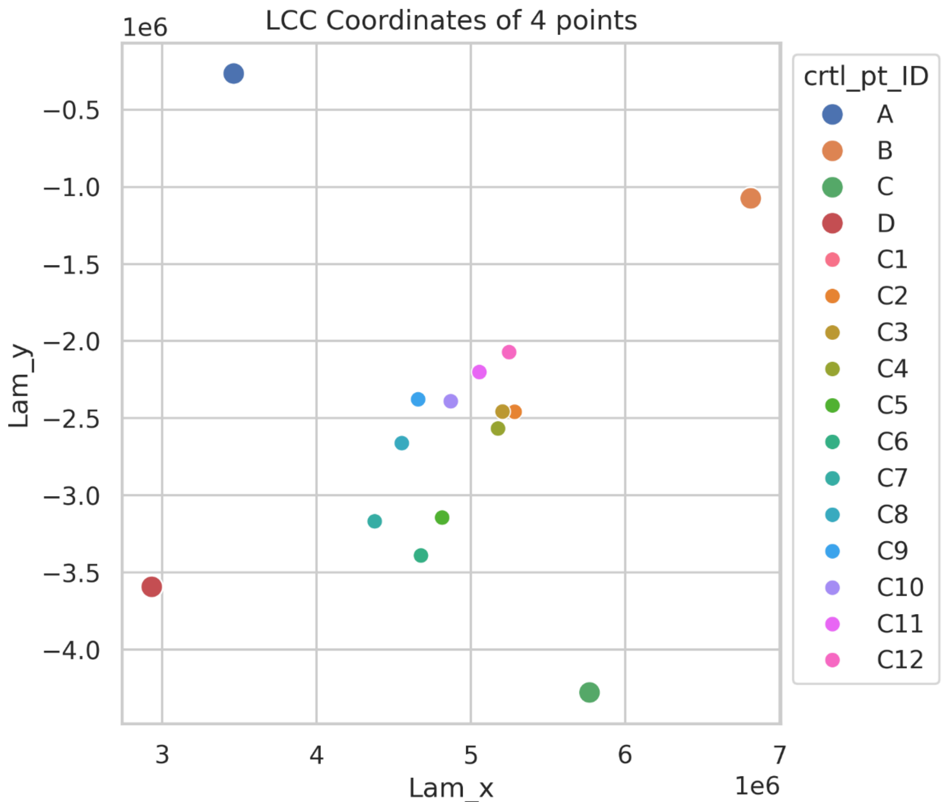


Figure 11



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