Portfolio Value for StableSwap: $x_0 + p \cdot x_1$ given $(D,\ p,\ \texttt{\_amp})$

1 Setup

For $N = 2$ coins, let $\alpha = 2 \cdot \texttt{\_amp}$ (this is `Ann` in the code, equal to $A \cdot n^n$ from the whitepaper). The StableSwap invariant is:

$$\alpha(x_0 + x_1) + D = \alpha D + \frac{D^3}{4\,x_0\,x_1}\,. \tag{1}$$

The price from `get_p` (computing $p = -dx_0/dx_1 > 0$ along constant D):

$$p = \frac{x_0\left(\alpha\,x_1 + \dfrac{D^3}{4\,x_0\,x_1}\right)}{x_1\left(\alpha\,x_0 + \dfrac{D^3}{4\,x_0\,x_1}\right)}\,. \tag{2}$$

2 Derivation

2.1 Step 1: Price equation

Cross-multiplying the price formula:

$$4\alpha\,(x_0x_1)^2(p-1) = D^3(x_0 - p\,x_1)\,. \tag{3}$$

2.2 Step 2: Key identity

Define $V = x_0 + p\,x_1$ (the target) and $Q = x_0 - p\,x_1$. Then:

$$V^2 = Q^2 + 4p\,x_0x_1\,. \tag{4}$$

2.3 Step 3: Express Q and x_0x_1 in terms of Γ

From the invariant, define:

$$\Gamma \;\equiv\; \frac{D^3}{4\,x_0x_1} \;=\; \alpha(S - D) + D\,, \tag{5}$$

where $S = x_0 + x_1$. So $x_0x_1 = D^3/(4\Gamma)$. From (3):

$$Q = x_0 - p\,x_1 = \frac{\texttt{\_amp}\,(p-1)\,D^3}{2\,\Gamma^2}\,. \tag{6}$$

2.4 Step 4: Formula for V^2

Substituting into identity (4):

$$\boxed{V^2 = \frac{p\,D^3}{\Gamma} + \frac{\texttt{\_amp}^2\,(p-1)^2\,D^6}{4\,\Gamma^4}} \tag{7}$$

2.5 Step 5: Second relation — V in terms of S

From $S = x_0 + x_1$ and $V = x_0 + p\,x_1$, using $x_0 = \frac{pS-V}{p-1}$, $x_1 = \frac{V-S}{p-1}$:

$$(1+p)\,V = 2pS - (p-1)\,Q\,. \tag{8}$$

Substituting Q from (6) and $S = \big[\Gamma + (2\,\texttt{\_amp} - 1)D\big]/(2\,\texttt{\_amp})$:

$$\boxed{V = \frac{p\big[\Gamma + (2\,\texttt{\_amp} - 1)D\big]}{\texttt{\_amp}\,(1+p)} - \frac{\texttt{\_amp}\,(p-1)^2\,D^3}{2(1+p)\,\Gamma^2}} \tag{9}$$

3 Result

Equations (7) and (9) jointly determine V and Γ. Eliminating V yields a single degree-6 polynomial in Γ—**there is no simple closed-form** for V in terms of D, p, `_amp` alone. The value must be found numerically.

3.1 Integral representation (Envelope Theorem)

A clean exact formula follows from the envelope theorem. Since $V(p) = x_0(p) + p\,x_1(p)$:

$$\frac{dV}{dp} = \frac{dx_0}{dp} + x_1 + p\,\frac{dx_1}{dp} = x_1\,,$$

because $dx_0 + p\,dx_1 = 0$ on the invariant curve at price p. Therefore:

$$\boxed{V(p) = D + \int_1^p x_1(s)\,ds} \tag{10}$$

where $V(1) = D$ (the balanced-pool value, since $x_0 = x_1 = D/2$ when $p = 1$).

4 Limiting cases

Regime	Formula	Note
$p = 1$ (balanced)	$V = D$	Exact for all A
`_amp` $\to 0$ (constant-product)	$V = D\sqrt{p}$	Recovers Uniswap
`_amp` $\to \infty$ (constant-price)	$V = D$	Only $p = 1$ is reachable

For finite `_amp`, V interpolates between $D\sqrt{p}$ and D. Larger `_amp` keeps V closer to D (less impermanent loss).

5 Notation summary

Symbol	Meaning
x_0, x_1	Token balances (`_xp[0]`, `_xp[1]`)
D	Invariant (from `newton_D`)
`_amp`	Amplification parameter $= A \cdot N^{N-1}$
α	$= 2 \cdot$ `_amp` (`Ann` in code)
S	$= x_0 + x_1$
Γ	$= D^3/(4\,x_0 x_1) = \alpha(S - D) + D$
p	Marginal price $= -dx_0/dx_1 > 0$
V	Portfolio value $= x_0 + p\,x_1$