CMSC 5718 INTRODUCTION TO COMPUTATIONAL FINANCE

Lecture 6

Outline

- Measuring portfolio performance
  - Performance attribution
  - Standard performance ratios
- The special case for hedge funds
- Measuring portfolio risk
  - Dispersion and other measures
- Value-at-risk

Application of asset pricing models in performance evaluation

- Given that there are many different kinds of trading strategies, how do we compare the performance of the various financial instruments?
  - One result that we want to achieve is to identify a good manager from others who get good investment results because of market or other random factors, i.e. we want to distinguish between skill and luck
- We can use the CAPM to calculate risk-adjusted performance

Examples of fund performance

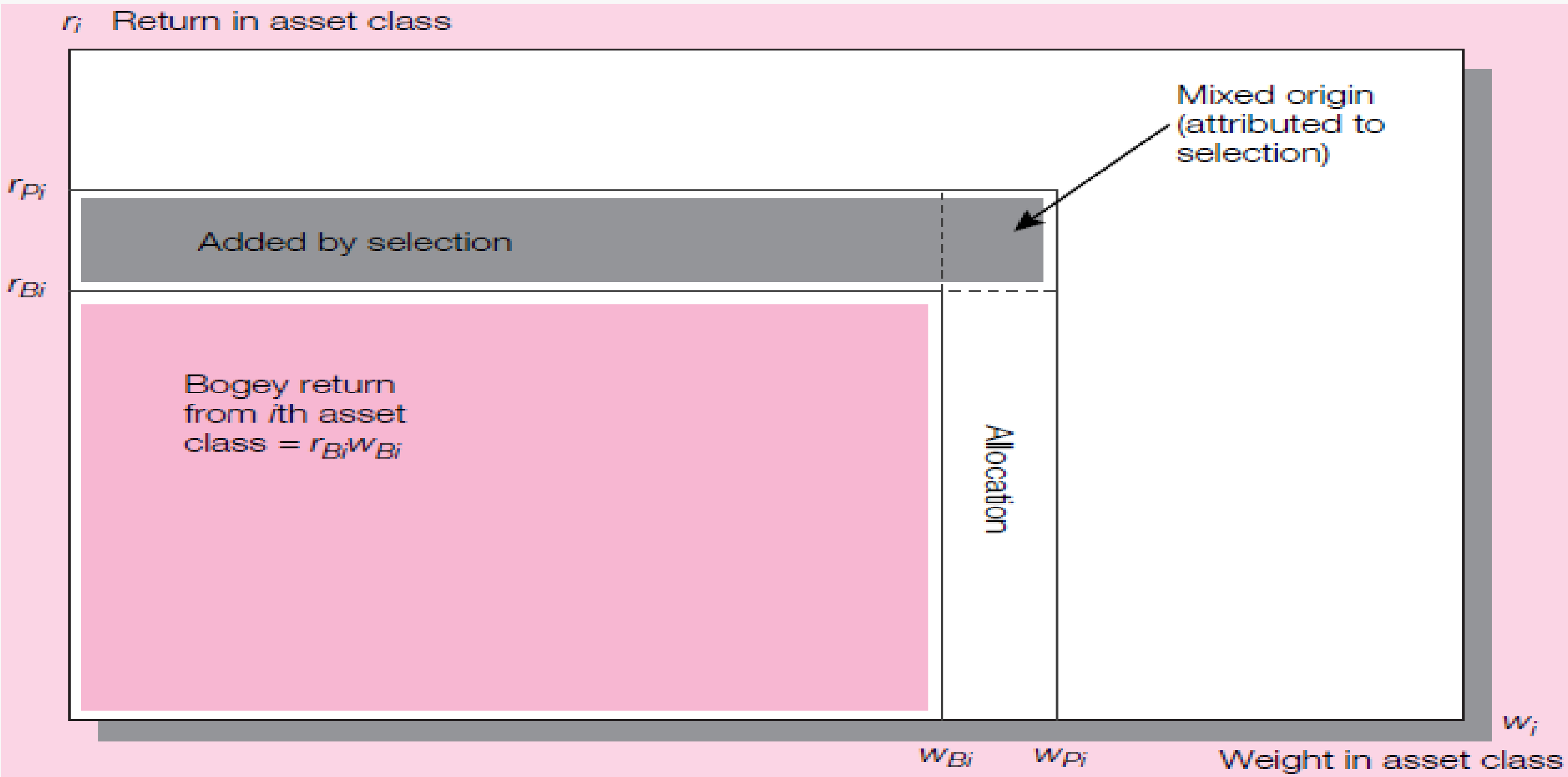
	產品風險級別 Product Risk Ratings	基金成立日 Inception Date	3M	2015	2014	2013	2012	2011	3年波幅 Ann. Volatility 3Y	3年夏普比率 Ann. Sharpe Ratio 3Y
多元入息資產 MULTI ASSET INCOME										
環球多元入息資產 Global Multi Asset Income										
貝萊德環球多元資產入息基金 A2(美元) BGF Global Multi-Asset Income A2 (USD)	3	28/6/2012	3.13%	-2.35%	4.11%	5.74%	NA	NA	5.03%	0.56
摩根全方位入息基金(港元) JPMorgan Multi Income Fund (HKD)	3	9/9/2011	3.63%	-1.67%	4.42%	6.01%	14.85%	NA	6.41%	0.57
股票：已發展市場 EQUITIES: DEVELOPED MARKETS										
環球股票 Global Equity										
聯博 – 低波幅策略股票基金 AD股(美元) AllianceBernstein Low Volatility Equity Ptfl AD (USD)	3	15/10/2013	1.29%	5.06%	9.43%	NA	NA	NA	NA	NA
富達基金 – 環球股息基金(美元) Fidelity Fds Global Dividend (USD)	4	30/1/2012	2.29%	1.58%	4.73%	26.12%	NA	NA	9.39%	0.63
北美股票 North American Equity										
聯博 – 精選美國股票基金A股(美元) AllianceBernstein Select US Equity Pf A (USD)	4	28/10/2011	2.55%	-0.20%	11.93%	29.35%	14.32%	NA	10.47%	0.71
摩根美國價值基金(美元) JPM US Value (USD)	4	19/10/2000	3.06%	-6.96%	13.35%	30.65%	13.13%	1.41%	11.20%	0.50
北美股票 – 增長 North American Equity – Growth										
美盛凱利美國進取型增長基金(美元) Legg Mason ClearBridge US Aggressive Growth Fund (USD)	4	20/4/2007	9.57%	-5.22%	13.62%	37.52%	18.69%	-2.64%	13.92%	0.39
富蘭克林美國機會基金A股(美元) Franklin U.S. Opportunities A (USD)	4	3/4/2000	7.05%	4.82%	6.71%	38.60%	9.33%	-3.88%	13.29%	0.47
歐洲股票 European Equity										
貝萊德歐洲基金A2(歐元) BGF European Fund A2 (EUR)	5	30/11/1993	-9.50%	10.97%	2.58%	21.78%	20.45%	-10.41%	13.12%	0.23

- Source: Funds Select, Fourth Quarter 2016, Standard Chartered Bank, Hong Kong

A simple performance attribution method

- Performance of a fund is measured against a benchmark, called the bogey portfolio, with fixed weights in each asset class
  - E.g. we can use broad market indices, like S&P 500, in each asset class to represent the passive strategies
  - One needs to decide on the "neutral" weights to be allocated to each asset class; this depends on the investor's risk tolerance
- The performance in each asset class is divided as:
  - Total return = return from asset allocation
    + return from security selection

A simple performance attribution method



- From Bodie, Kane and Marcus, Investments, 6th edition, McGraw Hill (2003)

A simple performance attribution method

- The returns of the managed portfolio and the bogey portfolio are:

$$r_P = \sum_{i=1}^{n} w_{Pi} r_{Pi}, \;\; r_B = \sum_{i=1}^{n} w_{Bi} r_{Bi}$$

- The difference between the two rates of return is

$$r_P - r_B = \sum_{i=1}^{n} (w_{Pi} r_{Pi} - w_{Bi} r_{Bi})$$

- Total contribution from asset class i is decomposed as:

$$\underbrace{w_{Pi} r_{Pi} - w_{Bi} r_{Bi}}_{\text{total contributi on}} = \underbrace{(w_{Pi} - w_{Bi}) r_{Bi}}_{\text{asset allocation}} + \underbrace{w_{Pi}(r_{Pi} - r_{Bi})}_{\text{security selection}}$$

A simple performance attribution method

- Bogey portfolio has 60% equity, 30% fixed income, 10% cash, whereas the managed portfolio has a 70/7/23 distribution
- The return of the bogey portfolio is 3.97%, managed portfolio is 5.34%, i.e. excess return is 5.34–3.97 = 1.37%
- Attribution of the returns is as follows (from Bodie et al. (2003)):

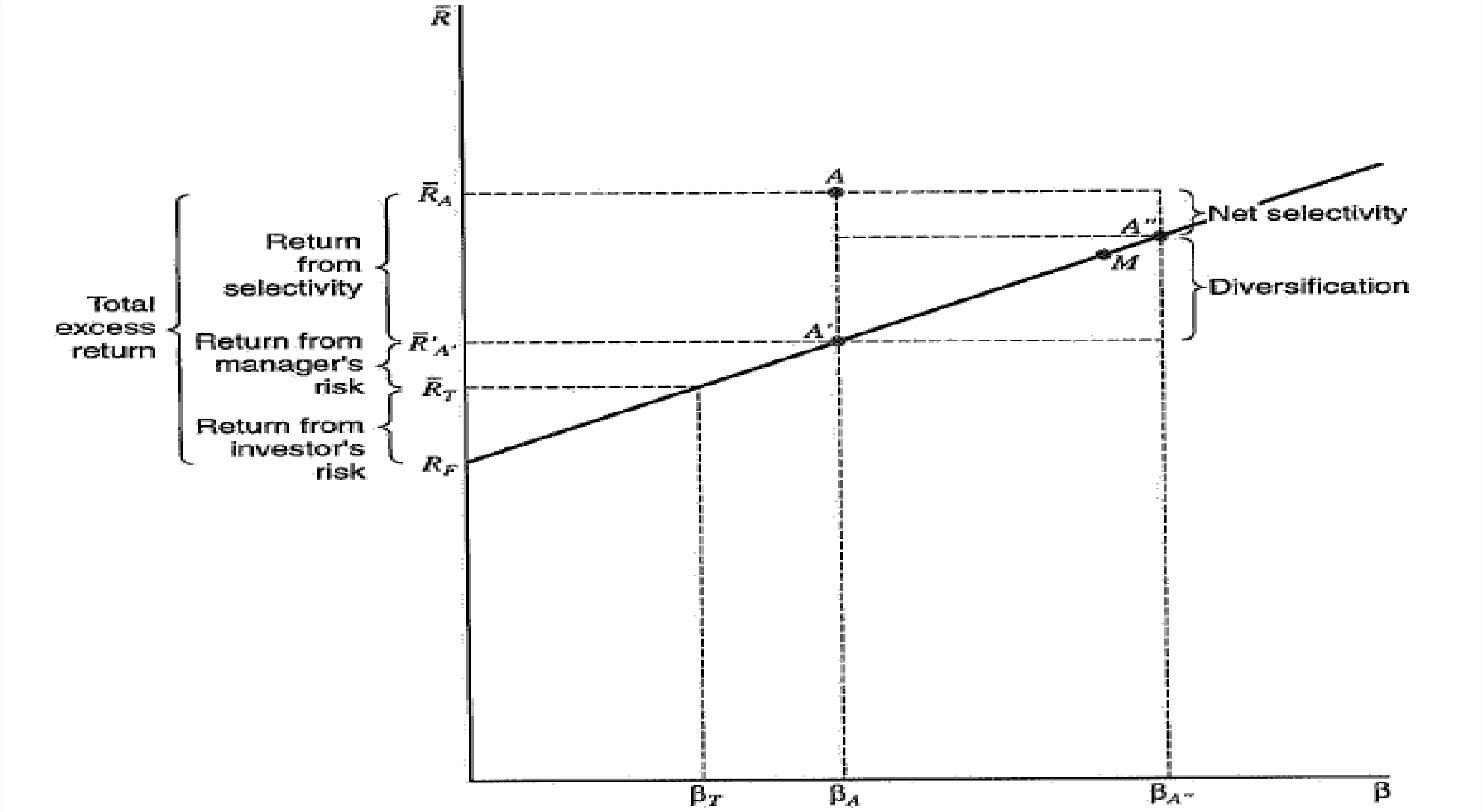
A. Contribution of Asset Allocation to Performance

Market	(1) Actual Weight in Market	(2) Benchmark Weight in Market	(3) Excess Weight	(4) Market Return (%)	(5) = (3) × (4) Contribution to Performance (%)
Equity	.70	.60	.10	5.81	.5810
Fixed-income	.07	.30	−.23	1.45	−.3335
Cash	.23	.10	.13	.48	.0624
Contribution of asset allocation					.3099

B. Contribution of Selection to Total Performance

Market	(1) Portfolio Performance (%)	(2) Index Performance (%)	(3) Excess Performance (%)	(4) Portfolio Weight	(5) = (3) × (4) Contribution (%)
Equity	7.28	5.81	1.47	.70	1.03
Fixed-income	1.89	1.45	0.44	.07	0.03
Contribution of selection within markets					1.06

Another performance attribution method



- Decomposition according to Fama (1972), from Elton et al. (2007)

Another performance attribution method

- Line R_F-M plots the return of all possible combinations of the riskless asset and the market portfolio
- A represents a portfolio with diversifiable risk
- Jensen's alpha = AA'= return from selectivity
- Form a portfolio A'' with the same total risk as the portfolio A, i.e. $\sigma_A^2 = \sigma_{A''}^2 = \beta_{A''}\sigma_M^2 \Rightarrow \beta_{A''} = \sigma_A^2 / \sigma_M^2$
- Extra return earn by portfolio A over the portfolio A'' (with the same total risk) = Net selectivity
- Return from selectivity = net selectivity + diversification

Another performance attribution method

- Secondly, the extra return from $\overline{R_{A'}} - R_F$ is decomposed into two components
- The investor may have a target risk level given by β_T, which will generate a return of $\overline{R_T}$
  - The extra return above the risk free rate = $\overline{R_T} - R_F$ is known as the "investor' risk"
- The remaining return $(\overline{R_{A'}} - \overline{R_T})$ is the return earned because the manager chose a different risk level than the target, known as the "manager's risk"

- Total excess return =

  return from selectivity + investor risk + manager risk

Performance attribution example

- Risk free rate R_F = 4%, return of market portfolio $\overline{R_M}$ = 12%, s.d. of market portfolio σ_M = 11%
- Beta of portfolio A β_A = 1.3, expected return $\overline{R_A}$ = 17.53%, s.d. of portfolio A σ_A = 14%
- Return of A from SML = 0.04 + 1.3x(0.12-0.04) = 14.4%
- Beta of portfolio A'': $\beta_{A''} = \sigma_A^2 / \sigma_M^2 = 0.14^2 / 0.11^2 = 1.62$
- Return of A'' from SML = 0.04 +1.62x(0.12-0.04) = 16.96%
- Return from selectivity = Jensen's alpha = 17.53-14.4 = 3.13%
- Net selectivity = 17.53-16.96 = 0.57%
- Return from diversification = 16.96-14.4 =3.13-0.57 = 2.56%
- Assume investor's target portfolio C has a beta = 1.2
- Return of C from SML = 0.04 +1.2x(0.12-0.04)= 13.6%
- Investor risk = 13.6 – 4 = 9.6%
- Manager risk = 14.4 – 13.6 = 0.8%

Performance from market timing

- In a successful market timing strategy, beta is higher than 1 when the market is bullish, but smaller than 1 when the market is bearish
- Instead of a linear model, the Security Characteristic Line is modified as (Treynor and Mazuy (1966)):

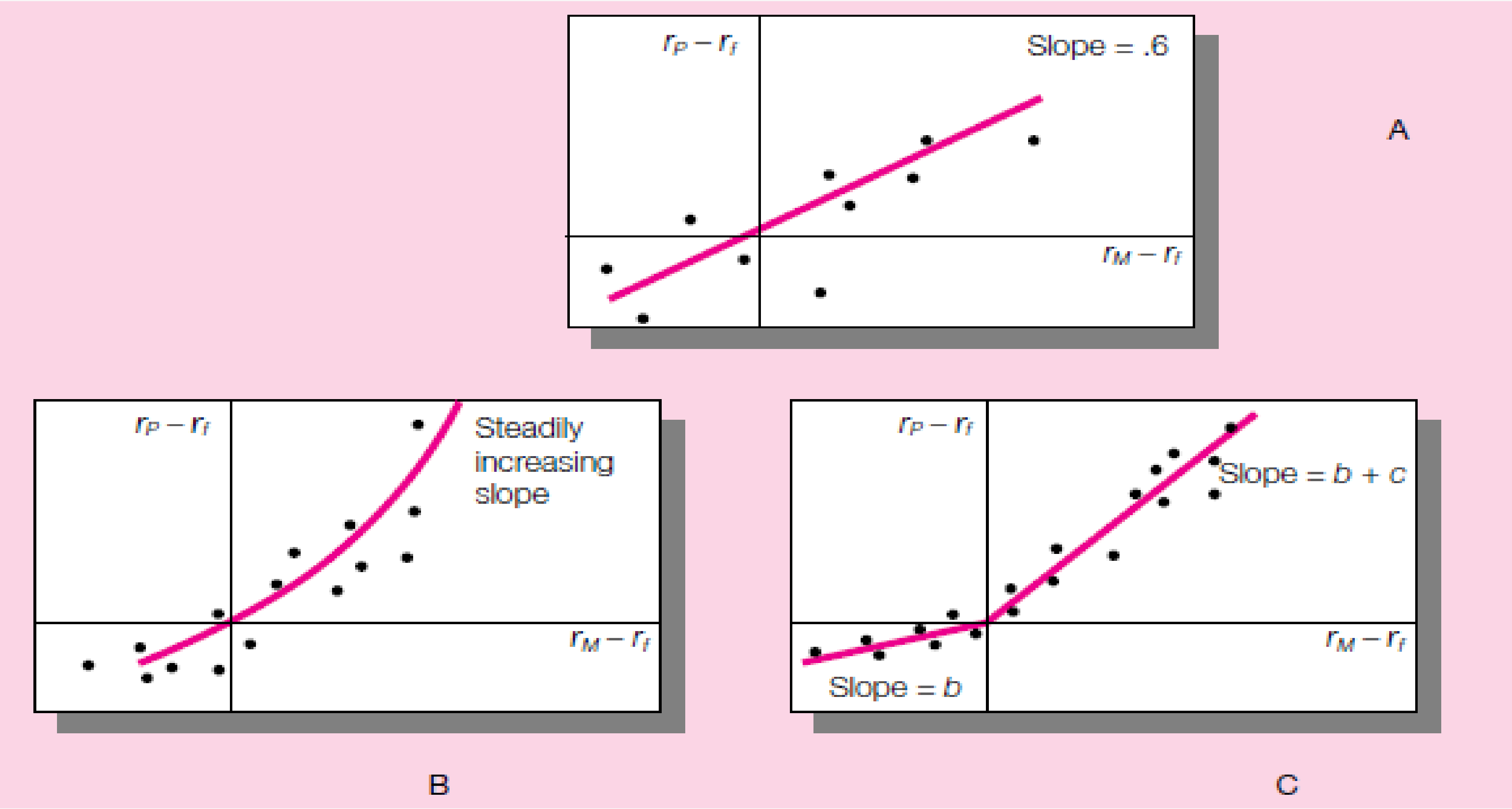
$$R_P - R_f = a + b(R_M - R_f) + c(R_M - R_f)^2 + e_P$$

  a, b, and c are constants estimated by regression
  - A positive c indicates evidence of timing ability
- Henrickson and Merton (1981) suggest a two-beta model:

$$R_P - R_f = a + b(R_M - R_f) + c(R_M - R_f)D + e_P$$

  $D=1$ if $R_M > R_f$, and $D=0$ if $R_M <= R_f$

Performance from market timing



- A: No market timing, constant beta; B: market timing with increasing beta; C: market timing with two values of beta (Bodie et al., 2003)

Standard performance measures

- From the Capital Allocation Line (CAL):

$$\text{Sharpe ratio} = RS = \frac{(\overline{r_A} - r_f)}{\sigma_A}$$

- From the Security Market Line (SML):

$$\text{Treynor ratio} = RT = \frac{(\overline{r_A} - r_f)}{\beta_A}$$

$$\text{Jensen's alpha } \alpha_A = \overline{r_A} - \left[r_f + (\overline{r_M} - r_f)\beta_A\right]$$

$$\text{Information ratio} = RI = \frac{\alpha_A}{\sigma_{e_A}} \text{ where } e_{\mathrm{A}} \text{ is a portfolio's tracking error}$$

When should we use each benchmark?

- Depends on the investment assumptions
  - Case 1: If the portfolio represents the entire investment of an individual, then total volatility matters, and the Sharpe ratio is appropriate
  - Case 2: If the portfolio is to be mixed with a passive market-index portfolio, the information ratio is more relevant as it gives the improvement in the Sharpe ratio of the overall portfolio
  - Case 3 If a portfolio is just part of a diversified portfolio, then systematic risk matters, and the Treynor ratio is more appropriate
- Jensen's alpha is widely used as it indicates a superior performance

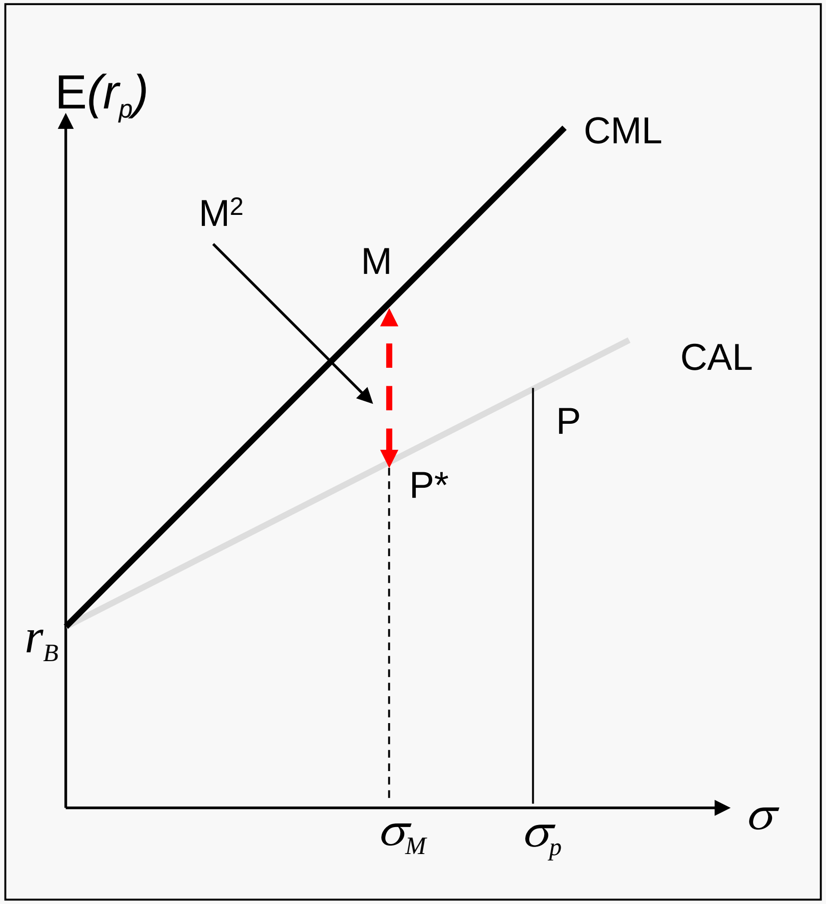
How should we use the benchmarks?

- In each measure, a higher value is better

- The benchmarks could be used to rank different portfolios

- However, it is sometimes difficult to interpret the meaning of each ratio directly
  - E.g. Say the Sharpe ratio of a market index is 0.7 and a portfolio has a Sharpe ratio of 0.65 (i.e. underperforms the market). What is the economic significance of a difference of 0.05?

The M^2 measure



- Popularized by Modigliani and Modigliani (1997)
- From portfolio P and the risk free asset, construct a portfolio P^* with the same σ as the market portfolio M
- Define M^2 as

  $$M^2 = r_{P*} - r_M = (S_P - S_M)\sigma_M$$

  where S_P and S_M are the Sharpe ratios of portfolios P and M
- Gives the excess return of the portfolio over the market portfolio
  - M^2 is negative in this example

Example

	Port P	Port Q	Market
Excess return (%)	2.76	7.56	1.63
s.d.	6.17	14.89	8.48
alpha	1.63	5.28	
beta	0.69	1.40	1.00
r2	0.91	0.64	1.00
sigma(e)	1.95	8.98	
Sharpe Ratio	0.45	0.51	0.19
Treynor Ratio	4.00	5.40	1.63
Information Ratio	0.84	0.59	
M2	2.16	2.68	

- Portfolio P has a lower beta but smaller residual risk (σ_e) than portfolio Q
- Both P and Q outperform the market (higher Sharpe ratios and positive alphas)
- Q has a higher Sharpe ratio, Treynor ratio and M^2 than P, therefore it is better in cases 1 and 3 (c.f. p.16)
- P is better if it is to be mixed with an index portfolio as it has a higher Information Ratio (case 2 in p.16)

Relationships between the ratios

$$RT_A = \frac{\overline{r_A} - r_f}{\beta_A} = \frac{\alpha_A}{\beta_A} + \left(\overline{r_M} - r_f\right) = \frac{\alpha_A}{\beta_A} + RT_M$$

- RT_M is the Treynor ratio of the market, with $\beta_M = 1$

$$RS_A = \frac{\overline{r_A} - r_f}{\sigma_A} = \frac{\alpha_A}{\sigma_A} + \frac{\beta_A\left(\overline{r_M} - r_f\right)}{\sigma_A} = \frac{\alpha_A}{\sigma_A} + \frac{\rho_{AM}\left(\overline{r_M} - r_f\right)}{\sigma_M} = \frac{\alpha_A}{\sigma_A} + \rho_{AM} RS_M$$

- RS_M is the Sharpe ratio of the market, ρ_{AM} is the correlation coefficient between the portfolio and market, which is often smaller than 1
- If the portfolio is well diversified and $\rho_{AM} \approx 1$, then

$$RS \approx RT / \sigma_M$$

Interpreting the ratios

- From the previous results, we can see that a higher α will lead to higher Sharpe and Treynor ratios

- We note the following:
  - A portfolio A with a positive α_A may not always result in a higher Sharpe ratio than the market portfolio

$$RS_A - RS_M = \frac{\alpha_A}{\sigma_A} + (\rho_{AM} - 1)RS_M$$

    which can be negative if ρ_{AM} is small
  - Sharpe and Treynor ratios may rank portfolios differently as α_A is used differently

Financial instruments with non-linear payoffs

- The standard ratios are appropriate for used in measuring portfolios with assets that have linear payoffs
- There exist a large class of instruments with non-linear payoffs, such as financial derivatives, which can give rise to different behavior for the ratios
- An important example is an option, where the holder has the right, but not the obligation, to choose whether to exercise the contract or not

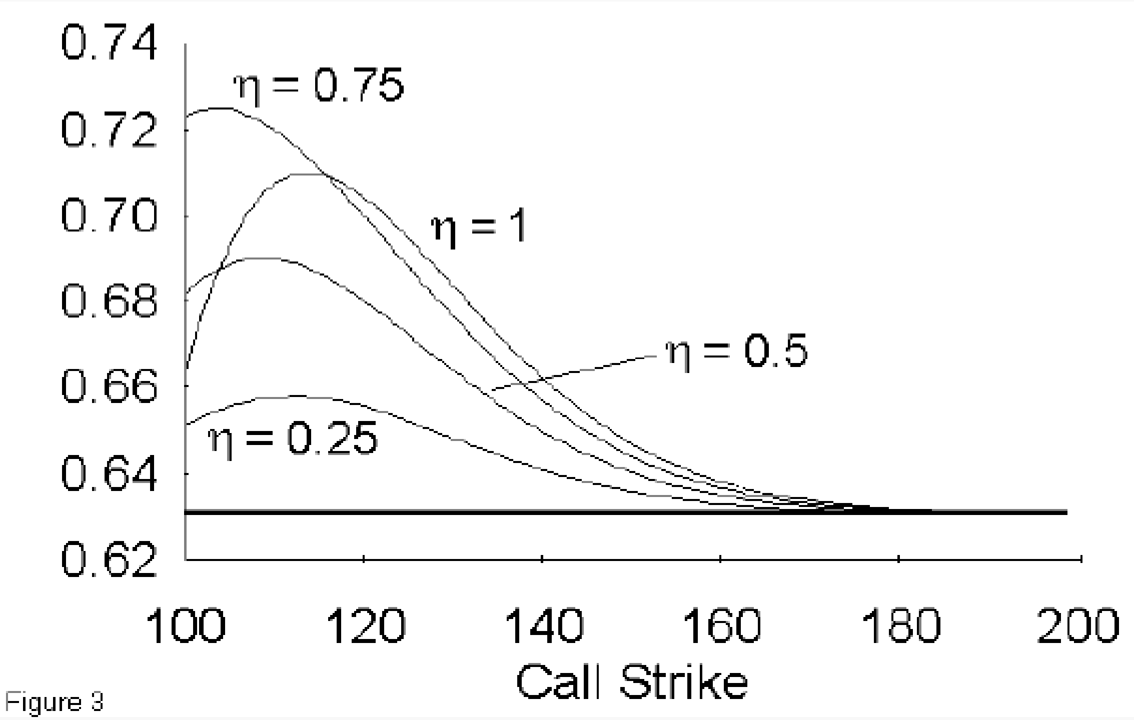
Call and put option examples

- European call option on HSBC, strike price 65
  - payoff formula: max(S_{final} – strike, 0)
  - at maturity, if spot price = 70, payoff per option is 70–65 = 5
  - if spot price = 60, option is worthless
- European put option on HSBC, strike price 65
  - payoff formula: max(strike – S_{final} , 0)
  - at maturity, if spot price = 70, option is worthless
  - if spot price = 60, payoff per option is 65–60 = 5

Manipulating the Sharpe ratio

- Goetzmann et al. (2002) show some examples where the Sharpe ratio may become misleading
- Recall that we want a strategy to have the Sharpe ratio to be as high as possible
- A great strategy with low Sharpe ratio
  - Assume a certain analyst can pick stocks that can always outperform the other companies within an industry
  - One strategy is to long the outperforming stocks, and short the underperforming stocks in each sector
  - *This strategy will always provide a positive return*; however, because there are variations in returns (i.e. high variance), the Sharpe ratio can be low because the variance appears in the denominator

Manipulating the Sharpe ratio



- The Sharpe ratio is especially misleading when applied to portfolios with options
- In this example, Sharpe ratio of the stock is 0.631
- A portfolio is formed that consists of long 1 share and short η calls
- A Sharpe ratio above 0.73 can be achieved
- From Goetzmann et al. (2002)

Dispersion measures

- Dispersion measures are measures of uncertainty, which consider both positive and negative deviations from the mean
- The most well known measure is the standard deviation of returns
- Other measures include:
  - Mean-absolute deviation (MAD): $E\left(\left|\sum_{j=1}^{n} w_j r_j - E\left[\sum_{j=1}^{n} w_j r_j\right]\right|\right)$
  - Mean-absolute moment: $\left\{E\left(\left|\sum_{j=1}^{n} w_j r_j - E\left[\sum_{j=1}^{n} w_j r_j\right]\right|^q\right)\right\}^{1/q}$

Alternative definitions of alpha and beta

- CAPM assumes that asset returns are normal and investors have mean-variance preferences (i.e. ignore skewness)
- For portfolios with arbitrary return distributions, the following modifications are used (Leland (1999)) :

$$CAPM\ \beta_P = \frac{\mathrm{cov}(r_P, r_M)}{\sigma_M^2},\ \text{Adjusted } B_P = \frac{\mathrm{cov}[r_P, -(1+r_M)^{-b}]}{\mathrm{cov}[r_M, -(1+r_M)^{-b}]}$$

- If excess return of the market is normal, b=1
- If excess return of the market is lognormal

$$b = \frac{\ln[E(1+r_M)] - \ln(1+r_f)}{\mathrm{var}[\ln(1+r_M)]}$$

- Adjusted Alpha is calculated using the CAPM formula, but with the adjusted B_p

Alternative definitions of alpha and beta

Long the market portfolio, short one call

Strike	E(r)	β	α	adj B	adj A
90	5.51%	0.038	0.24%	0.073	0.00%
100	6.76%	0.163	0.62%	0.251	0.00%
110	8.61%	0.394	0.85%	0.515	0.00%
120	10.27%	0.650	0.72%	0.753	0.00%
130	11.30%	0.838	0.57%	0.900	0.00%
140	11.77%	0.939	0.20%	0.967	0.00%

Long the market portfolio, long one put

Strike	E(r)	β	α	adj B	adj A
90	11.49%	0.962	-0.24%	0.927	0.00%
100	10.24%	0.837	-0.62%	0.749	0.00%
110	8.40%	0.606	-0.84%	0.485	0.00%
120	6.73%	0.351	-0.72%	0.247	0.00%
130	5.70%	0.163	-0.44%	0.101	0.00%
140	5.24%	0.062	-0.19%	0.034	0.00%

- Computed assuming a lognormal portfolio with expected return 12%, s.d. 15%
- α is computed with a risk free rate of 5%
- The factor b is 3.63
- From Leland (1999)

Downside measures

- Instead of allocating risks equally between upside and downside returns, downside risk measures only take into account of the losses with respect to a certain per-defined level
- An example is the Roy's safety-first criterion: portfolio optimization is based on minimizing the probability that the portfolio return will be lower than a threshold R_0
- By Tchebycheff's inequality, it can be shown that

$$P(R_P < R_0) \le \frac{\sigma_P^2}{\overline{R_P} - R_0}$$

where $P()$ denotes a probability function

Downside measures

- The Sharpe ratio is based on a dispersion risk measure around the mean, thus variations above or below the mean have equal contributions
- The following two measures only take into account of the asymmetry and the downside risks
  - Semi-variance

$$SV(R) = E\left[\left[(E[R]-R)^{+}\right]^{2}\right]$$

  - Lower partial moment

$$SV(R) = E\left[\left[(E[R]-R)^{+}\right]^{p}\right]$$

where p is an integer > 2

Sortino ratio

- A very popular ratio used for comparing hedge fund performances
- Only takes into account of the loss expectations

$$Sor(R) = \frac{E(R) - L}{\sqrt{E\left[\left[(L-R)^{+}\right]^{2}\right]}}$$

  where L is a minimal acceptable return level
- Conceptually it is similar to the Sharpe ratio
- A few choices of L
  - To control the loss risk, $L = 0$
  - The riskless rate r_f can be used as the benchmark, i.e. $L = r_f$
  - L can be chosen as the mean expected returns of other funds

Example of hedge fund performance

Illustration 5: Hedge fund strategies' risks for the period from 1999 to 2008

	Risk Dimension				Risk-Adjusted Performance	
Reference period: January 1999-December 2008	**Maximum Drawdown (in %)**	**Volatility (in %)***	**Downside Risk (in %)***	**Modified Value-at-Risk (in %)*****	**Sharp Ratio*/****	**Sortino Ratio*/****
Convertible Arbitrage	29.27%	6.74%	8.69%	3.55%	0.24	0.26
CTA Global	11.68%	8.80%	4.85%	3.52%	0.48	1.01
Distressed Securities	22.60%	5.88%	5.56%	2.50%	1.04	1.22
Emerging Markets	34.54%	11.60%	9.04%	5.05%	0.65	0.91
Equity Market Neutral	11.08%	3.17%	4.34%	1.28%	1.05	0.91
Event Driven	20.07%	5.83%	5.34%	2.56%	0.84	1.04
Fixed Income Arbitrage	17.60%	4.21%	6.44%	2.03%	0.36	0.33
Global Macro	7.92%	5.27%	2.62%	1.56%	0.95	2.16
Long/Short Equity	21.04%	7.60%	5.21%	3.14%	0.51	0.86
Merger Arbitrage	5.65%	3.58%	2.95%	1.31%	1.21	1.68
Relative Value	15.94%	4.52%	5.27%	2.08%	0.81	0.82
Short Selling	36.30%	17.74%	11.31%	7.91%	-0.01	0.05
Funds of Funds	20.22%	6.18%	4.78%	2.43%	0.50	0.78

- Source: Veronique Le Sourd, Hedge Fund Performance in 2008, EDHEC Risk and Asset Management Research Centre, Feb 2009

What is Value-at-Risk (VaR)?

- ***"An attempt to provide a single number summarizing the total risk in a portfolio of financial assets for senior management"***

  – Hull (2003)

- A typical statement is:
  - "We are X % certain that we will not lose more than V dollars in the next N days"
  - X is the confidence interval; usually X=95 or 99
  - N is the holding period; N=1 day or 10 days are common
  - V is the VaR

What are the VaR parameters?

- *Normal distribution is commonly used* because of the Central Limit Theorem and its analytical tractability
- **Holding period** reflects the liquidity risk; should roughly equal to the time needed to liquidate the portfolio
- **Confidence level** is arbitrary; a consistent measure should be used to compare the change in VaR on a daily basis

VaR for linear portfolios

- For a portfolio with no options

$$\text{VaR} = a \times \text{expected \% move in portfolio value}$$

- If we assume a normal distribution, at 99% confidence interval, a = 2.33; at 95% confidence interval, a = 1.96

$$\text{expected \% move in portfolio value} = N \times \sigma_{\text{p}} \sqrt{t}$$

  where N is the portfolio value, σ_p is the portfolio s.d. and t is the holding period (in yrs)

- If we assume assets prices follow normal distributions, the portfolio variance is given by

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{j=1}^{N} \sum_{\substack{k=1 \\ k \neq j}}^{N} w_j w_k \sigma_{jk}$$

How do we compute VaR?

- Mark to market of the current portfolio (e.g. $100 Million)
- Measure the variability of the risk factors (e.g. 15% per annum)
- Set the time horizon or the holding period (e.g. 10 business days, out of 252 business days per year)
- Set the confidence interval (e.g. 99%, or 2.33σ for a normal distribution)

$$\text{Value-at-Risk} \quad V = N \times \sigma_p \times \sqrt{t} \times a$$

$$= 100 \times 0.15 \times \sqrt{\frac{10}{252}} \times 2.33$$

$$= \$6.96 \text{ Million}$$

VaR for portfolios with derivatives

- A complex (but realistic) topic
- Because of non-linear payoff, the risk profile could be highly irregular
- If the portfolio contains lots of option positions, the only sensible method is to perform a full scale scenario analysis, which is very computationally intensive
- However, a number of simplifications has been made, but we need to understand the limitations of each method

Delta-normal method

- Replace each option position by its delta equivalent (c.f. lectures 7 to 9)

$$\Delta = \frac{\delta P}{\delta S} \Rightarrow \delta P = \Delta \delta S = \Delta S \delta x$$

- δx is the % change in the stock price in one day
- For a portfolio with n stocks

$$\delta P = \sum_{i=1}^{n} \Delta_i S_i \delta x_i$$

- We could then use the equation on p.35, using δP to replace the term $N\sigma_p$

Delta normal method: example

- Delta of HSBC options = 800 shares
- Delta of Sun Hung Kai (SHK) options = 1200 shares
- HSBC share price = \$130, SHK share price = \$72
- Volatility of HSBC = 25%, SHK = 30%, correlation = 0.60
- Portfolio standard deviation =

$$\sigma_{HSBC} = 800 \times 130 \times 0.25 = 26000$$
$$\sigma_{SHK} = 1200 \times 72 \times 0.30 = 25920$$
$$\sigma_P = \sqrt{26000^2 + 25920^2 + 2 \times 26000 \times 25920 \times 0.6}$$
$$= 46439$$

- VaR at 99% confidence interval is $2.33 \times 46439 \times \sqrt{1/252} = \$6816$

Historical simulation method

- Use the daily historical return and apply to the current market prices in order to generate a possible future scenario
- Simple to implement if data are readily available
- Allows non-linearities and non-normal distributions and correlation behaviour
- Difficulty in getting sufficient data
- One sample path used only, hence it could only be used as a reference and should be supplemented with other measures
- *History may not be a reliable guide of the future!*

Historical simulation example

Reference date	2/15/2013		
spot fx	1.6850	Position	
R(USD)	2.700%	USD fwd	16,760,000
R(GBP)	4.200%	GBP fwd	-10,000,000
days to maturity	90	Current MTM (USD)	-29,659

	days to				Absolute daily change			Simulated movements				
	maturity	spot FX	R(USD)	R(GBP)	FX % retur	R(USD)	R(GBP)	FX	R(USD)	R(GBP)	MTM	P&L
7/29/2010	90	1.5425	4.8750%	5.6250%				1.6850	2.7000%	4.2000%	-29,659	
8/1/2010	87	1.5360	4.8125%	5.6875%	-0.00421	-0.0625%	0.0625%	1.6779	2.6375%	4.2625%	43,613	73,272
8/2/2010	86	1.5355	4.8125%	5.6250%	-0.00033	0.0000%	-0.0625%	1.6774	2.6375%	4.2000%	45,891	2,278
8/3/2010	85	1.5573	4.8125%	5.5000%	0.01420	0.0000%	-0.1250%	1.7012	2.6375%	4.0750%	-195,480	-241,371
8/4/2010	84	1.5357	4.7500%	5.4375%	-0.01387	-0.0625%	-0.0625%	1.6776	2.5750%	4.0125%	37,677	233,157
8/5/2010	83	1.5436	4.7500%	5.5625%	0.00514	0.0000%	0.1250%	1.6862	2.5750%	4.1375%	-43,761	-81,439
8/8/2010	80	1.5394	4.8750%	5.6250%	-0.00272	0.1250%	0.0625%	1.6816	2.7000%	4.2000%	-2,708	41,053
8/9/2010	79	1.5280	4.8750%	5.5000%	-0.00741	0.0000%	-0.1250%	1.6692	2.7000%	4.0750%	115,608	118,316
8/10/2010	78	1.5370	4.8750%	5.5000%	0.00589	0.0000%	0.0000%	1.6790	2.7000%	4.0750%	17,553	-98,055
8/11/2010	77	1.5580	5.0000%	5.5625%	0.01366	0.1250%	0.0625%	1.7019	2.8250%	4.1375%	-212,715	-230,268

- The P&L is ranked and the 95% worst case is selected as the VaR

Monte Carlo simulations

- Using a mathematical model (e.g. Geometric Brownian motion), generate paths of asset prices (say 100,000 paths)
- For each asset path, calculate the mark-to-market of the portfolio
- Rank the P&L of each path; the 99 (or 95) percentile return is the VaR
- Very computational intensive
- Strong assumptions in the underlying model (i.e. model risk is high)

Coherent measure of risk

- Let X and Y be two portfolios and $\rho()$ is the risk measure
- Conditions of coherence
  - Monotonicity: $\text{if } X \leq Y, \rho(X) \geq \rho(Y)$
  - Translation invariance: $\rho(X+k) = \rho(X) - k$
  - Homogeneity: $\rho(b \cdot X) = b \cdot \rho(X)$
  - Sub-additivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$

- VaR is not a good risk measure as it does not satisfy the last property!

- From P. Artzner, F. Delbaen, J.-M. Eber, D. Heath, "Coherent Measures of Risk", Mathematical Finance, 9 (1999) 203-28

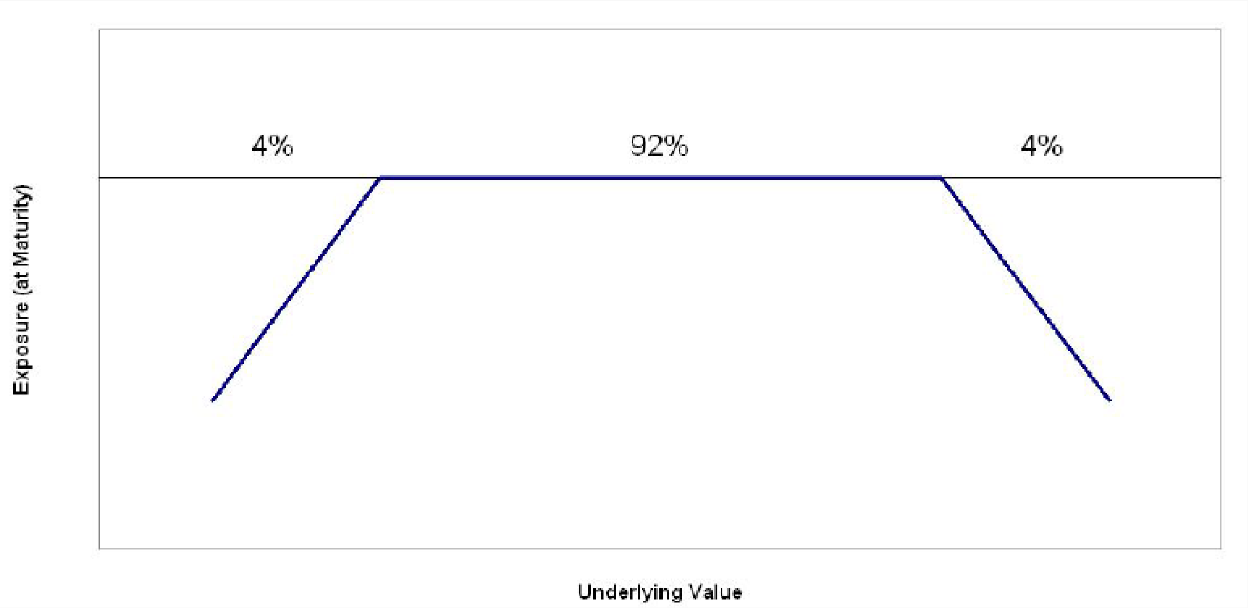
VaR and subadditivity

- Supposedly, given the effect of diversification, a portfolio with assets X and Y should have risks that are lower than the plain sum of the risks of these assets

- It is possible that the VaR of a portfolio can be higher than the sum of the VaRs of the individual assets of the portfolio

- Example (from Artzner et al. (1999)
  - Trader A sells an out-of-the-money call, with a probability of 4% that the call be in-the-money
  - Trader B sells an out-of-the-money put, with a probability of 4% that the put be in-the-money

VaR and subadditivity



- The 95% VaR of each individual position is 0 since there is only a 4% probability that there will be a loss (VaR(A)+VaR(B)=0)
- The 95% VaR of the portfolio is higher than 0, since the portfolio only has a probability of 92% that it will not end up with a loss

A further example

- Consider the following :
  - Each of two independent projects has a probability 0.98 of a loss of $1 million and 0.02 probability of a loss of $10 million
  - The 97.5% VaR for each project is $1 million

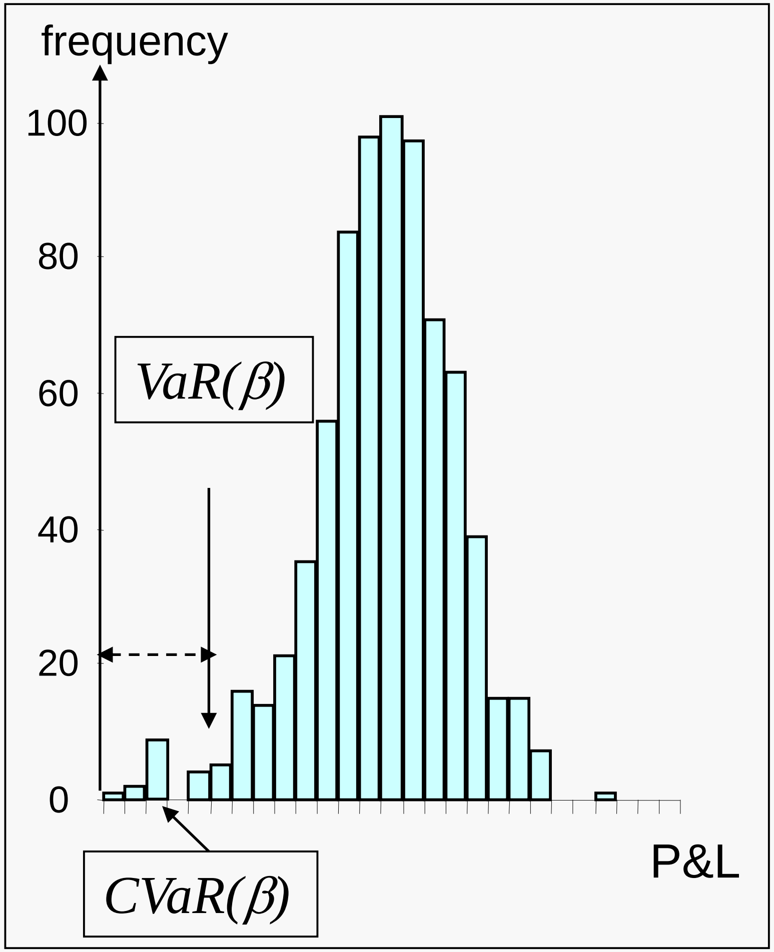
- When combining the two projects into a single portfolio, the loss distribution can be calculated as follows:
  - Probability = 0.02 x 0.02 = 0.04% of a loss of $20 million
  - Probability = 2 x 0.02 x 0.98 = 3.92% of a loss of $11 million
  - Probability = 0.98 x 0.98 = 96.04% of a loss of $2 million

- The 97.5% VaR of the portfolio is thus $11 million, which is much higher than the sum of the individual VaRs

Conditional value-at-risk (CVaR)

- Also known as the Expected shortfall
- VaR is the “best outcome of a set of bad outcomes on a bad day”
- CVaR is the “average bad outcome on a bad day”
  - Measures the expected amount of losses in the tail of the distribution of possible portfolio losses
- Calculating CVaR can be computationally more intensive and requires a more detailed description of the loss distribution

Conditional value-at-risk (CVaR)



- Measures the expected portfolio loss rather than the quantile
- Value-at-Risk :
  - Pr(loss <= VaR(β)) = β
  - e.g. β = 99%
- Conditional VaR
  - CVaR(β) = E(loss | loss> VaR(β)) >= VaR(β)

Possibility of extreme events

- Normal distribution implies a very small probability for extreme moves (e.g. 3 standard deviations)

- Extreme moves do occur much more frequently (e.g. stock market crash in 1987, Asian crises in 1997, 9/11 event in 2001, financial crisis of 2007/2008, etc.)
  - “Black Swan” events

- Other fat-tailed distributions are used, e.g. Student t distribution, Extreme Value Theory
  - Not so analytically tractable

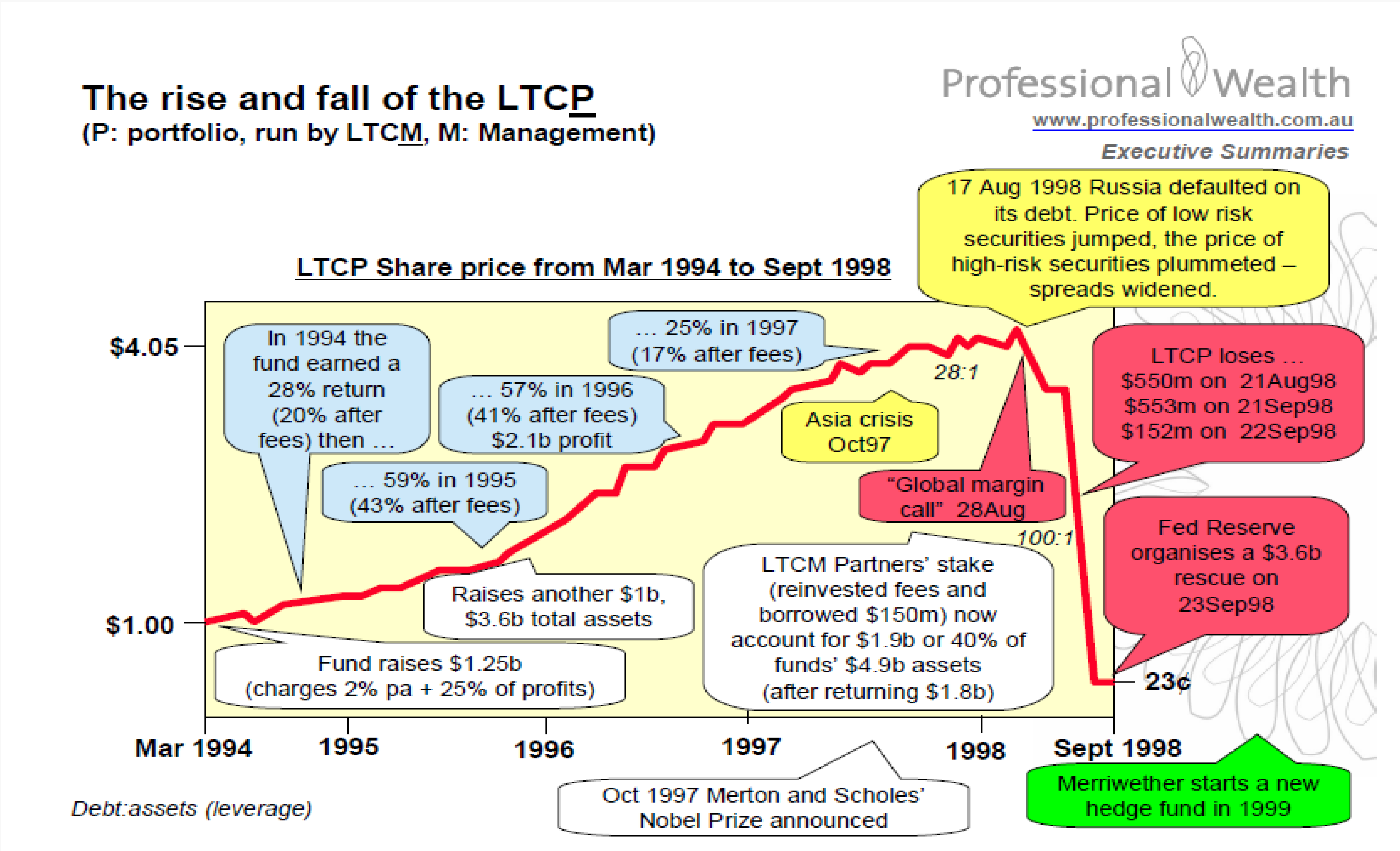
Case study: rise and fall of LTCM

- Long Term Capital Management (LTCM) was a high profile hedge fund, established in 1994
  - Partners include a former head trader from Salomon Brothers, (future) Nobel prize winners, and a former deputy governor of the Federal Reserve Board
  - Very successful in 1994-1996; performance not as good in 1997
  - Capital at US$ 4 Billion at the start of 1998

Case study: rise and fall of LTCM

- **Events in 1998**
  - Salomon Brothers, which allegedly had similar positions as LTCM, liquidated its positions in May 1998, causing sharp marked-to-market losses in LTCM
  - Downturn in mortgage-backed securities market in June 1998 led to cut backs in LTCM's liquid positions (to reduce leverage)
  - Russia defaulted in August 1998
  - LTCM's faced margin calls and were forced to reveal its positions; more severe marked-to-market losses followed (single day loss of US$550 Million on Aug 21 and Sep 21, 1998)
  - Lost all the $4 Billion by late Sep 1998; the New York Federal Reserve organized a bailout at end of Sep 1998, to avoid systemic risk

Case study: rise and fall of LTCM



Case study: rise and fall of LTCM

- **Lessons**
  - Liquidity: unable to unload illiquid positions – had the strategy be sustainable, the losses could be much smaller
  - VAR: should use back-testing to validate results
  - Correlation risk: change in correlation in severe conditions; should check sensitivity of portfolio due to parameter changes

- P.S. had they been able to survive Sep 1998, the strategies rebounded and made back all the money in 1999

How should we use VaR?

- The VaR calculated is not a magic number; it should be used as one of the measures in managing risks
- 99% confidence interval = 1% of the time, or about 3 days per year, the loss could be greater than the calculated VaR
- Need to understand thoroughly the assumptions behind the VaR number
- *Should monitor the trend of the VaR of a portfolio*