1. Introduction and Fundamental Financial Concepts

1.1. The scope of this course

Computational finance is not a well defined subject area and many different topics can be discussed. In this course, we aim to learn some theories and try to apply these models and algorithms in managing the risks and making decisions regarding the trading of financial assets. In particular, we will introduce models in two main areas:

- Modern Portfolio Theory: concern with the equilibrium pricing of assets
- Pricing of derivative instruments and related concepts

Topics such as technical trading rules, time series analysis/forecasting models and high frequency trading algorithms will NOT be covered.

1.2. The different financial markets

1.2.1. Some financial jargons

We first look at some terms that often appear in financial conversations.

Buy/sell: The meanings should be obvious
Long/short: *Long* is just another term for buy, and *short* equals sell
Bid: The price that the buyer is prepared to pay
Ask: The price that the seller is willing to sell

For example, if HSBC stock price is quoted at \$63.00/63.05, it means that the bid price is \$63.00 and the ask price is \$63.05. No trade would occur unless a buyer emerges and is willing to pay the asking price, or a seller emerges who is willing to accept a lower price than the current asking price (i.e. \$63.05).

Take a "position": enter into a trade
Close a "position": enter into trade(s) to neutralize an existing position

Note that there is a difference between closing an existing trade and entering into an offset position with another party. For example, if Bank A starts with a long position in a financial contract with Bank B, and now no longer wants to bear the risk of having this position, it could either: i) go to Bank B and negotiate a close-out price, and close the existing trade; ii) go to Bank C and enter into an opposite trade with the same contract terms. However, in case (ii), while Bank A may not be further exposed to market fluctuations, technically it will still have two trades (with both Bank B and Bank C) remaining in its portfolio and some residual risks remain (e.g. it is possible that one of the parties can go bankrupt).

Risk (風險): This is a complex concept, and we will soon look at some more refined definitions. Basically we can say that if there is uncertainty in the outcome of an investment, with the possibility of suffering a loss, then the position is a risky position.

Hedge (對沖): Engage in a trading strategy to reduce the fluctuation in profit or loss. Normally this cannot be perfectly done unless identical positions are entered. Note that in the market there is a kind of company called the "Hedge Fund" (對沖基金) , which may not hedge most of its positions at all.

Arbitrage（套戥）: Engage in a trading strategy which is supposed to generate a "riskless" profit, i.e. no matter how the market will move in the future, a guaranteed profit can be made. Usually this is achieved by simultaneously entering into multiple trades in order to capture the inefficiency in the markets. Occasionally such opportunities can arise.

"P&L": A short form of "profit and loss"

1.2.2. Types of financial markets and their characteristics

The four major types of financial markets (or asset classes) are:

- Currency (Foreign Exchange, or FX) market 外匯: e.g. US dollar against Euro
- Fixed Income market (including the interest rate market) 定息工具
    - A significant sub-market under this area is credit trading 信貸
- Equity (or stock) market 股票
- Commodities market 商品 : e.g. gold, silver, coffee

The most active market is currency (in terms of monetary amount), followed by interest rate. Stock markets are not as active as currency and interest rate, although more people pay attention to the stock market than the others. The markets in the United States are the biggest in the world. In terms of total capitalization (i.e. size of the companies), the combined size of the New York Stock Exchange & NASDAQ is US$ 24.0 trillion (Dec 2013), whereas the average turnover US$166.4 billion (2013). In the fixed income market, the amount of all outstanding US government bonds (treasuries) is US$ 11.59 trillion (Q3 2013), with daily turnover US$545.4 billion (2013). For the US corporate bond market: outstanding US$ 9.586 trillion (Q3 2013) and daily turnover US$18.1 billion (2013). The average daily turnover in the FX market globally is US$5.3 trillion (April 2013). It is noted that 87% of all trades involved the US dollar.

In terms of comparison, we also give some statistics for the markets in Hong Kong and China. Total market capitalization of the Hong Kong listed stocks was HK$ 24.8 trillion (Dec 2016), and the average daily turnover was HK$ 66.9 billion (2016). The total market capitalization of Chinese listed stocks in Shenzhen and Shanghai was RMB 50.8 trillion (Dec 2016), and the average daily turnover was around RMB 500 billion (2016). In the Hong Kong interest rate market, the total outstanding amount of Exchange Fund Bills and Notes was HK$ 963.1 billion (Dec 2016), with average daily turnover HK$ 9.96 billion (Dec 2016). If we look at the FX activities in Hong Kong, the daily turnover was US$ 437 billion (April 2016).[1]

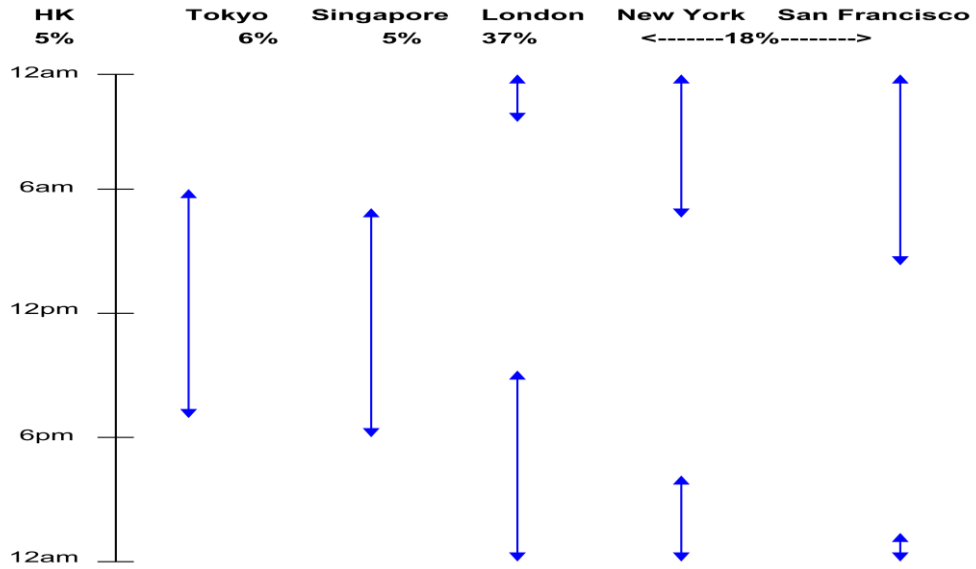
Another category which has seen tremendous growth in the last thirty years is the derivatives market 衍生工具市場. However, its nature is different from the markets listed

[1] Sources: Bank for International Settlements; World Federation of Exchanges; SIFMA; Hong Kong Exchange, HKMA

above, because derivatives is a sub-group which exists under each of the asset market, i.e. there can be currency derivatives, interest rate derivatives, equity derivatives, and commodity derivatives.

Currency Market

The global currency market consists of two main sectors: spot and forward markets. The main players are the central banks, major FX trading houses (mostly banks), and some customer demands. The *spot FX* market is a truly global and 24-hour market, because trading activities for the active currency pairs are not limited to one location, which means that the market is always open during the day.



A typical trading day lasts for 11 hours in a single location. It may start in the Asian time zone at 7am in Tokyo (6am in Hong Kong; Singapore is in the same time zone as Hong Kong). In the afternoon in Asia, the European markets will open, and the most significant centre is London (7am there is 3pm in Hong Kong). During the London afternoon, it is the morning in the US market (7am in New York is 8pm in Hong Kong and 3pm in London). The US late afternoon will again overlap with the morning in Asia on the next trading day.

The percentages under each centre in the above diagram represent the market share of these centers as of year 2010. London dominates this market, partly because of its advantage in geographical location, having significant overlaps with both Asia and US markets during the normal trading hours. Finally, the market is open on almost every week-day during the year. The only "global" holiday where no trading occurs is January 1.

Spot FX trading is a contract between two parties, i.e. the trading is not conducted through any intermediate party (c.f.. an Exchange, as in the case of stocks). In a typical trade, one of the parties will buy currency A and sell currency B at an agreed exchange rate at a certain notional amount. The trade is to be settled at *spot date*, usually two business days after the trade date. For liquid currency pairs, this is closest to the theoretical "perfect" market, and the bid/ask spread is very tight. For example, we can see a price of US dollar against Japanese Yen at a price quote of 116.75/77. Note that the quoting conventions for different currency pairs are fixed. Say if we see a rate for Australian dollars (AUD) at 0.7350, we understand that the convention is for AUD 1 = USD 0.7350, not the

other way round. Of course, we can express this price at USD 1 = AUD 1/0.7350 = AUD 1.3605, but no one in the market will quote a price like that.

Instead of the trade being settled at spot date, it can be settled at a future date, known as the *maturity date* (normally less than one year from today). This type of trade is known as the *forward FX*. A key characteristic is that there is no cost to enter into this trade on the trade date if it is traded at the market quoted price. Because of this feature, it is very convenient to use it as a speculation tool.

Hong Kong dollar is pegged to the US dollar at USD 1 to HKD 7.80. Supposed that some speculators think that the peg would be changed, and that HK Dollar would appreciate (go up in value). One way to express this view is to enter into a spot FX position, by buying HKD and selling USD. However this strategy requires substantial commitment in USD upfront. On the other hand, a typical strategy is to enter into a forward transaction, to buy HKD in (say) 6 months. There is no initial amount required, and the position could be closed out at any time before maturity.

USD/HKD 12-month FX forward (Data as of May 5, 2014; source: Bloomberg LP)

From the data above, we can see that while the HK dollar is fixed at the pegged rate, the forward price can still fluctuate a lot. During the Asian financial crisis in 1998, the 12-month HKD forward actually traded at around USD 1 = HKD 8.5000; while in late 2004, it traded at around USD 1 = HKD 7.6200.

USD/CNY FX forward (Data as of May 5, 2014; source: Bloomberg LP)

<HELP> for explanation.

USDCNH 99) Chart Spot/Forwards Monitor

China Offshore Spot

Market Type Regular Source BGNL BGN London

	Term	Pts Time	Pts Bid	Pts Ask	Spread	Outrt Bid	Outrt Ask	Time
1)	SPOT	22:43	6.2423	6.2430	0.0007	6.2423	6.2430	22:43
2)	ON*	22:43	2.97	4.53	1.56	6.2416	6.2426	22:43
3)	TN*	22:43	1.32	2.32	1.00	6.2421	6.2429	22:43
4)	SN							
5)	1W	22:43	12.50	15.50	3.00	6.2436	6.2446	22:43
6)	2W	22:43	27.68	32.32	4.64	6.2451	6.2462	22:43
7)	3W							
8)	1M	22:43	61.00	69.00	8.00	6.2484	6.2499	22:43
9)	2M	22:43	123.00	133.00	10.00	6.2546	6.2563	22:43
10)	3M	22:43	182.00	192.00	10.00	6.2605	6.2622	22:43
11)	4M							
12)	5M							
13)	6M	22:43	342.50	357.50	15.00	6.2766	6.2788	22:43
14)	9M	22:43	487.50	502.50	15.00	6.2911	6.2933	22:43
15)	12M	22:43	605.00	625.00	20.00	6.3028	6.3055	22:43
16)	15M							
17)	18M							
18)	2Y	19:20	1,027.26	1,097.74	70.48	6.3450	6.3528	22:43

* Near Leg Outrights Precision 4

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2014 Bloomberg Finance L.P.
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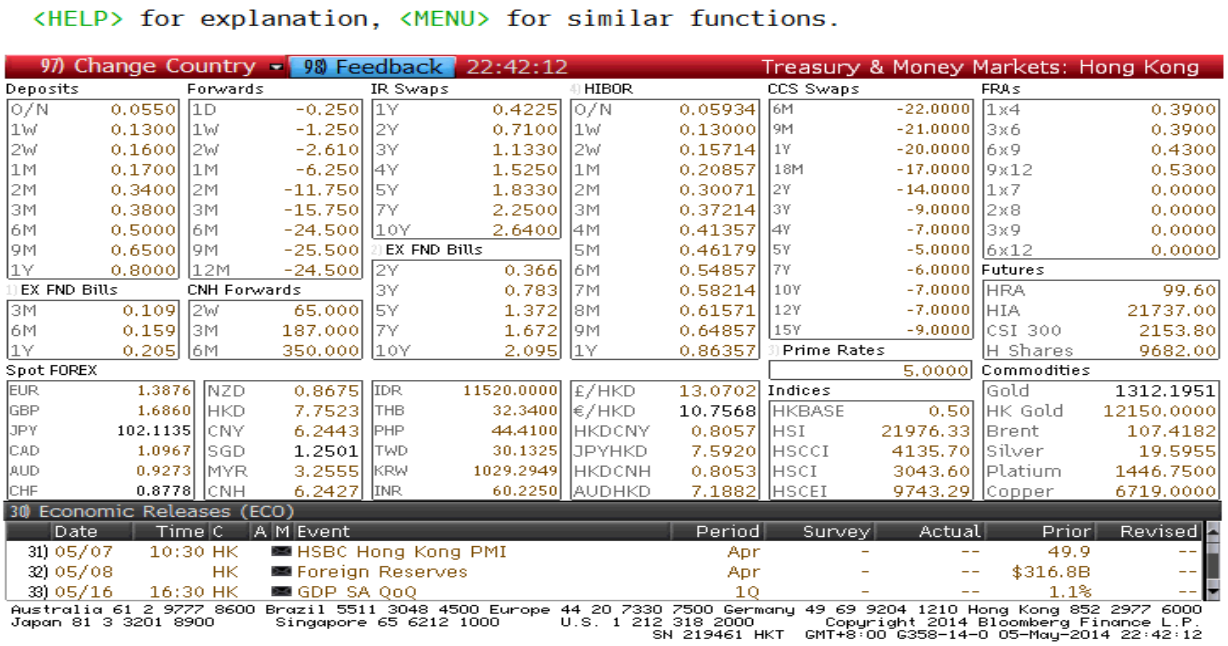
For the Chinese Yuan, the market consensus in May 2014 was that the currency will weaken in the next year, reversing the trend of expecting the Yuan to appreciate against the USD in the last few years.

Fixed Income market

Interest rate instruments are a means for governments and companies to borrow money. There are many different kinds of interest-rate related instruments, so one must be careful with the meaning of the quoted rate. For example, it is not enough to say that 1-year interest rate of USD is 0.75%, because the rate can refer to a few different types of financial instruments (as seen below).

In one of the sectors, often known as the fixed income market, the typical instruments are known as bills, notes and bonds. These terms usually refer to instruments issued by governments. The contract terms are similar, and the only difference is the time to maturity: bills have maturity less than 1 year, notes have maturity between 1 to 10 years, and the maturity for bonds is 10 years and above, which can sometimes go to 30 years or beyond. Examples of government-issued bonds include the US Treasuries and the Exchange Fund Bills and Notes from Hong Kong. Some HKD market instruments are shown below (data as of May 5, 2014; source: Bloomberg LP):



Instruments issued by corporates (i.e. private companies) are usually known as "bonds", irrespective of the time to maturity. An example of a typical bond has the following terms:

- Issuer: HSBC Holdings PLC
- Currency: USD
- Issued Date: May 3, 2006
- Maturity: May 2, 2036
- Coupon: 6.50%
- Payment Frequency: Semi-annual
- Face Value: USD 1000
- Credit Rating: A3 / A–
- Current price (as of 4/9/2014) = 128.32, equivalent to a yield of around 4.45%

Equity market

Although we will mostly use the equity market as an example in the remaining part of this course, we don't need to know the details of how the market works. Only two facts will be mentioned here. Firstly, stock trading is conducted through an exchange, i.e. you are trading (buying or selling) against an exchange, and the exchange will find the opposite party who is willing to sell / buy the stocks from you. While we normally think about trading stocks, actually many kinds of instruments are traded in an exchange. The most common types are:

- Stocks 股票
- Exchange Traded Funds (ETFs), especially based on Stock Market Index 指數基金
- Some derivative Instruments 衍生工具 e.g. warrants, index futures 窩輪，期指

On the other hand, some instruments are traded between two contract parties, such as:

- Mutual Funds基金
- Some derivative instruments, e.g. options, swaps 期權，掉期合約

- Structured products 結構性產品 e.g. Equity Linked Note 股票掛鈎高息票據

Secondly, we can look at a simple example regarding the different ways of making money in the stock market. Let's say HSBC shares are trading at HKD 64 per share when you enter into a position.

- If you are **bullish** (i.e. you think the shares would go up), then you should go long
- If you are **bearish** (i.e. you think the shares would go down), you should go short

Consider the following positions and scenarios:

Position (shares)	**P&L**	
	Share price = $60	**Share price = $70**
Long 10000	–$40000	+$60000
Short 4000	+$16000	–$24000

We can easily see that we can make money if we:

- Take a long position in stocks, and share price goes up;
- Take a short position in stocks, and share price goes down (note there may be regulations regarding short sell and not all shares in the markets can be short).

Alternatively, we can also try to construct a "synthetic" position in stocks, and hopefully the market view is correct, e.g. by taking a long position in a stock index (say Hang Seng Index or the Tracker Fund (2800)) and short most of the component shares in the index. (Construction of stock indices will be discussed in Topic 5.)

1.3. Fundamental financial concepts

1.3.1. Risk and return

Formulation of financial theory depends critically on the concept of the relationship between risk and return. In the example below, which strategy would you choose?

Strategy A:

- 50% chance of earning a 10% return
- 50% chance of losing 5%

Strategy B:

- 50% chance of earning a 15% return
- 50% chance of losing 12%

We can easily calculate the expected return of *A* is 2.5% (0.5 x 0.1 + 0.5 x (–0.05)) and for *B* is 1.5%. However, one may argue that the (high) possibility of losing 12% seems to be a bit excessive, and so this choice may not be attractive. From this simple example, we can see that higher absolute return may not always be the best strategy; we need to consider the expected return AND risk together.

Many important finance theories are based on the fact that people make rational decisions. A rational investor always wants to achieve *the highest return with the minimum risk.* But there have been challenges to this simple and beautiful theory. Let's look at the following scenarios.

Situation 1:
Expected return of a lottery is \$10; there is a small chance that you would earn nothing. It costs you \$9 to play this game. How many times would you want to play?

Situation 2:
Expected return of an investment is \$1 Million; there is a small chance that you would earn nothing, and it costs you \$900,000 to invest in this project. How many times would you want to invest?

The expected return in both these examples is about 11% higher than the cost, which means that in the long run, one should expect to make a return of 11%. The question is, would your answer be different in these two situations? Is it because of the high initial cost in the second case, and the fear of a loss of \$900,000 makes you reluctant to invest in the choice more frequently?

This has actually been a long standing problem, which was first noticed by Daniel Bernoulli in 1738. In the famous St. Petersburg Paradox, a coin is tossed until the first head appears. Let n be the number of times a tail appears before the first head is tossed. In this game, the payout is 2^n. For example, if there are 3 tails before a head appears, the payout is 2^3 = \$8. Now how much money would you want to pay to play the game?

To arrive at a mathematically correct answer, we need to calculate the expected payoff, which is given as:

No. of tails before a head appears	Probability	Payoff	Expected payoff
0	$(½)^1$	$\$2^0 = \$1$	½ x \$1 = \$½
1	$(½)^2$	$\$2^1 = \$2$	$(½)^2$ x \$2 = \$½
2	$(½)^3$	$\$2^2 = \$4$	$(½)^3$ x \$4 = \$½
3	$(½)^4$	$\$2^3 = \$8$	$(½)^4$ x \$8 = \$½
n	$(½)^{(n+1)}$	$\$2^n$	$(½)^{(n+1)}$ x $\$2^n$ = \$½

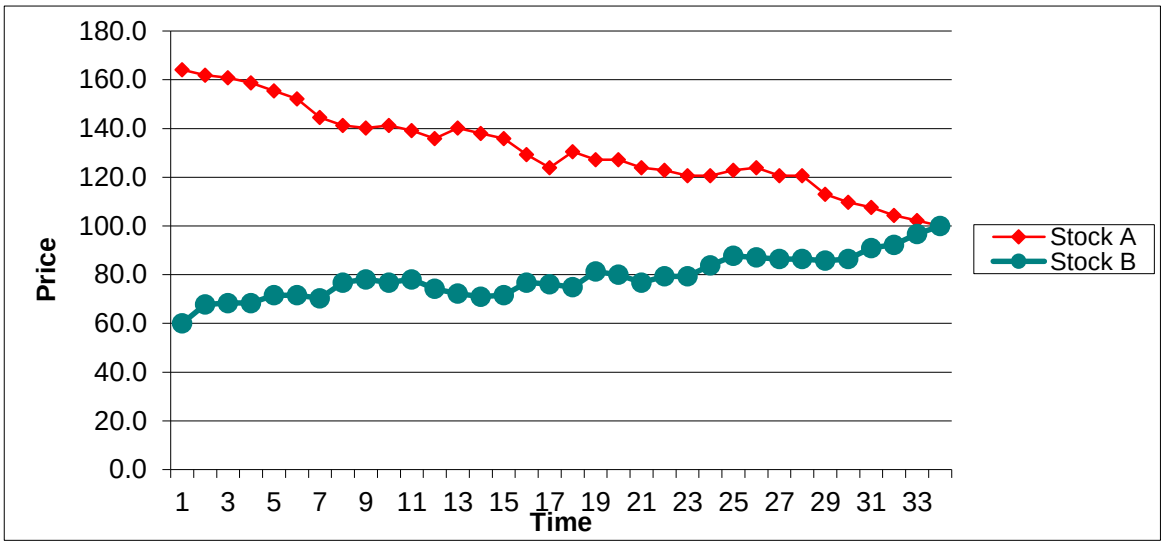
$$\text{Expected payoff} = \sum probability \times payoff = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots = \infty \ !$$

The fair price of this game should be close to the expected payoff, but most people would not pay a lot of money to play. The lesson is that even though we may have a correct mathematical model, the actual trading decision is more complex. We will come back and look at a solution to this problem when we talk about utility functions in the next chapter.

Another problem refers to a commonly encountered situation. Let's say that the stock prices of companies *A* and *B* are both equal to \$100 today, i.e. price of stock *A* = price of stock *B* = \$100. Furthermore, it is given that the expected standard deviation of price movement of stock *A* is equal to the expected standard deviation of price movement of stock *B*. Is it more likely for stock *A* or stock *B* to move to \$102 tomorrow?

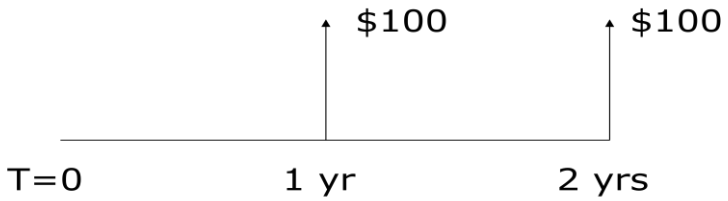
A celebrated finance theory called the Efficient Market Hypothesis (EMH) claims that all currently known information is captured by the stock price, and price movements follow

a "random walk" unless new information arrives. Therefore the chance of either of this stock moving to \$102 is going to be the same. However, would your answer be different if you are shown the following graph?



1.3.2. Time value of money

How do we compare cashflows occurring at different times?



The horizontal line is "time line" and the cashflows are represented by arrows lying above the line. In this example, assume that the interest rate is 5% per annum. If you need \$$P$ to get \$100 in 1 year, the formula to calculate \$$P$ is:

$$100 = P \times (1+5\%)^1, \text{ or } P = \frac{100}{(1+5\%)} = 95.24$$

Similarly, you need \$$Q$ to get \$100 in 2 years:

$$100 = Q \times (1+5\%)^2, \text{ or } Q = \frac{100}{(1+5\%)^2} = 90.70$$

In general,

$$P = 100 \times df \text{ where } df = \frac{1}{(1+r_t)^t}$$

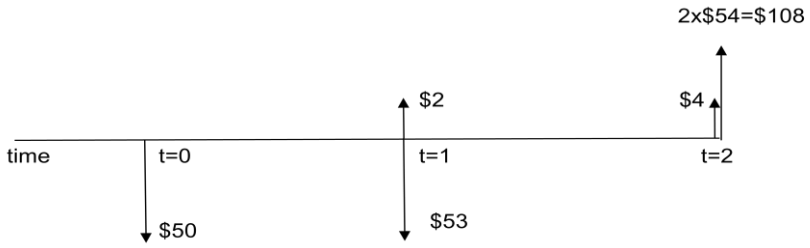
By definition, df is the discount factor at time t, and P is the present value of the 1-year cashflow.

1.3.3. Calculation of returns

The return of an investment is given by the simple formula: (end value – initial value) / (initial value). For example, if the initial investment is $1000 and after one year, this position is sold at $1200, the return is 20% p.a (= $1200 / $1000 – 1). However, in the case when there are multiple trades occurring at different times, there are different ways to represent the return of the investment.

We start with an example. The portfolio has the following cashflows:

- Bought 1 share at $50 at time 0
- Bought 1 share at $53 at time 1-year
- Sold 2 shares at $54 at time 2-year
- Receive $2 dividend at time 1-year
- Receive $4 dividend at time 2-year



We can calculate the dollar-weighted return, or more commonly known as the Internal Rate of Return (IRR) of an investment. This is a representation of the economic reality in terms of the cashflows. By equating the present values at time t=0,

$$50+\frac{53}{1+r}=\frac{2}{1+r}+\frac{(108+4)}{(1+r)^2}\Rightarrow r=7.117\%$$

Alternatively, we can ignore the number of shares held in each period and calculate the time-weighted return. In the previous example, the return in year 1 = (53+2)/50 – 1 = 10%, and the return in year 2 = (54+2)/53 = 5.66%. The arithmetic average of the 2-year return = (10%+5.66%)/2 = 7.83%, and the geometric average return = $[1.10\text{x}1.0566]^{0.5} - 1 = 7.81\%$.[2] In an extreme example, say the stock price starts at $100, goes up 100% in year 1 to $200, and drops 50% to $100 in year 2. The arithmetic average return is (100-50)/2 = +25%, while the geometric average return is 0 ($100 becomes $100 in 2 years).

Although it seems that the IRR is a better way to represent the actual investment result of an investor, it may not always be the benchmark quoted in the financial market. The reason is that while the IRR takes into account of the timing of the cash flows, a fund manager usually does not have any control on this timing (which depends on investors' subscriptions). Therefore time-weighted return can be more relevant to the fund manager's skills.

If we agree with this logic, we still need to choose between the arithmetic and geometric return. Geometric return takes into account of the holding period, and it calculates the historical performance accurately, i.e. this return accurately represents what the investor would get at the end of the investment period. However, for the prediction of

[2] Note that the geometric average must be smaller than the arithmetic average by a well known mathematical property.

future 1-period performance, the arithmetic average is the unbiased estimator. In the extreme example given earlier, if there is a 50% chance for each scenario, the expected stock price is (200+50)/2 = 125, which represents a return of 25%, as indicated by the arithmetic average return.

To conclude, it is noted that different methods of calculating the investment return exist, and which method is best depends on how the return is to be used in practice.