8. Practical issues in Hedging

8.1. Introduction

Hedging is an important concept in derivatives pricing and trading. Before we move to the more technical discussion of these concepts, we would first want to give a more general definition. When we say "the risks have been hedged," it is supposed to mean that there will be **no profit or loss** (P&L) when certain parameters move. In other words, if you manage to make money while you claim that the risks have been hedged, you must be doing a bad job! Furthermore, we need to refer to hedging against certain parameters, e.g. stock price, interest rates, volatility, and so on. A portfolio could usually not be completely hedged and there would be some residual risks. In order to make money from a position that would benefit from a certain view, one must leave the exposure to that parameter unhedged and the other risks are supposed to be hedged.

When people use the term hedging, it can be referred to two different cases. Firstly, we can talk about the optimal hedging of a portfolio, and the aim is to reduce the fluctuation in P&L during times of uncertainty. What we mean is, if we are not certain of the directional movement of the underlying variables, we may want to hedge the portfolio "approximately." This strategy is often used in managing a portfolio where the aim is to achieve outperformance of the underlying basket against an index.

The other case of hedging refers to the optimal hedge of an option position. According to the pricing theory, the option price is equal to the hedging cost. We need to be able to find the correct strategy so that the option position would result in no profit / loss when underlying parameters move. Note that for a trader managing a derivative portfolio, it is normal for the trader to adopt certain market views – the unhedged position(s) is/are the view taken by the trader.

8.2. Hedging Using Index Futures

To hedge the risk in a (long) portfolio with futures contracts, the number of contracts that should be shorted is

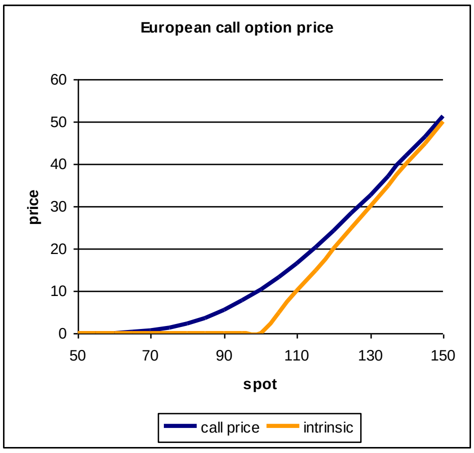
$$\beta \frac{V_A}{V_F}$$

where V_A is the value of the portfolio, β is its beta, and V_F is the value of one futures contract. Note that β is only a statistical measure; even if we hedge with this ratio, in the short term, the futures contract may not move synchronously with the portfolio which would result in fluctuations in profit or loss.

Example

Hang Seng Index futures contract is trading at 23000 and the value of a Hong Kong stock portfolio is HK$15 million, with beta of the portfolio = 1.5. The notional value of each futures contract is 23000 x $50 = $1150000 ($50 being the value of 1 point). Therefore the theoretical number of futures contracts to hedge the portfolio is: 1.5 x 15000000 / 1150000 = 19.56. Since we can only trade a round number of contracts, 19 or 20 contracts would be acceptable.

8.3. Option Risk Parameters



The above diagram shows a price profile of a typical European call option. As we can see from the above diagram, the interesting feature for most options is that the relationship between the option price and the underlying spot price is not linear. We can calculate the sensitivities of the option price to various parameters, commonly known as option "Greeks."

8.3.1. Common option sensitivities for vanilla options (Greeks)

If C is the option price and S, σ, t, and r take their usual meaning, some of the common sensitivities are defined as:

First order derivatives:

Delta	= Change in option price / change in share price;	$\Delta = dC/dS$
Vega	= Change in option price / change in implied volatility;	$v = dC/d\sigma$
Theta	= Change in option price / change in time to maturity;	$\Theta = dC/dt$
Rho	= Change in option price / change in interest rate;	$\rho = dC/dr$

Second order derivative:

Gamma = Change in delta / change in share price; $\Gamma = d\Delta/dS = d^2C/dS^2$

Obviously, shares are linear instruments, and therefore only delta is non-zero (delta = 1). A share position has no gamma, vega, rho or theta, i.e. gamma=vega=rho=theta=0. On the other hand, most options are sensitive to all the other parameters. Therefore, later when we look at the issue of option hedging, delta risk can be hedged with the underlying shares, whereas other types of option risks can only be offset with other instruments that are exposed to these risks, which means that one must use options to hedge an option position.

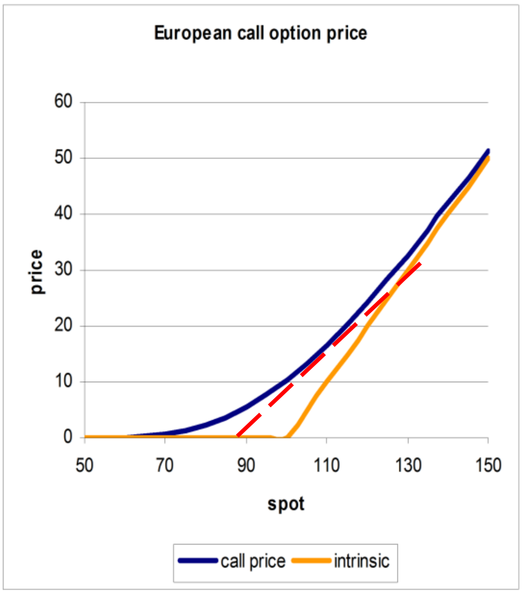
Black-Scholes Greeks

For a European call option on a non-dividend paying stock, under the Black-Scholes framework we have the following formulas:

$$\Delta = N(d_1)$$
$$\nu = S_0\sqrt{T}N'(d_1)$$
$$\Theta = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$
$$\rho = KTe^{-rT}N(d_2)$$
$$\Gamma = \frac{N'(d_1)}{S_0\sigma\sqrt{T}}$$

These results are obtained from a straightforward differentiation of the Black-Scholes formulas. Using the put-call parity, we can easily obtain the corresponding formulas for a put.

8.3.2. What is option delta?



Strike 100, maturity 3 months, r=2%, volatility 50%

As defined above, delta is the sensitivity of option price to share price, or we can write $\Delta = dC/dS$ where C is the option price and S is the underlying share price. For example, if delta = 0.74, it means that if share price moves up by \$1, i.e. $dS = 1$, we can calculate $dC = 0.74 \times 1 = \$0.74$, i.e. option price would go up by \$0.74. Delta can be interpreted as the slope of the tangent in the Option price–underlying price diagram.

In risk management, we often use the delta to represent the "share equivalent" of the option position (c.f. the delta-normal method in section 4.2.2). For example, if we are long 10000 options and the delta is 0.74, the position is treated as equivalent to long 7400 shares.

What is the Black-Sholes delta?

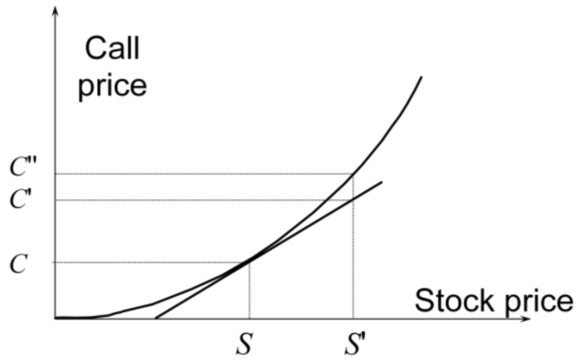
Under the Black-Scholes framework, supposedly if we keep the portfolio to be delta neutral everyday (i.e. delta of the portfolio plus any hedging instrument is always kept at zero), *the final amount in our account is the same as the option price*. Thus result is based on a few strong assumptions used in deriving the Black-Scholes formulas, such as the continuous re-balancing of the portfolio, interest rate and volatility are both constant and known. These assumptions are hard to fulfill in real life situations. For example, if we use a wrong volatility to calculate delta, we may not be able to recover the original option premium even though other conditions have been satisfied. As we have seen from the last topic, volatility is not a constant and it is impossible to predict the future volatility with accuracy.

Example of delta calculation

The delta of a European call option (Δ) with dividend yield d is: $\Delta = \exp(-dt)N(d_1)$.
If spot price = \$100, strike = \$100, time = 1, volatility = 30%, interest rate r = 3%, dividend yield d = 1%, we could calculate Δ = 0.5799. (c.f. section 6.2.1. for the notation of r and d) When spot price moves to \$120 and other parameters remain constant, Δ becomes 0.7872.

Note that *the delta for a vanilla option is always between –1 and 0 (for a put), 0 and 1 (for a call)* (check the limits given by the yellow lines in the diagram above).

8.3.3. Gamma



In the above section, we use delta to represent the share equivalent position of an option. However, because of the non-linear pricing profile, the error of using delta to represent the position gets bigger if the spot price moves away from the current price. Gamma measures the hedging error caused by the curvature.

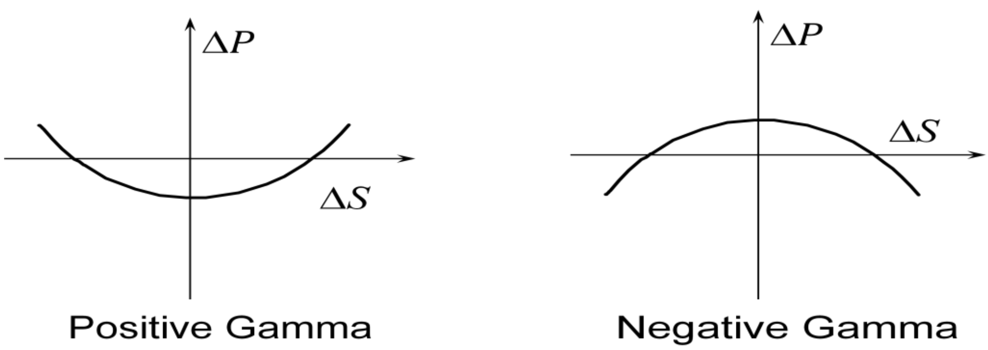
In the above diagram, when spot moves from S to S', the price change predicted by using delta alone would be C', whereas the correct option price should be C''. An adjustment term from gamma would make the approximated change much closer to the real value.

If P is the option price, a theoretical expression can be seen from a simple Taylor series expansion:

$$\Delta P = \frac{\partial P}{\partial S}\Delta S + \frac{\partial P}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 P}{\partial S^2}(\Delta S)^2 + \frac{1}{2}\frac{\partial^2 P}{\partial t^2}(\Delta t)^2 + \frac{\partial^2 P}{\partial S \partial t}\Delta S \Delta t + \ldots$$

Ignoring second order terms, this expansion links the various Greeks together, i.e.:

Change in option price = delta x dS + theta x dt + ½ gamma x $(dS)^2$

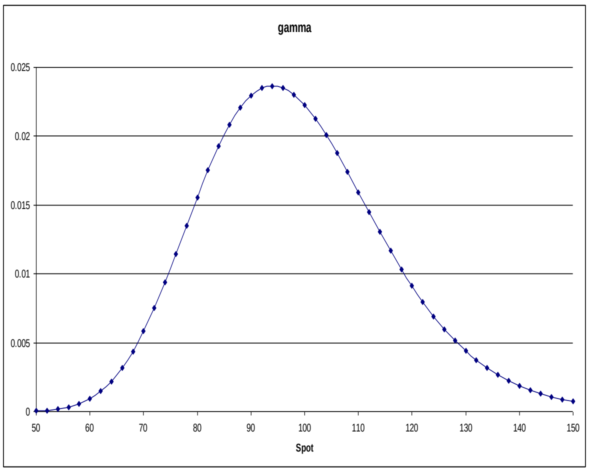


Portfolios can have positive or negative gammas, as depicted in the diagrams above. In addition, it is common to form a delta neutral portfolio, so that we can write:

$$\Delta P \approx \Theta\, \Delta t + \tfrac{1}{2}\Gamma \Delta S^2$$

Spot	Delta	Gamma
40	2.5	0.66
60	71.2	7.7
80	301.6	14.7
100	580.8	12.9
120	780.5	7.7
140	887.8	3.7
160	936.9	1.6

- Long 1000 European call option
- Maturity 1 year, interest rate 3%, strike 100, volatility 30%

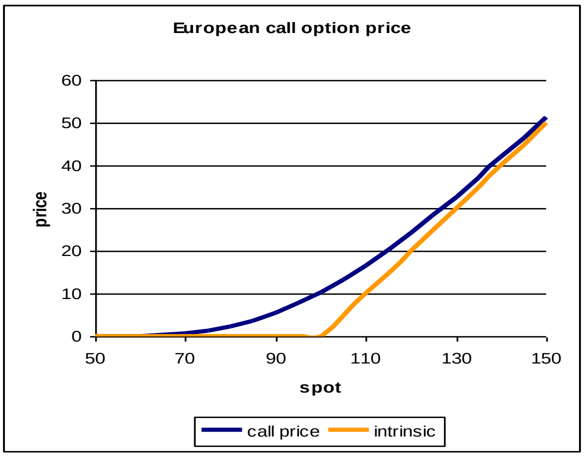


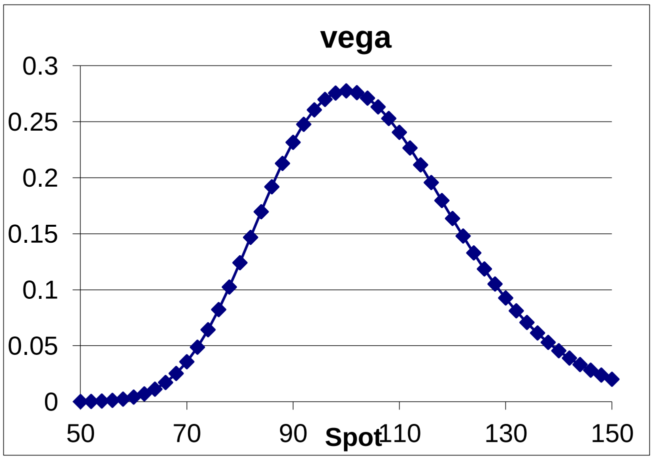
- European call, Maturity 6 months, volatility 25%, strike 100, rate 3%

In the above example, when stock price goes up, delta (expressed in shares) becomes higher, i.e. the position is equivalent to holding more shares. Conversely, when stock price goes down, delta becomes smaller. We can also attribute the effect to come from a positive gamma. This is a desirable position for the trader to have, as it limits the downside risk but will fully participate in the upside. In other words, higher delta means higher gain when stock price goes up, and vice versa. Of course this position is not free, and can usually only be achieved by holding a long option position, where there is a cost to enter into this position.

Mathematically speaking, we can show that gamma is highest when spot price is near to at-the-money. We can also see this effect in the earlier option price-spot price profile, where the curvature is greatest near the to the current spot price level.

8.3.4. Vega, theta and rho





Vega and theta can also be explained from the price/spot diagram for a call option. Basically, because of the non-symmetric payoff pattern, volatility increase means that there is more chance of higher payoff (because spot price can move higher). However, the maximum downside remains to be 0 (when option is out-of-the-money). As the option price is the probability-weighted payoff, it is evident that higher volatility would lead to a higher theoretical option price. Similar to gamma, vega of an option is highest when the spot price is near to the strike.

Components of option theta

The blue line in the above diagram represents the option price profile before maturity, whereas the orange line is the intrinsic value (or the payoff profile at maturity). In order to explain theta, we notice that the blue line would move towards the orange line as the option approaches maturity. Given that there is less chance for the spot price to move substantially beyond the current spot price, there is a reduction in time value of the option as it moves towards maturity (c.f. section 7.3.2.). Hence the theta of a long option position is often negative, which is also known as the *time decay* of an option. Another explanation of a negative theta is due to the reduced possibility of the spot price moving higher because of less time to maturity, thus lowering the expected payoff.

We also need to understand the effect due to funding cost. Let's assume we have a long position of 1000 shares and the current stock price is $100. If the stock price remains at $100 tomorrow, on paper there is no P&L, but in terms of the real costs, there would be a loss of one day of interest for $100,000. The correct calculation is $100 – $100 / (1+$r$/365) per share where r is the annualized interest rate.

We can apply this analysis to an option example. Given the following parameters: spot price = $100, strike = $50, rate = 5%, volatility = 10%, dividend = 0, European call. If the time to maturity is 365 days, the option price = $52.3810 (from Black-Scholes model). Given that the option is deep in-the-money, we can estimate its price from a cash flow analysis, i.e. calculate the forward value of $100, deduct the strike, and discount back to today, so that the option price is roughly:

$$= \{100 \times (1+0.05) - 50\} / (1+0.05) = 100 - 50/1.05$$

One day later, the time to maturity is 364 days, and the option price = 52.3747 (from Black-Scholes model, assuming parameters remain unchanged). This is roughly given by:

$$= 100 - 50/(1+0.05\times 364/365)$$

Obviously, the change in option price (i.e. the theta) comes from the 1-day discounting effect of the strike if option is deep-in-the-money.

A peculiar situation that can happen in practice is as follows. Occasionally some option positions are already deep-in-the-money and the trader has fully hedged the positions. For example, the portfolio is long deep-in-the-money call and short shares at 100% delta. This is a riskless position and should generate no P&L. However, in the trading system, it can show a P&L hit everyday if funding cost is not taken into account. The reason is because even if the stock price remains unchanged, the option position would show a small loss (due to negative theta) but the stock position would show no P&L.

Another look at theta

Theta can be mis-understood even by professional option traders, and one confusion comes from the difference between the settlement day calendar. For a trader, a common empirical definition of theta is the P&L difference between last night's closing portfolio value and the value of the portfolio before market opens today.

Let's say yesterday is Jan 28, 2014 (Tues) in Hong Kong and Jan 29 (Weds) is not a holiday. According to the discounting calculation in the section above, normally we would expect there will be one day of 'time decay' which would give rise to a negative P&L if we hold a long position in an option. In practice, a trading system would need to use the exact date schedule, i.e. cashflows would be discounted from settlement day to settlement day.

On Jan 28, assuming settlement is T+2, settlement day for the position is Jan 30. On Jan 29, because Jan 31 and Feb 3 are holidays in Hong Kong, the correct settlement day is Feb 4. If we use the example above, there is a 5-day time decay between Jan 28 and 29 (because the change in the discounting effect lasts from Jan 30 to Feb 4)!

If we use the option pricing formulas correctly, theta actually comes from two components. Firstly, in calculating the forward price of the underlying, the calendar days *between current date and maturity* are taken into account. Secondly, for the change of discounting effect, it needs to be derived from the calendar days *between settlement dates.* In the example above, if the maturity date of the option is Oct 30, 2014, we will have the following day schedule:

Current date (A)	**T+2 (B)**	**Maturity (C)**	**T+2 (D)**	**days for calculating forward (C)–(A)**	**days for calculating discounting (D)–(B)**
Jan 28, 2014	Jan 30, 2014	Oct 30, 2014	Nov 3, 2014	275	277
Jan 29, 2014	Feb 4, 2014	Oct 30, 2014	Nov 3, 2014	274	272
Movement				1	5

We can see that for the first effect, a movement of one day forward would account for a 1-day theta move of the change of forward, but the second effect represents a 5-day move of the change of discounting. Under normal circumstances, the first effect would have a bigger impact on option prices. However, for options that are deep-in-the-money, the second effect becomes dominant.

Components of option rho

In order to understand rho, it is instructive to look at Black (1976) again. The European call price is given by:

$$C = \exp(-rt)\left[FN(d_1) - KN(d_2)\right]$$
$$d_1 = \frac{\ln(F/K) + \sigma^2 t/2}{\sigma\sqrt{t}}, d_2 = d_1 - \sigma\sqrt{t}$$

We can identify two effects of *r*. Firstly, forward F involves *r* (=*S*exp(*rt*) if interest rate is not stochastic). Secondly, *r* appears in the term used for discounting the expectation of the payoff, i.e. exp(–*rt*). If interest rate goes up, F goes higher, which means that the terms in the bracket would become bigger. At the same time, the discounting factor is smaller, but this impact on the overall price is less. Therefore call price is more expensive, which means that the position has a positive rho.

8.3.5. Vanilla option characteristics and calculation

	Long call	Long put	Short call	Short put
Delta	+	–	–	+
Gamma	+	+	–	–
Vega	+	+	–	–
Theta	–	–	+	+
Rho	+	–	–	+

It would be convenient to summarize the properties of the basic options. For example, if we have a long European call position, the delta, gamma, vega and rho are all positive whereas the theta is negative. Knowing the properties of individual positions can help to understand how a portfolio behaves when more than one position is added together.

Sometimes there are no explicit formulas for the calculation of the risk parameters, whereas a model is available for calculating the option price. We can then make use of this pricing model to generate the risk parameters by first principles. For example, vega is defined as the change in option price divided by the change in volatility. We want to price a European call option with the following parameters:

- Spot price = \$100, strike = \$100, rate = 5%, maturity = 1 year, dividend = 0
- Volatility = 30%, price = 14.1730 (from Black-Scholes model)
- Volatility = 31%, price = 14.5529 (from Black-Scholes model)

Therefore the vega for this option = 14.5529 – 14.1730 = 0.3799 (approximately). In addition to the original price, only one additional call to the pricing routine is required (at volatility = 31% in this example). A slightly more accurate method is the center-difference approach which involves two calls to the pricing function. Vega is given by the difference between the prices obtained at volatility 30.5% and 29.5%, i.e. vega = Price(volatiliy+0.5%) – Price(volatility–0.5%). This method can also be used to obtain the other greeks.

8.4. Practical hedging issues

8.4.1. How should the portfolios be hedged?

By definition, if the portfolio is perfectly hedged, we could NOT make any money! Traders would only want to enter into trades such that they could earn money when they express some market views. Therefore hedging is only needed when one is not sure of market conditions. For example, we have bought some shares and are bullish in the long run, but there may be some short term uncertainty. We may consider shorting some futures as a hedge (c.f. section 8.2). More often, option traders can trade volatility, i.e. they want to express a view in volatility but not the direction of the market. One way to achieve this is to hedge the directional move of the underlying share price (i.e. have a delta neutral portfolio) but leave the volatility risk (vega) unhedged.

Just to recap the definitions, a portfolio is *delta-neutral* if there is no P&L movement when spot price moves; *gamma-neutral* if delta is unchanged when spot price moves, and *vega-neutral* if there is no P&L movement when implied volatility changes. Option portfolios are rarely neutral to all these parameters. Using the example from above, if we think that the volatility would increase but are unsure whether the market would go up or down, we could keep the portfolio delta neutral but vega positive in order to capture some profit if our market view is correct.

Static and dynamic hedge

We can distinguish the cases between a static hedge and a dynamic hedge. Static hedging means that once the portfolio is hedged, there is no need for any re-balancing before the maturity of contract. This is rare unless one has identified an arbitrage strategy. It is much more common for option positions to be dynamically hedged. The hedging portfolio has to be adjusted depending on market conditions. For example, one has to buy or sell the underlying shares to hedge delta, which would create trading profit/loss. As explained in the section below, theoretically this hedging cost is equal to the option premium.

Option portfolios are often exposed to risks coming from delta, gamma and vega, among others. Obviously only instruments with gamma could hedge gamma exposures, and only instruments with vega could hedge vega exposures. Therefore, we could hedge the delta with shares, but could only hedge vega and gamma with (any) option. From the earlier sections, we also note that gamma and vega profiles could change dramatically with time and spot price. When there is a choice of different hedging instruments, one has to consider the effectiveness of the hedge, as well as other practical issues, e.g. liquidity, bid/ask spread, transaction costs (see the section on second order changes below).

8.4.2. Issues in option hedging

Dynamic delta hedging is a key concept in the Black-Scholes framework. For example, an issuer sells 100,000 HSBC call options. Assuming that other pricing parameters are known, when spot price = \$90, delta is 55% (calculated from Black-Scholes formula). Following this result, in order to keep a delta neutral position the issuer needs to buy 55,000 shares (= 100000 x 0.55) at \$90 or better when the option is traded.

On day 2, assume that the spot price has now become \$93. The delta is now 61% (calculated using the new parameters). Therefore the issuer needs to buy another 6,000 shares at \$93 or better. It is because the new delta is 61,000 shares but the issuer already has 55,000 shares; therefore only another 6,000 shares need to be bought.

On day 3, let's say the spot price drops to \$88, and the delta is now 51% (again re-calculated from the formula). The issuer needs to sell 10,000 shares at \$88 or better to maintain a delta-neutral position (61,000 – 51,000 = 10,000 shares).

In the following table, we use a similar example as above, but the issuer sold 1000 options initially and received some option premium P. The delta hedging operation with the share account and the balance in the bank account on a daily basis is shown below:

	Initial Position	Share Price	Delta Required	New Trade	Amount	Cash Account Balance
	(A)	(B)	(C)	(D)=(C) - (A)	(E) = – (D) x (B)	(F) = (F)previous + (E)
Day 1	0	90	550	550	-49500	-49500
Day 2	550	93	610	60	-5580	-55080
Day 3	610	88	510	-100	8800	-46280

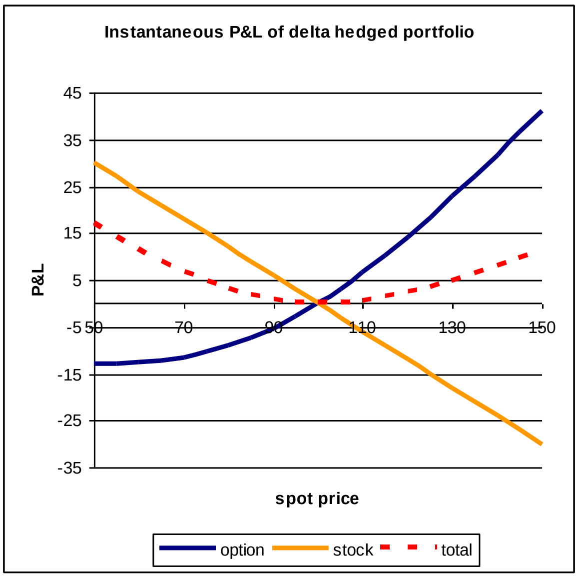
The process is repeated everyday until the option maturity. This strategy seems to lead to a certain loss, because basically we are buying shares at a high price and selling them at a low price. However, if the Black-Scholes theory is correct and the parameters are stable, *the final amount in the cash account should be equal to the initial option price, no matter whether the option is out-of-the-money or in-the-money at maturity*.

We can add three elements to make this calculation slightly more realistic. Firstly, interest cost is not negligible. In the above example, we need to borrow money to buy shares. From day 1 to day 2, the change in the account balance should take into account of the interest cost. For example, if the annual interest rate is 4%, the account balance should become –5580 – 49500x(1+0.04/365) = –55085.42 (assuming a 1-day difference in the settlement calendar). Secondly, buying and selling shares would incur transaction costs, which need to be taken away from the account balance (note that it is a *cost* no matter whether we are buying or selling shares). Thirdly, one might need to add back the effects due to dividends. In the case of a short position, the accrued dividends should be paid to the stock lender, so this amount has to be deducted from the account balance.

In practice, the Black-Scholes hedging scheme cannot work so perfectly. We already include transaction costs in the calculations above, but other factors which may affect the accuracy of the hedging operation include:

- Wrong estimate of volatility
- Intra-day movement (notice that volatility is usually measured based on closing prices)
- Gap movement in underlying price
- Liquidity constraints, i.e. it may not be possible to execute a big trading order without affecting the stock price.

P&L profile of delta-hedged position



The example in the previous paragraph looks at the effect of the P&L when the hedging action is continued everyday. Now we examine the hedging issue from a different angle. Assume the current spot price is $100 and we have a portfolio with long 1 European call option and short Δ shares, so that the portfolio is delta-neutral. This diagram shows the instantaneous P&L of this portfolio when underlying spot price moves, where the aggregate profile is shown as the red dotted line. Because the price profile (the blue line) is a convex function of the spot price whereas the hedge (the orange line) is a straight line, if stock price has suddenly moved away from $100, we would always make a profit! The profit increases if the stock price movement is bigger. This seems too good to be true, but of course there is no free lunch - *if stock remains unchanged tomorrow, there would be a loss on time value* (i.e. maturity is reduced everyday).

P&L attribution

Instrument	position	stock price ($)	volatility	MTM price	delta (shares)	vega ($)	P&L ($)
Day 1							
Call option	10000	100	25%	11.344	5788	3878	0
shares	–8000	100		100	–8000	0	0
total					–2212	3878	0
Day 2							
call option	10000	105	26%	14.894	6527	3797	35500
shares	–8000	105			–8000		–40000
total					–1473	3797	–4500

In this example, the portfolio has long 10000 call options and short 8000 shares as hedge. The portfolio is not delta or vega neutral: initially we are short delta (–2212) and long vega ($3878). When stock price and volatility both go up, the change in P&L can be positive or negative, depending on whether the delta effect or the vega effect is more dominant.

Assume that the stock price has increased by $5 and the volatility also moves up by 1% after 1 day. When delta and vega of the portfolio are available, we can make use of these parameters to estimate the P&L change when the parameters move (without using a pricing model to re-price all the positions). A rough estimate of the P&L is given as follows:

- P&L from delta = –1800 x 5 = –$9000; –1800 is roughly the average of –2212 and –1473
- P&L from vega = $3800, given that the volatility has moved up 1%.
- total = –9000+3800 = –$5200 (compared to the actual P&L of –$4500 from the pricing model).

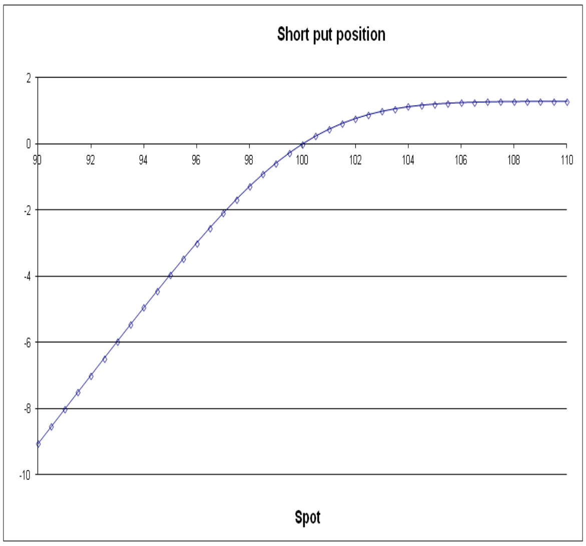
8.4.3. Hedging strategies

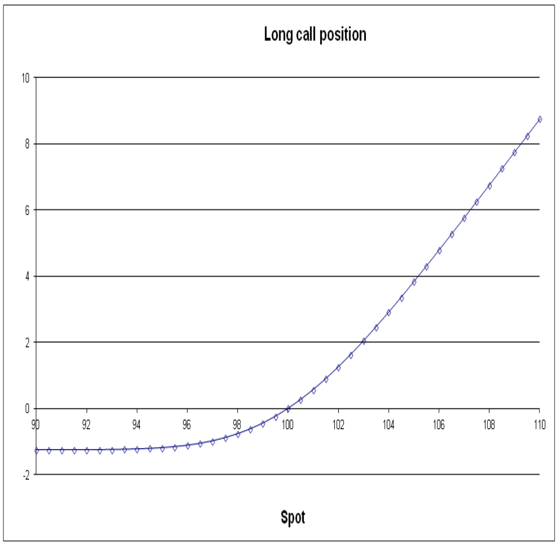
While the above sections try to analyze the risks of individual option positions, it is rare to hedge a single position. In practice, risks of a portfolio with many option positions would be managed together. The traders would need to keep within trading limits (typically with limits in delta, gamma, vega, theta). Typically traders would apply some kind of hedge to the first order price risk (i.e. delta), and trade the risks that come from the change in delta or adopt volatility trading strategies. Furthermore, unlike the case for demonstrating the Black-Scholes framework above, a hedge is only applied for short term and seldom throughout the life of an option, because supposedly the option position can be closed at any time.

A main difficulty in option risk management is the problem of aggregating the risks. Even if we consider the trading of a stock option portfolio with stocks from a single country, usually there are two levels of aggregation. Firstly, the aggregation can be performed based on each single underlying. For example, there can be many different options on a stock index with different strikes and maturities. The risks of options with all these different characteristics are simply added together, to form the total delta, gamma, vega, theta for this underlying. This is a crude method because risk parameters can change rapidly and changes in the market parameters can affect options with different characteristics in uneven ways (see an example in the section on second order changes below).

The second level of aggregation is to calculate the summary risk parameters across all underlyings, normally on a per country basis. For example, we can have a delta of $2 million (using a dollar amount instead of number of shares to represent the delta) for the Hong Kong portfolio. This is supposed to mean that when the market moves up 1%, the profit of this portfolio is roughly $20,000. Of course, this is only an approximation at best, since the inherent assumption is that the same shock (% move) is applied to all the underlyings. In reality, stocks would have different movements everyday. However, aggregate risks on a country level can be useful if a trading desk has a portfolio with positions in many different countries.

Using delta to represent position risk

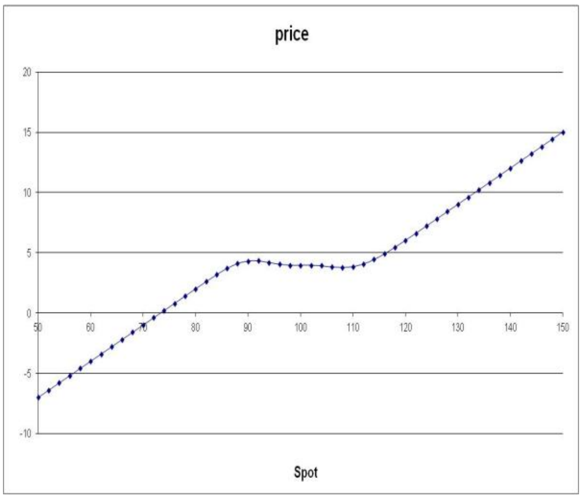


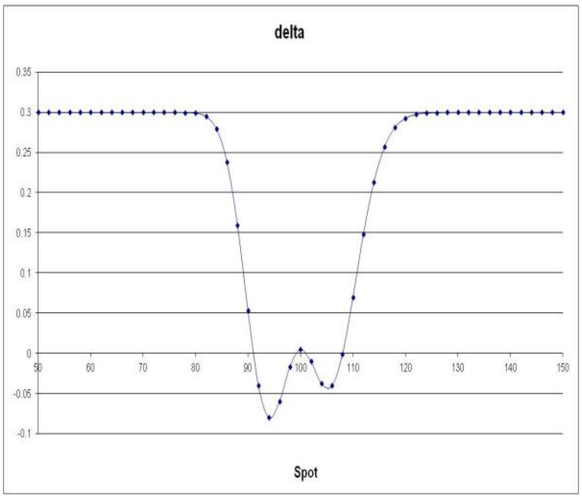


One must be beware of reading the risk parameters directly and not understand how they can move when underlying conditions change (even if second order parameters such as gamma are included). In both diagrams above, the delta at the current spot price (assume = $100) is the same; it is obvious that their risk profiles are very different. One method to deal with this problem is to separate between upside and downside risks, but it means that more calculations have to be performed to generate the risk parameters and aggregating them can still be a challenge.

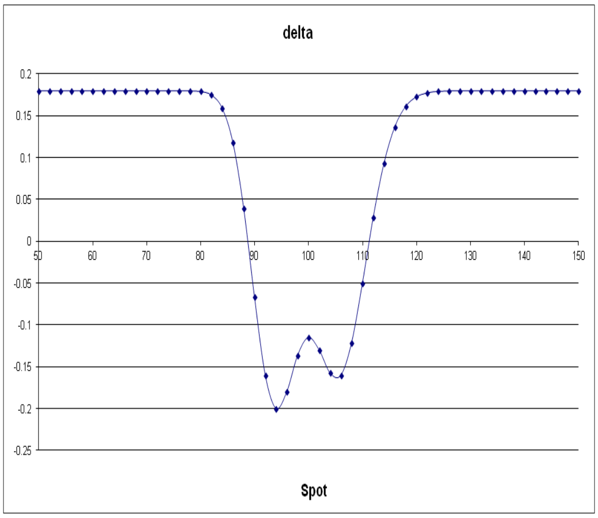
Delta hedging example

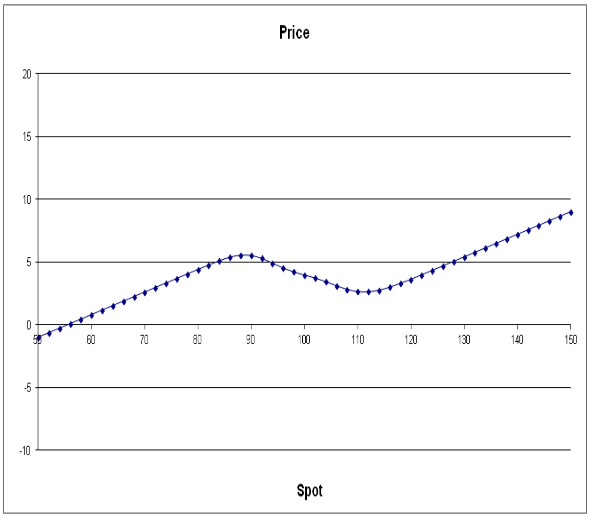
Traders may not always want to keep the portfolio exactly delta neutral (even if they don't want to express a view in the underlying change). Consider the following positions: short a 92 strike put, long a 96 strike call, short a 104 strike call, and long a 108 strike call. Current spot price = $100, r=1%, σ=16%, T=0.083 (1 month); delta of this portfolio = 0.7.





If we hedge the portfolio to make it delta neutral by shorting 0.7 share, the resulting price and delta profiles are shown in the above diagrams. It is evident that when the spot price lies between \$90 and \$100, the hedge is pretty effective and the change in price of the resulting portfolio is quite small. However, for big movements in the stock prices, the portfolio resembles a long stock position (with a constant delta of 0.3 for spot below \$80 and above \$120). There can be big variations in the portfolio prices.

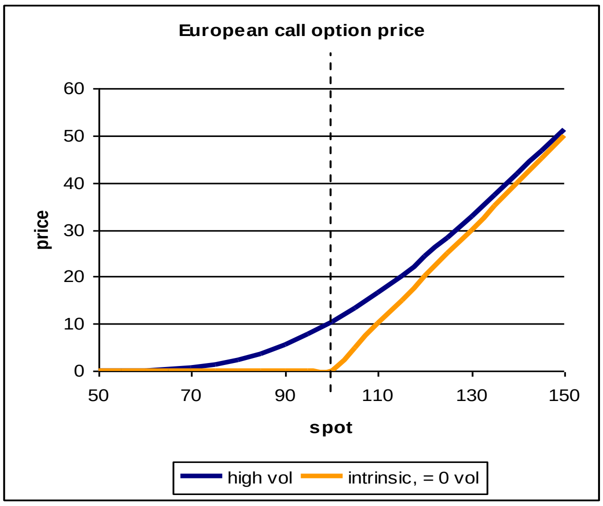




Sometimes traders would adopt a modified delta hedge. Instead of hedging with a delta of –0.7, the hedge applied is –0.82. It can be seen that while there may be some residual P&L around the current spot, the overall P&L is limited across a wider range of spot movement; this is useful especially when the spot is volatile.

Volatility and delta

In real market conditions, volatility is not a constant and sometimes we need to change the volatility that we use to price the options. If the portfolio is initially delta-neutral and we change the volatility marking, how would delta move?



The answer depends on whether the option is in-the-money or out-of-the-money. We know that as volatility decreases, the option profile would move towards the intrinsic value, i.e. the blue line would move towards the orange line if volatility becomes zero. Remember that delta is the slope of the tangent of the option payoff. In the case of a call option, if the option is in-the-money (i.e. the right hand side of the dotted line), the slope of the tangent would increase when the blue line moves towards the yellow line (from a positive number smaller than one towards one). On the other hand, when the option is out-of-the-money, the slope of the tangent would decrease (from a positive number smaller than one towards zero).

Second order changes

Assume the spot price is \$100, and we have the following position
- European call, strike \$100, maturity 1 year, volatility 25%, rate 4%; vega = 0.3831

We would like to hedge so that the portfolio is vega neutral, and the following instruments are available:
- Option A, European call, strike \$100, maturity 1 month, volatility 30%, rate 4%; vega = 0.1148
- Option B, European call, strike \$100, maturity 2 years, volatility 23%, rate 4%; vega = 0.5194

All these options are based on the same underlying stock. In order to match the vega of the original position, we could hedge with either 0.3831 / 0.1148 = 3.337 of option A, or 0.3831 / 0.5194 = 0.7376 of option B. The question is, which one should we choose as a better hedge?

From the trading point of view, option A is usually more liquid (because of the shorter maturity) and is preferable if we only need to consider the ease of entering and exiting the position. However, another consideration is the effectiveness of the hedge when market parameters move. The simplest case is if spot price stays at \$100, and both A and B seem to work fine. However, if spot price moves to \$110, the vegas would become (calculated from the Black-Scholes answer and based on the new parameters):
- Vega of original position = 0.3519
- Vega of A = 0.0400; net vega = 0.3519 – 3.337x0.04 = 0.2184
- Vega of B = 0.4869; net vega = 0.3519 – 0.7376x0.4869 = –0.0072

Therefore we can see that if we hedge with option A, there will be a big residual vega if underlying spot price moves away from the current level. So if both the spot price and volatility moves, say the volatility moves down while the spot is at \$110, hedging with option A would suffer a bigger loss than hedging with option B. The lesson is that the Greeks could change when underlying market parameters change at the same time. Hence it is advisable to perform some scenario analysis to check whether the hedge parameters change much when certain other parameters move.

The risk parameters only represent a snapshot of the risks at the current parameters. Even if the risks are well hedged, there may be higher moment exposures and there may not be enough time to adjust the hedges (gap moves). We already show an example of a modified hedge above. For managing the risk of complex portfolios, different scenario reports have to be generated at least on a daily basis. For example, we can shift forward the time to maturity (i.e. reduce the time to maturities of all positions), shift the underlying spot price levels and the volatilities of the underlyings, and observe the change in the risk parameters accordingly. This would help to reduce the "surprise factor" when unpredictable market conditions occur, and the trader would be able to have a better understanding of the characteristics of the portfolio.