Additional information on the table in course notes section 9.4.3.

9.4.3. Daily hedging demand

Assume that the NAV of a fund at time t_n is $A(t_n)$ with leverage x (which means that original exposure $L(t_n)$ is $xA(t_n)$), and the return for the period between t_n and t_{n+1} is r_n. Before re-hedging (at time t_{n+1}), the exposure is:

$$L_{BR}(t_{n+1}) = L(t_n)\,[1 + r_n] = xA(t_n)[1 + r_n]$$

whereas the NAV of the fund is

$$A(t_{n+1}) = A(t_n)[1 + xr_n]$$

which would require a new exposure of

$$L(t_{n+1}) = xA(t_{n+1}) = xA(t_n)[1 + xr_n]$$

Therefore the re-hedging that needs to be carried out

$$\Delta(t_n) = L(t_{n+1}) - L_{BR}(t_{n+1})$$
$$= xA(t_n)[1 + xr_n] - xA(t_n)[1 + r_n] = (x^2 - x)A(t_n)r_n$$

Case 1	$S(t_n)$	$A(t_n)$	$L(t_n)$	$L_{BR}(t_n)$	$\Delta(t_n)$
n=0	100	100	200		
n=1	90	80	160	180	–20
n=2	99	96	192	176	+16
Case 2					
n=0	100	100	-100		
n=1	90	110	-110	-90	–20
n=2	99	99	-99	-121	+22
Case 3					
n=0	100	100	-200		
n=1	90	120	-240	-180	–60
n=2	99	96	-192	-264	+72

The hedging action is shown in the examples above. The parameters are:

- Case 1: $x = 2$; Case 2: $x = -1$; Case 3: $x = -2$
- index returns on day 1 and 2 are –10% and +10%

We use case 1 as an example to illustrate how the numbers in this table are calculated. In order for it to return 2 times the performance of the index (because $x = 2$), we need the portfolio to have an exposure that is equal to 2 times the asset level, e.g. by holding a position in a futures contract with the required notional amount. That's why $L(t_n)$ would need to be maintained at 2x $A(t_n)$. At the first time step when n=0, the asset level is 100, and $L(t_0) = 200$. At time n=1, the stock index $S(t_n)$ has moved down 10% (from 100 to 90). Hence the exposure $L(t_0)$ would have the same performance, i.e. moved down 10% from 200 to 180, which is denoted as $L_{BR}(t_1)$ above (the exposure before re-hedging). This loss of 20 (=200-180) would be reflected in the change in the asset level $A(t_1)$, so that $A(t_1)$ is now 80 (=100-20).

Imagine an investor starts to hold a position in this ETF at n=1. Currently the asset level is $A(t_1)$=80. Using the logic as above, the required exposure in this fund $L(t_1)$ should be 2x $A(t_1)$ = 160. However, from the period above, we know that the exposure level (before re-hedging) is 180, so the proper action to take is to reduce the exposure from 180 to 160, i.e. $\Delta(t_0)$ = –20.

At n=2, the index has moved up 10% (from 90 to 99). Hence $L_{BR}(t_2)$ = 160 x 110% = 176, and $A(t_2) = A(t_1)$ + 16 = 96. The new required exposure $L(t_2)$ is 96x2 = 192, and the re-hedging action is thus 192–176 = +16.

Basically, the re-hedging required = $(x^2 - x)A(t_n)r_n$. At n=0, r_0 =–10% and x=2, the re-hedging required is equal to (4 –2)x100x(–10%) = –20.