CMSC 5718 INTRODUCTION TO COMPUTATIONAL FINANCE

Lecture 7

Outline

- Derivative instruments
  - Forwards
  - Futures
  - Options
- Some properties of options
- Option strategies

- Background reading: John Hull (2009, or other editions), Chapters 1-10

Financial derivatives: definition

- A contract “derived” from some underlying asset prices
  - For example, there are derivatives on stocks, FX rates, interest rates, and derivatives on derivatives
- Payoff could be **linear** or **non-linear**
  - Linear: swaps, futures
    - If underlying asset price moves by 1 unit, the derivative instrument’s value would move linearly
  - Non-linear: options
    - If underlying asset price moves by 1 unit, the derivative instrument’s value could sometimes move dramatically
- Could involve multiple assets/currencies

Characteristics of derivatives

- Provide instruments for **risk transfer**
- Could be highly **leveraged**
- Market completeness
  - Provide instruments for investors with different risk profiles
- Listed instruments could be very liquid; very efficient as a speculative tool
  - Typical volume of HSI futures: about 120,000 contracts per day, roughly equivalent to HKD 140 billion (more than 100% of cash market turnover per day)

"Fair value" and "no arbitrage"

- Arbitrage: the possibility of setting up a trading strategy such that a riskless profit could be earned
  - E.g. At 3:10pm HK time, HSBC is trading at HKD 64 in HK and GBP 6.77 in London. The GBP/HKD rate is 9.65.
  - Strategy: sell *N* shares in London; convert the money into HKD; buy *N* shares in HK. Net profit is HKD 1.3305 per share before cost (= 6.77 x 9.65 – 64.0)
  - Note the transaction cost and convertibility issues
- **If an instrument is trading at its "fair value", arbitrage does not exist**
- Models are used to find the "fair value" of an instrument

Forward contracts

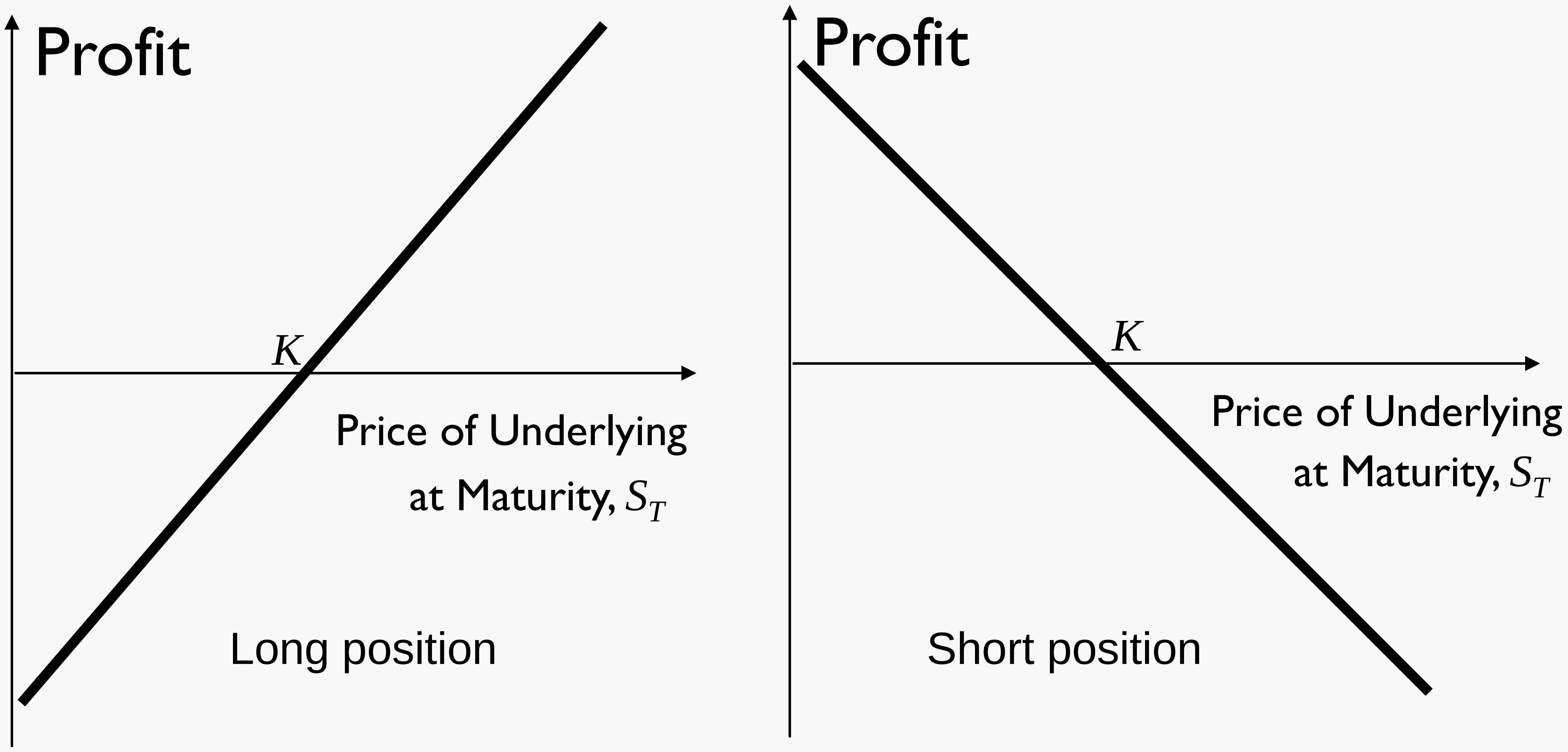
- A contract between two parties (not through an exchange)
- The forward price for a contract is the delivery price that would be applicable to the contract if it was negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero)
- The forward price may be different for contracts of different maturities

Currency forward example

- Usually no initial cash exchange, i.e. trade at the forward price
  - FX forward
    - A agrees to buy GBP 1 Million from B in 3-months, in exchange for USD 1.24 Million
  - Forward Rate Agreement (FRA)
    - A agrees to borrow USD 1 Million from B for 6-months at 0.75% p.a., and the loan would start 1-year from now

- Trades related to interest rates are usually *cash-settled*
  - e.g. for the FRA above, if 6-month interest rate resets at 1.19% in 1 year's time, then A would earn a profit of USD 1000000 x (1.19 − 0.75)% x 0.5 = USD 2,200 payable at time 1.5 year, or 2200 / (1+0.0119 x 0.5) = USD 2,186.99 at time 1 year

P&L of forward positions



- Note that the payoff is linear to the price movement

Pricing of forward contracts

- Cost-of-carry model
- Stock (index) forwards

$$F = Se^{(r_c - d_c)t} \text{ or } F = S[1 + (r_a - d_a)t]$$

  - r_c is the interest rate and d_c is the dividend yield, given in the continuous compounding convention; the conversion is given by
$$e^{r_c t} = (1 + r_a)^t, e^{d_c t} = (1 + d_a)^t$$
  - where r_a is the interest rate in annual compounding convention
- Currency forwards

$$F = Se^{(r_2 - r_1)t}$$

Example

- Spot Gold Price is US$1240
- 1-year US$% interest rate is 1.5%
- Therefore S = 1240, t = 1, r = 0.015, d = 0
- Forward price of gold in 1 year

  F = 1240 x (1+0.015) = 1258.60
- Note that this price is often different from the market price

Interest rate parity

- Interest rate of currency 1 = r_1
- Interest rate of currency 2 = r_2

- Spot FX rate is
  1 unit of currency 1 = S units of currency 2

- At time t,
  - Currency 1 becomes $(1+r_1 t)$
  - Currency 2 becomes $S(1+r_2 t)$

- **Forward FX rate F is thus $S(1+r_2 t)/(1+r_1 t)$**

Interest rate parity

- $F = S(1+r_2t)/(1+r_1t)$
- An illustration of an "arbitrage" relationship
- Given any 3 of the 4 variables from F, S, r_1, r_2, the 4th variable could be obtained
- If the equation doesn't hold, riskless profit could be made

A simple arbitrage trade

- Assume the following market rates:
  - Spot FX EUR 1 = USD 1.3585
  - 12-month forward FX is 1.3677 (market price)
  - USD interest rate 3.10%
  - EUR interest rate 2.30%

- Theoretical 1-year forward FX rate is $1.3585 \times (1+0.031 \times 1)/(1+0.023 \times 1) = 1.3691$
  - Not equal to the market rate of 1.3677, therefore arbitrage exists

A simple arbitrage trade

- On Spot date
  - Borrow EUR 1@2.3% for 1 year
  - buy USD 1.3585 (spot FX transaction)
  - Deposit USD 1.3585@3.1% interest for 1 year
  - Enter into forward FX, buy EUR/sell USD 1.4006@ 1.3677
- After 1 year
  - USD amount becomes 1.3585 x (1+3.1%x1) = 1.4006
  - Settle the forward FX trade by selling USD 1.4006 and receiving EUR 1.4006/1.3677 = 1.0241
  - Need to repay EUR loan = EUR 1 x (1+2.3%x1) = 1.0230
- Net profit = EUR 1.0241 – 1.0230 = EUR 0.0011
- Note that there is **no cost involved** when entering the trade (e.g. could we borrow EUR 50 Million??)

Futures contracts

- Contracts exist for different asset classes, examples include
  - Equity: S&P 500, Dow Jones, N225, HSI
  - Interest rates: US Treasury bonds, Eurodollar (LIBOR)
  - FX: USD/JPY
  - Commodity: Corn, wheat, gold, coffee, electricity, pork belly

- There are contracts for other (and more complex) types of underlyings, e.g.
  - Futures on a volatility index, such as VIX in the US
  - Futures on the forecast dividend of index stocks, e.g. for HSI and HSCEI in Hong Kong

Index futures: characteristics

- **Standardized contract**, e.g. Hang Seng Index (HSI) futures has a size of HKD 50 per point
- **Daily mark-to-market**: P&L allocated to settlement account everyday
- **Margining** : no need to pay a full amount of the underlying components to trade a contract
- **Fixed maturities**, c.f. forward contracts
- Electronic trading/outcry/transparency
- Liquidity: usual much more liquid than the underlying stocks
- Very little settlement risk (dependent on the creditworthiness of the exchange)

Fair value of index futures

$$F = (S - D)(1 + r_t t)$$

- F: fair value of the futures
- S: today's index level
- r_t: interest rate from today to futures expiry
- t: time from today to futures expiry
- D: present value of all dividends paid before futures expiry (equivalent index points)
- Note the slightly different form compared to the equation as shown on slide 9

Fair value of index futures

- Today is January 23, 2017
- Assume spot HSI level S = 23,050, interest rate r = 1%
- February futures expires on February 27, 2017, i.e. t = 35/365 = 0.09589 years (35 days between Jan 23 and Feb 27)
- *Estimated dividends* before futures expiry = 97.5 index points
- Fair value F = (23050 – 97.5) x (1+0.01x0.09589) = 22975.51
- Market price of February futures = M
- If $M > F$, futures is trading at a premium
- If $M < F$, futures is trading at a discount
- e.g. if M = 22995, it should be considered as trading at a premium although it is below the spot HSI level of 23050

Options

- A contract between 2 parties
- The holder of the option has the right, *but not the obligation*, to enforce the contract
  - Contrast this with forwards, when both parties are obliged to fulfill their contracts
- The option buyer usually needs to pay a fee (known as the option price or option premium)
  - Contrast this with forwards traded at the market price, where there is no initial cost

Some option terminology

- Call option: holder has the **right to buy** a pre-determined amount of the underlying asset at the strike (or exercise) price by a certain date
- Put option: holder has the right to sell a pre-determined amount at the strike price by a certain date
- Option premium: the price paid by the option buyer to the option seller to purchase the right
- Option maturity date: last date at which the contract must terminate

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European and American options

- European option: option holder could only exercise his/her right at maturity

- American option: option holder could exercise the right at any time up to maturity

- The holder of an American option has more choices, therefore this option could NEVER be cheaper than the corresponding European option; usually it is more expensive

Option examples

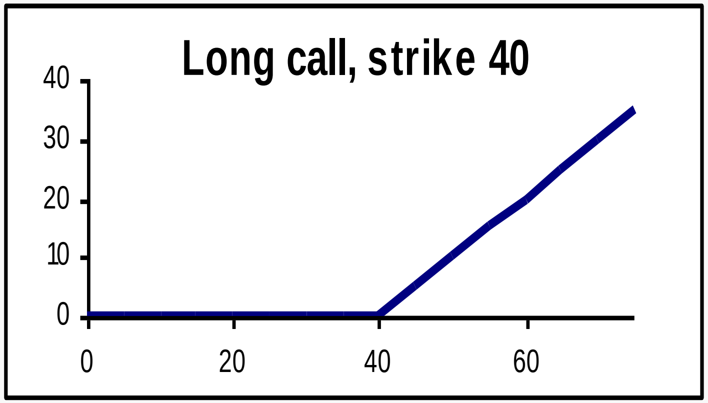
- **(stock option)** Long 10,000 European call options on Hang Seng Bank, strike HKD 150, maturity March 30, 2017
- **(FX option)** Long USD 5 Million notional American call USD/put JPY option, strike 115.00, maturity June 30, 2016
- **(Interest rate cap)** For every quarter in the next 3 years, if 3-month LIBOR is higher than 1%, the holder gets (3-month LIBOR–1)% on a notional of USD 10 Million

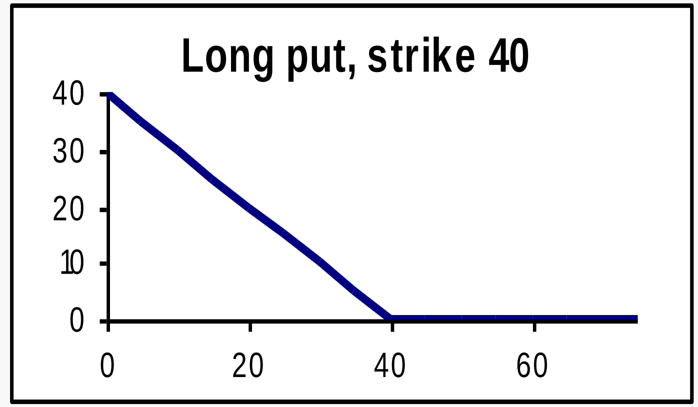
Call and put option examples
(c.f. Lecture 6, slide 23)

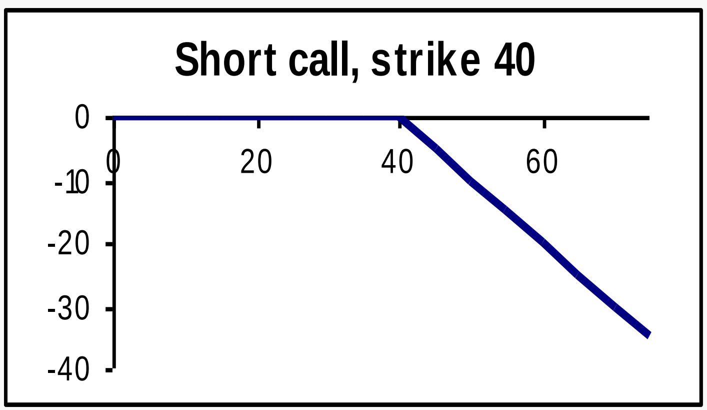
- European call option on HSBC, strike price 65
  - at maturity, if spot price = 70, payoff per option is 70–65 = 5
  - if spot price = 60, option is worthless
  - payoff formula: max(S_{final} – strike, 0)

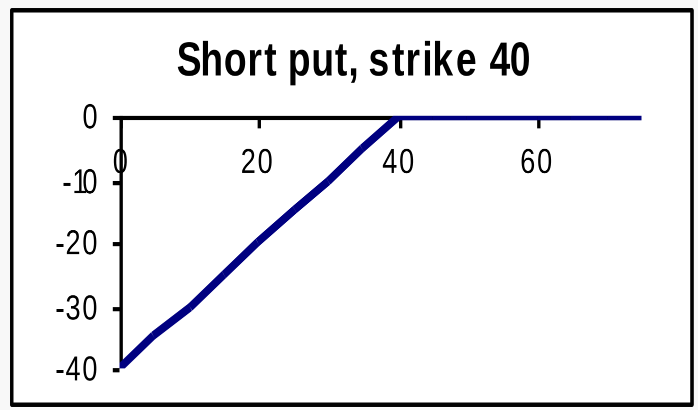
- European put option on HSBC, strike price 65
  - at maturity, if spot price = 70, option is worthless
  - if spot price = 60, payoff per option is 65–60 = 5
  - payoff formula: max(strike – S_{final} , 0)

Payoff profiles at maturity



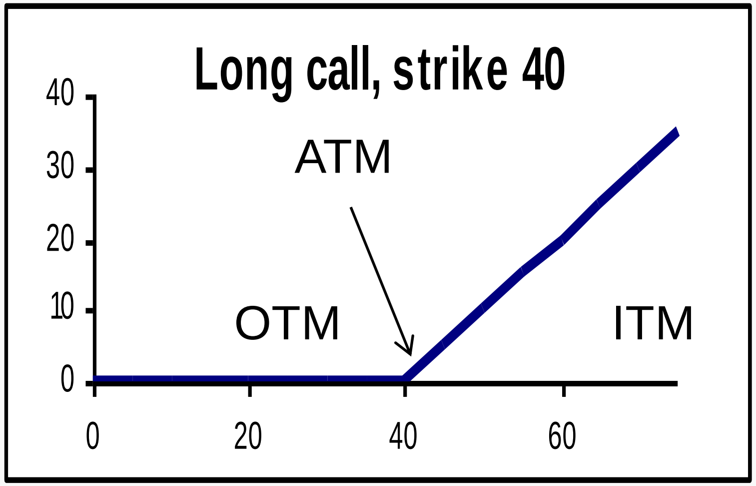


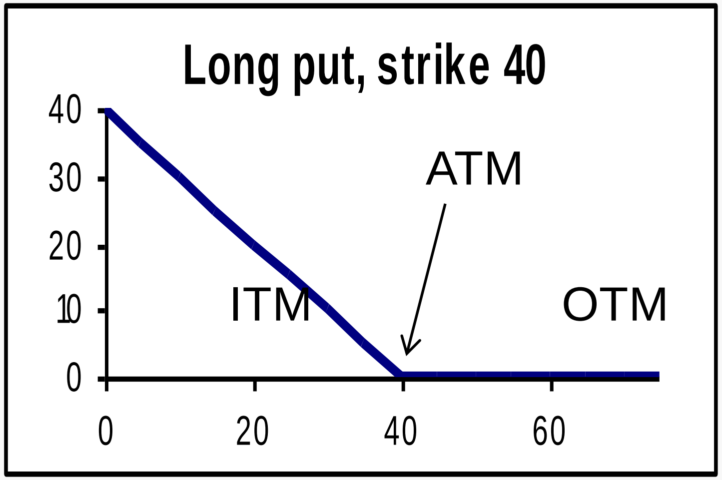




- These are profiles without taking into account of the initial option premium

Moneyness





- Moneyness: *at-the-money* (asset price = strike), *in-the-money* (payout of option is positive), *out-of-the-money* (payout of option is 0)
  - e.g. call option strike = 40; spot price = 37 (out-of-the-money), 40 (at-the-money), 50 (in-the-money)
  - e.g. Put option strike = 40; spot price = 35 (in-the-money), 40 (at-the-money); 45 (out-of-the-money)

Option settlement example

- European call option, strike $75, contract for 2,000 shares
- Let's say share price at maturity is $80 and the option buyer decides to exercise
- Physical settlement:
  - Option buyer pays 2000x75 = $150,000 to option seller and obtain 2,000 shares
  - Option buyer could decide to hold these 2,000 shares or sell them in the market (@$80 per share) for a profit
- Cash settlement:
  - Option seller pays 2000 x (80 – 75) = $10,000 to option buyer

Types of options

- Simple call and put options are often known as "plain vanilla" options
  - They can be either European or American style

- There are many types of options with more exotic (non-standard) payoff formulas:
  - Typical ones include Digital (Bet), Barrier, Asian (Average), quanto

Listed and OTC options

- Many options are traded in the over-the-counter (OTC) market, e.g. between a company and an investment bank; terms are tailor-made and agreed between the two parties

- There is a very liquid listed market in many countries, with standardized terms
  - e.g. Listed HSI options: European calls and puts, 1 index point = HKD 50, similar maturities as HSI futures (with more maturity months), strikes are fixed by the exchange at 200 points intervals

Examples of HSI listed options

Contract	Bid	Ask	Last Traded	High	Low	Volume	Prev. Day Settlement Price	Net Change	Prev. Day Open Interest
C Jan-17 - 21600	1198	-	1245	1245	1245	1	1173	72	557
P Jan-17 - 21600	6	7	7	10	6	779	8	(1)	5483
C Jan-17 - 21800	681	-	1050	1050	1050	1	980	70	1,033
P Jan-17 - 21800	9	10	9	16	9	481	15	(6)	4209
C Jan-17 - 22000	781	-	880	880	769	7	785	95	1,522
P Jan-17 - 22000	16	17	16	27	16	851	27	(11)	6565
C Jan-17 - 22200	499	-	673	673	586	15	603	70	1,268
P Jan-17 - 22200	26	29	28	45	27	1565	45	(17)	2256
C Jan-17 - 22400	480	-	498	516	436	44	433	65	1,379
P Jan-17 - 22400	50	53	52	77	48	1,607	77	(25)	2,522
C Jan-17 - 22600	261	-	335	359	270	285	290	45	1,987
P Jan-17 - 22600	88	96	92	133	85	2387	131	(39)	2,324
C Jan-17 - 22800	206	216	211	224	155	1,003	175	36	2,861
P Jan-17 - 22800	156	164	160	220	145	1,056	216	(56)	945
C Jan-17 - 23000	112	123	114	124	80	3123	93	21	3751
P Jan-17 - 23000	257	268	270	347	250	250	335	(65)	620
C Jan-17 - 23200	54	57	53	61	38	1479	45	8	2615
P Jan-17 - 23200	-	-	424	435	393	71	480	(56)	266
C Jan-17 - 23400	20	26	23	29	16	1086	18	5	3668
P Jan-17 - 23400	523	-	-	-	-	0	655	N/A	27

- As of Jan 17, 2017 (HSI closed at 22840.97, up 122.82 points from the previous day; HSI January futures closed at 22854, up 90 points from the previous day); source: www.hkex.com.hk

Some properties of options

- The fair pricing of options require the specification of the statistical distribution of the underlying price
- However, it is possible to derive some properties of options which are independent of the distributional assumptions
- The classical exposition of the put-call parity was given in Stoll (1969) and many properties of options were described by Merton (1973)

Notations

- c : European call option price
- p : European put option price
- S_0 : Stock price today
- K : Strike price
- T : Life of option
- σ: Volatility of stock price
- C : American Call option price
- P : American Put option price
- S_T :Stock price at option maturity
- D : Present value of dividends during option's life
- r : Risk-free rate for maturity T with continuous compounding

- Note that S_0, K, T, σ, r, D would affect stock option prices as seen today
- To simplify notation, only the relevant arguments are specified in some of the slides that follow (e.g. $c(K)$ instead of $c(S_0, K, T, \sigma, r, D)$)

Basic properties

- Non-negativity of option prices

$$C \geq 0, P \geq 0, c \geq 0, p \geq 0$$

  - Reason: payoffs of options are non-negative

- American and European options

$$C \geq c, P \geq p$$

  - Reason: An American option has all the rights of the European option, plus the privilege of early exercise – this right has a non-negative value

Option prices with different initial stock price levels

$$C(S_2) > C(S_1),\ \ S_2 > S_1$$
$$c(S_2) > c(S_1),\ \ S_2 > S_1$$
$$P(S_2) < P(S_1),\ \ S_2 > S_1$$
$$p(S_2) < p(S_1),\ \ S_2 > S_1$$

- Reason: payoff of a call option increases with increase stock price, it also has a strictly higher chance to be exercised; similar argument holds for put options

Option prices with different strike levels

$$C(K_2) < C(K_1),\ \ K_2 > K_1$$
$$c(K_2) < c(K_1),\ \ K_2 > K_1$$
$$P(K_2) > P(K_1),\ \ K_2 > K_1$$
$$p(K_2) > p(K_1),\ \ K_2 > K_1$$

- Reason: payoff of a call option decreases with increase strike price, it also has a strictly less opportunity to be exercised; similar argument holds for put options

Concept check

- Other parameters being equal, which option price is higher?
- (i)
  - Call option with stock price = 100, or
  - Call option with stock price = 110
- (ii)
  - Put option with strike price = 80, or
  - Put option with strike price = 70

Put-Call Parity (no dividend case)

- Consider the following 2 portfolios:
  - Portfolio A: one call option c + cash Ke^{-rT}
  - Portfolio B: one put option p + one share S_0
- At maturity

Stock price at expiry	$S_T < K$	$S_T \geq K$
Portfolio A	$0 + K = K$	$(S_T - K) + K = S_T$
Portfolio B	$(K - S_T) + S_T = K$	$0 + S_T = S_T$
Result of comparison	$V_A = V_B$	$V_A = V_B$

- As the two portfolios would always have the same value, hey must therefore be worth the same today. This means that

$$c + Ke^{-rT} = p + S_0$$

Put-Call Parity (no dividend case)

- Example
  - S_0=31, K=30, r=10%, T=0.25, c=3.00, p=2.25
  - $c + Ke^{-rT} = 3 + 30e^{-0.1\text{x}0.25} = 32.26$
  - $p + S_0 = 2.25 + 31 = 33.25$

- Strategy
  - Buy call, sell put, sell stock
    - upfront cash = $-3+2.25+31 = 30.25$
  - Invest this amount for 3 months to get $30.25e^{0.1\text{x}0.25} = 31.02$
  - At maturity, if S_T>30, the call is exercised; if S_T<30, the put will be exercised by the buyer; in either case, the result is that 1 share would be purchased at a price of 30, which can be used to close out the existing short stock position
  - Guaranteed net profit is thus 31.02 – 30 = 1.02

Early Exercise

- Usually there is some chance that an American option will be exercised early

- An exception is an American call on a non-dividend paying stock
  - Mathematically, this should never be exercised early; the option should be sold rather than exercised
  - In practice, it depends whether it is possible to close out the option at the theoretical price

Early Exercise (deep in-the-money)

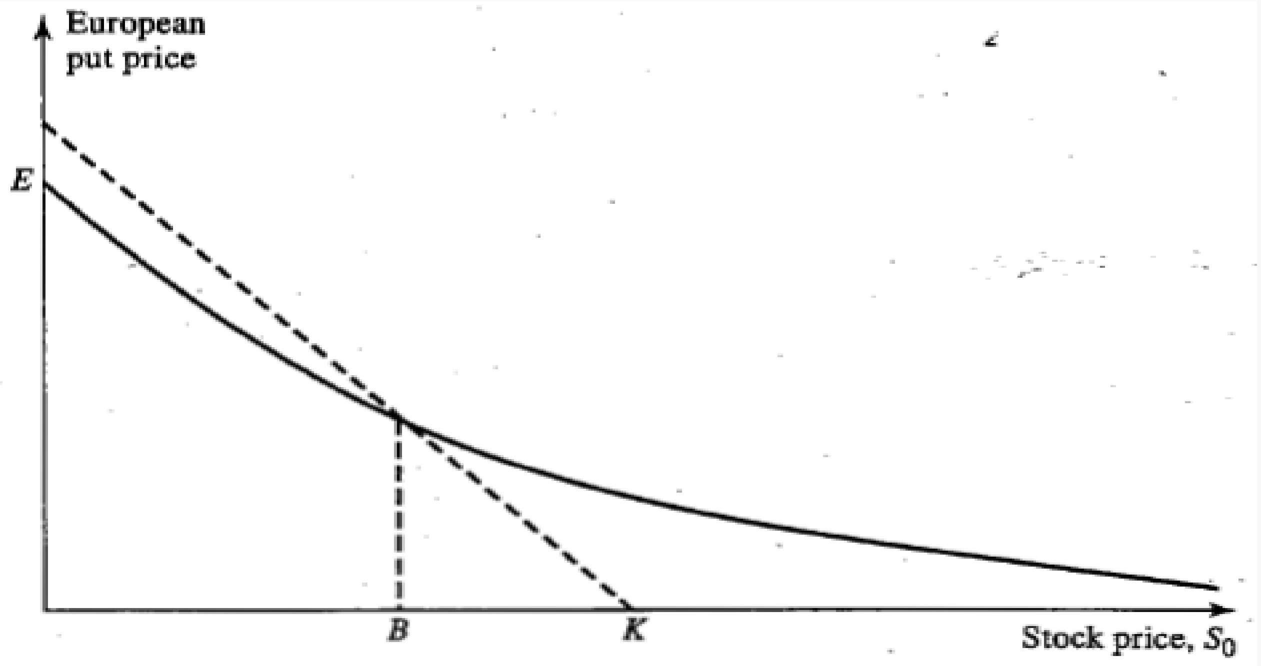
- For an American call option:
  - S_0 = 100; T = 0.25; K = 60; D = 0
  - Should you exercise immediately?

- What should you do if
  - (i) you want to hold the stock for the next 3 months?
  - (ii) you do not feel that the stock is worth holding for the next 3 months?

Reasons for not exercising a call early (no dividend)

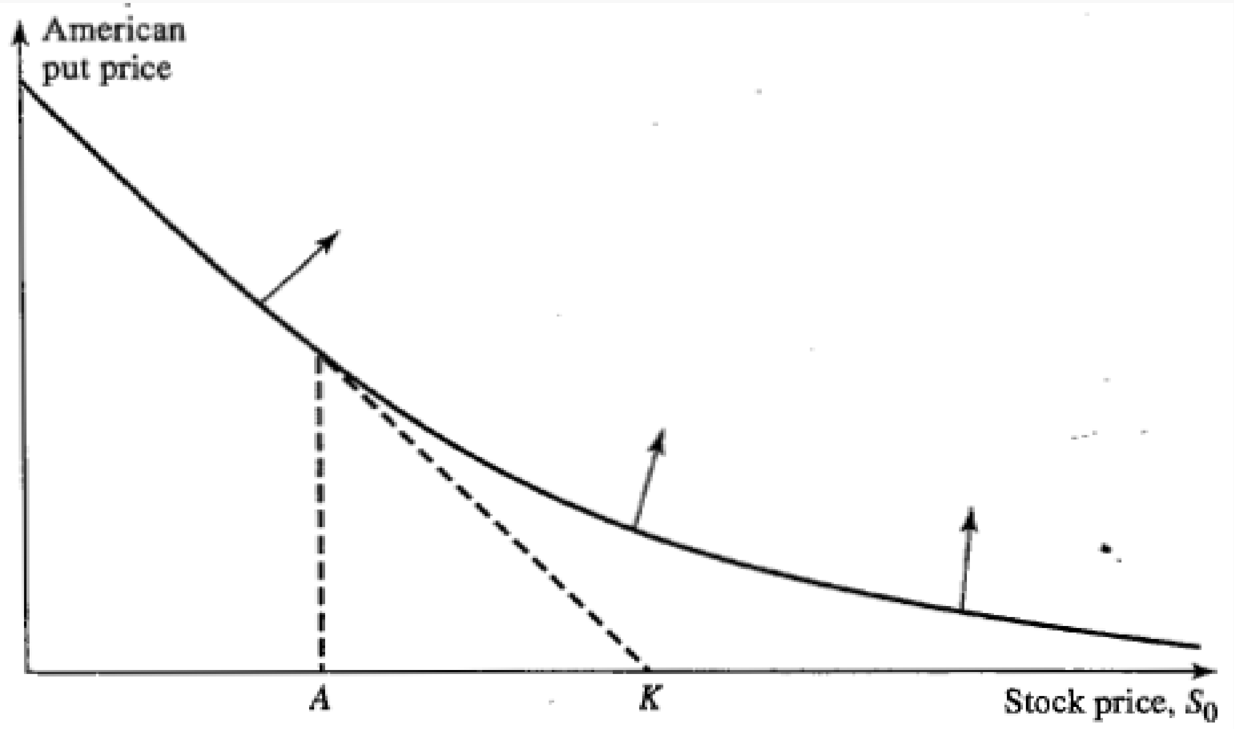
- Case (i)
  - No income is sacrificed, since there is no dividend before maturity
  - If the option is exercised immediately, the strike price has to be paid; by delaying the exercise, payment of the strike price is made at maturity (c.f. present values of cash flows at different times)
  - Holding the call provides insurance against stock price falling below strike price
- Case (ii)
  - Selling the option will provide more income than the intrinsic value

Should Puts Be Exercised Early ?



- The above shows the price behavior of a European put
- Are there any advantages to exercising an American put when
- S_0 = 60; T = 0.25; r=10%
- K = 100; D = 0

American put option and early exercise



- If $r > 0$, at a sufficiently low stock price, it is optimal to exercise the put early, i.e. $P > p$
  - Reason: try the extreme case, where stock price drops to 0

Option strategies

- We concentrate on the payoff profiles at maturity
- Combinations of a number of calls/puts and the underlying
- Capture the specific views of investors
  - we don’t want to pay for something that is unlikely to happen

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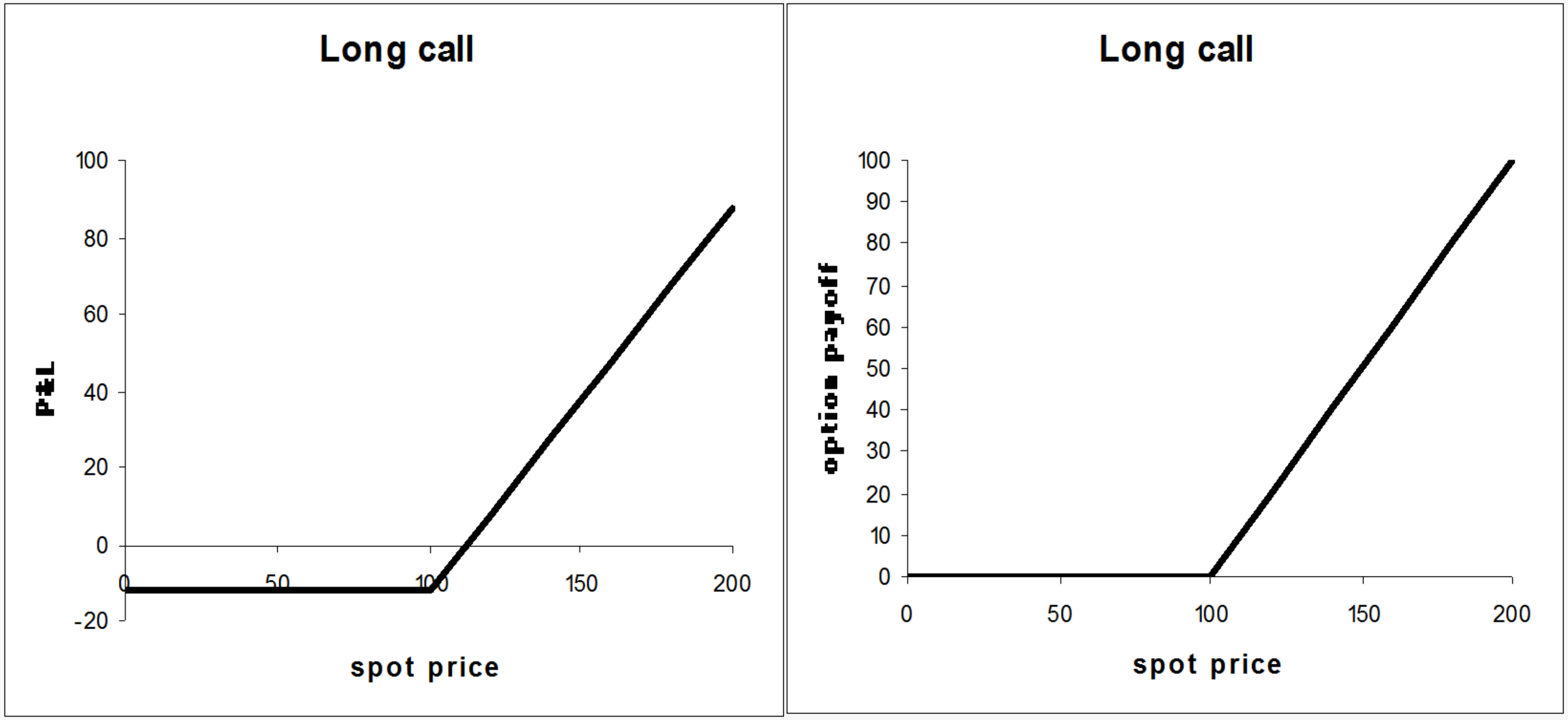
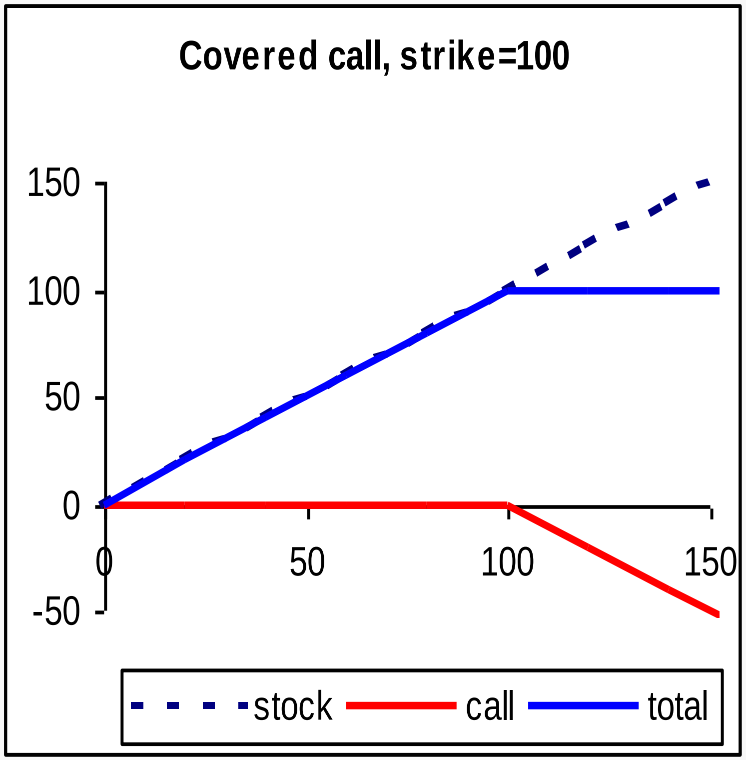
Representation of payoffs



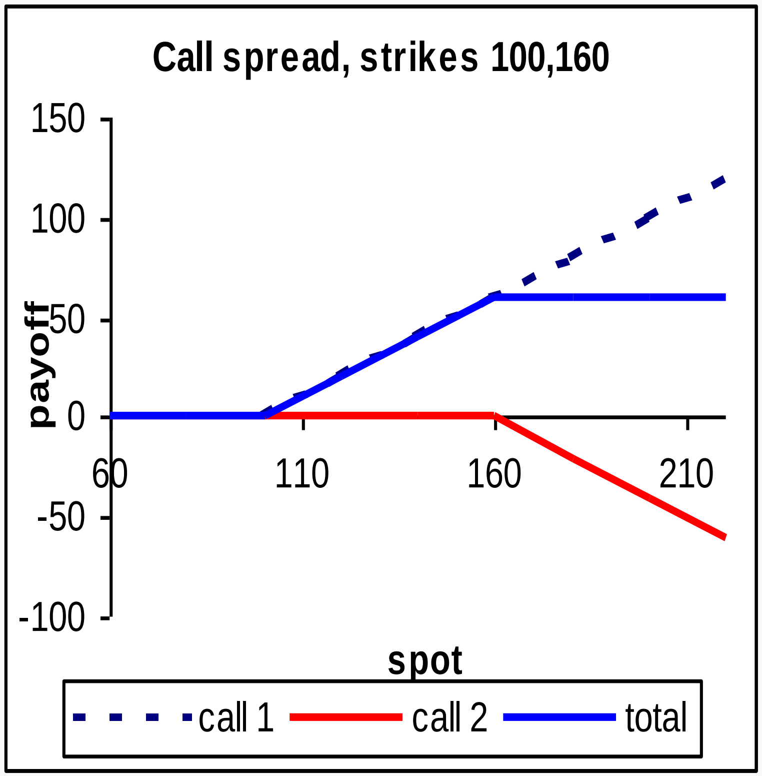
- In some textbooks, payoff diagrams include the cost of the option (as in the left diagram); an alternative representation is shown on the right diagram, where only the option payoff is displayed
- In the following diagrams, the convention in the right diagram is used

Covered call



- **Covered call**
  - Long 1 share, short 1 call
- Anticipate that the stock price would not go very much higher
- Collect option premium, but give up some upside potential

Call spread / put spread



- Also known as bull/bear spreads
- **Call spread**
  - earn a profit if stock price moves up
  - Long 1 call at strike K_1, short 1 call at strike K_2 where $K_2 > K_1$
- **Put spread**
  - earn a profit if stock price moves down
  - Long 1 put at strike K_1, short 1 put at strike K_2 where $K_2 < K_1$ (compare with short 1 call at strike K_1, long 1 call at strike K_2 where $K_2 > K_1$)
- Limit the upside potential, hence cheaper than a single call or a single put

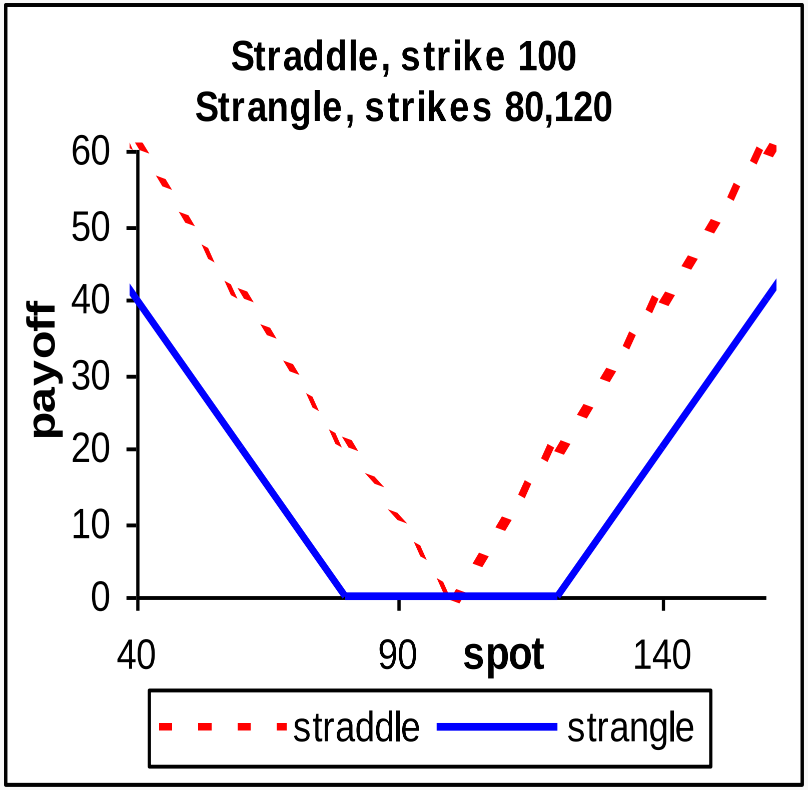
Call spread: analysis of payoff

- Let $K_2 > K_1$; $c(K_2) < c(K_1)$, thus a long call spread position will require an initial premium

- At maturity

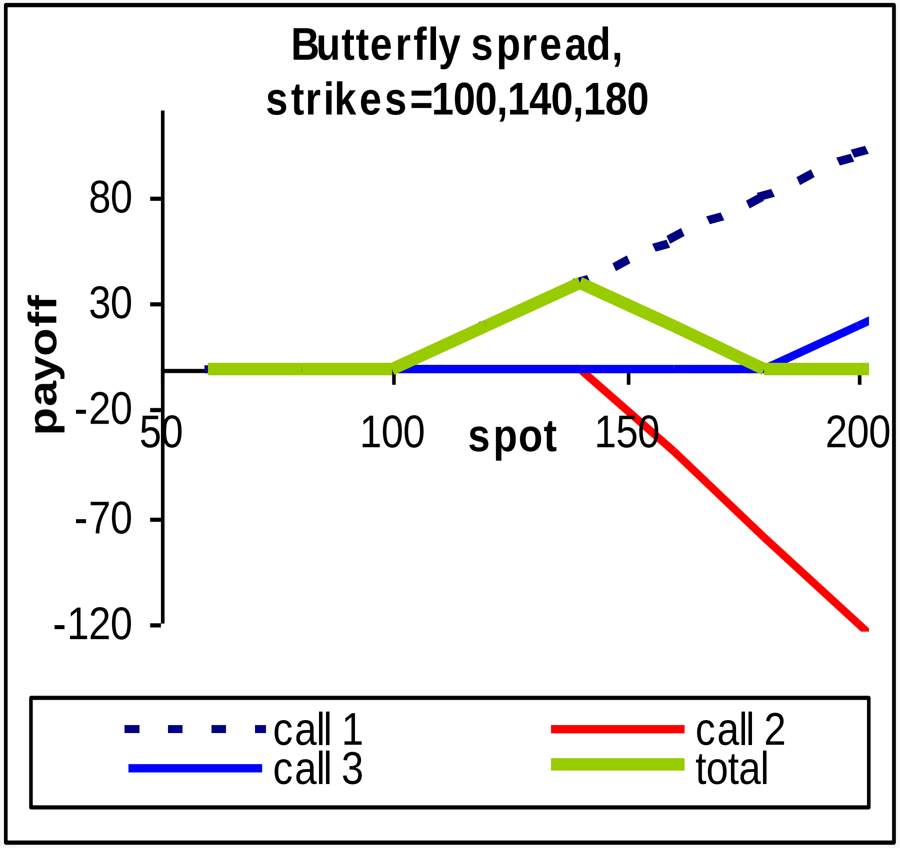
Stock price at expiry	$S_T \leq K_1$	$K_1 < S_T < K_2$	$K_2 \leq S_T$
Payoff from long call position	0	$S_T - K_1$	$S_T - K_1$
Payoff from short call position	0	0	$K_2 - S_T$
Total payoff	0	$S_T - K_1$	$K_2 - K_1$

Strangle / straddle



- **Straddle**
  - Long 1 call at strike K_1, long 1 put at strike K_1
- **Strangle**
  - Long 1 call at strike K_1, Long 1 put at strike K_2 where $K_1 > K_2$
- Expensive
- Profitable if the stock price has moved away from the strike(s)

Butterfly spread



- **Butterfly spread**
  - Long 1 call at strike K_1, long 1 call a strike K_3, short 2 calls at strike K_2 where $K_2 = (K_1 + K_3)/2$
  - K_1=100, K_2=140, K_3=180 in the example on the left
- Maximum profit when final spot is at K_2
- Compare with short straddle, the risk is lower, but is more costly to put on the trade

Butterfly spread: analysis of payoff

- Let $K_3 > K_1$ and $K_2 = (K_1 + K_3)/2$
- It can be proved from the convexity property of options that

$$c(K_2) \le 0.5c(K_3) + 0.5c(K_1) \Rightarrow 2c(K_2) \le c(K_3) + c(K_1)$$

  i.e. long butterfly spread will require an initial premium
- At maturity

Stock price at expiry	$S_T \le K_1$	$K_1 \le S_T \le K_2$	$K_2 \le S_T \le K_3$	$K_3 \le S_T$
Payoff from $c(K_1)$	0	$S_T - K_1$	$S_T - K_1$	$S_T - K_1$
Payoff from $c(K_3)$	0	0	0	$S_T - K_3$
Payoff from $-2c(K_2)$	0	0	$-2(S_T - K_2)$	$-2(S_T - K_2)$
Total payoff	0	$S_T - K_1$	$K_3 - S_T$	0

What is the option premium?

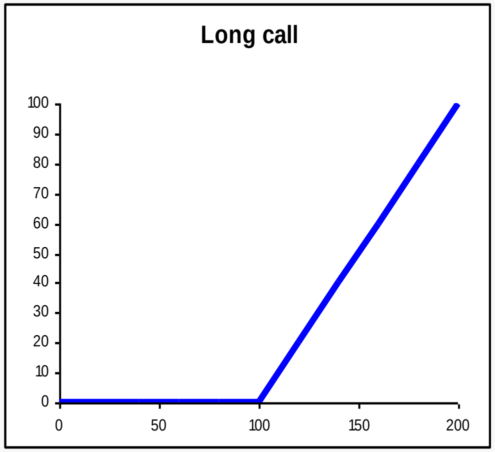
- Fair price equals the expected payoff of the option (under some technical conditions)
  - As a private investor, whether the trade should be entered or not is based on the subjective probabilities of the expected payoff, which is not necessarily the fair price
  - E.g. fair price of a butterfly strategy is $10, market price is $12, but if you think that there is a high probability of making more than $12, you may want to engage in this strategy

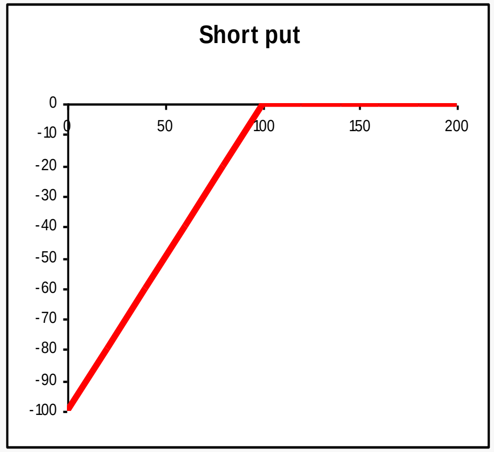
- Fair price also equals the expected hedging cost
  - If we could find a hedging strategy which could replicate the payoff of the option under any circumstances, then the cost of the strategy should be the same as the cost of the option (the principle of no arbitrage)

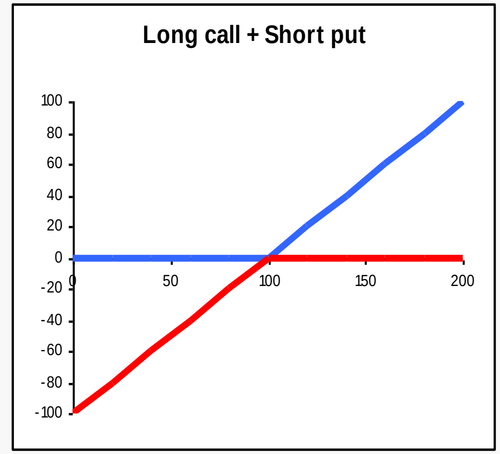
Constructing the payoffs

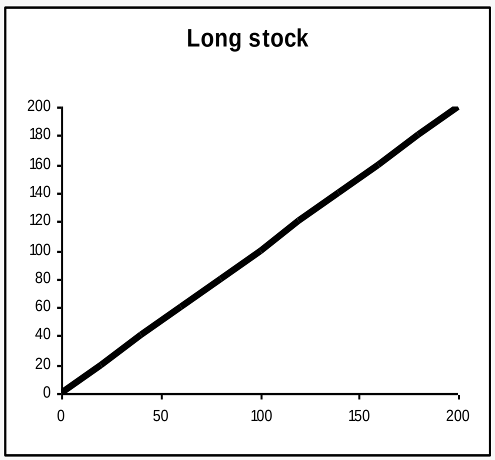
- Theoretically, any payoff pattern at maturity could be constructed
- Tradeoff between cost and payoff
- Could pose hedging difficulties for the strategy seller (hence higher cost)
  - Sometimes we may not be able to find a seller of options (at an attractive price)
- Could use a combination of the underlying (e.g. stocks), bonds, calls and puts so as to replicate the payoff patterns of other simple strategies

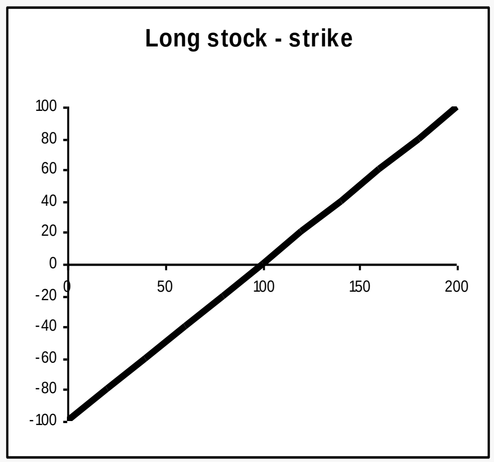
Synthetic forward, call and put











payoff profiles
at maturity

Synthetic forward, call and put

- **Put-call parity**
  - At maturity: $C – P = S_T – K$
  - Today: $C_0 – P_0 = S_0 – Ke^{–rT}$
- **Synthetic forward**: Long 1 call, short 1 put, both at strike K
- **Synthetic call**: Long 1 share, long 1 put, borrow cash $Ke^{–rT}$
- **Synthetic put**: Long 1 call, short 1 share, deposit cash $Ke^{–rT}$

Example: short synthetic call

- Current share price = \$40
- Strike price = \$40
- Price of put option = \$4.233
- Time to maturity = 1 year
- 1 year interest rate = 2.5% (simple compounding)

- Short call option created by:
  - Sell 1 share, sell 1 put, deposit cash $K/(1+rT)$

- Net call option premium received
  = 40 + 4.233 – 40/(1+0.025x1) = \$5.209