Assignment suggested answer

Q1

1.1 Vector BA = <11 – 1 , 6 – 1> = <10,5>

1.2 Length of BA = $\sqrt{10^2 + 5^2} = \sqrt{125} = 11.18$

1.3 Unit vector of BA = <10/11.18,5/11.18> = <0.89, 0.45>

Using the line equation, B+tBA = <1,1> + t <0.89, 0.45>

C is at where t = 2.0, so

C = <1,1> + 2.0 <0.89, 0.45> = (2.78, 1.9)

1.4

The rotation matrix for rotating in clockwise is :

$$\begin{bmatrix} \cos(-45) & -\sin(-45) \\ \sin(-45) & \cos(-45) \end{bmatrix} = \begin{bmatrix} \cos(45) & \sin(45) \\ -\sin(45) & \cos(45) \end{bmatrix} = \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix}$$

To rotate point B

$$\begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.707*1 + 0.707*1 \\ -0.707*1 + 0.707*1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.414 \\ 0 \end{bmatrix}$$

So, point B will rotate to (1.414,0)

1.5

The rotation about point B in ϕ, will consist of following steps:

A. Translate in –B (i.e. (-1,-1))

B. Rotate ϕ degree

C. Translate back in B (i.e. (1,1))

So, the transformation matrix will be formed by multiplying the 3 matrices mentioned above

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \cos\phi & -\sin\phi & -\cos\phi+\sin\phi \\ \sin\phi & \cos\phi & -\cos\phi-\sin\phi \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & -\sin\phi & -\cos\phi+\sin\phi+1 \\ \sin\phi & \cos\phi & -\cos\phi-\sin\phi+1 \\ 0 & 0 & 1 \end{bmatrix}$$

Q2

2.1

Vector AB = <2.5 –(-1), 6.0-4.0, 2.0-4.0 > = < 3.5 , 2.0, - 2.0>

Vector AC = <4.5-(-1), 0.0 – 4.0, 3.0-4.0> = < 5.5, -4.0, -1.0>

2.2

Length of AB = $\sqrt{3.5^2 + 2.0^2 + 2.0^2} = 4.5$

Unit vector of AB = <3.5 /4.5 , 2.0 /4.5, -2.0/4.5> = < 0.7778 , 0.4444, -0.4444>

Length of AC = $\sqrt{5.5^2 + 4.0^2 + 1.0^2} = 6.8739$

Unit vector of AC = <5.5 /6.8739 , -4.0 /6.8739, -1.0/6.8739> = < 0.8 , -0.5819, -0.1455>

Angle between AB and AC = acos(unitAB dot unitAC)

= acos(< 0.7778 , 0.4444, -0.4444> dot < 0.8 , -0.5819, -0.1455>)

= acos(0.7778 * 0.8 + 0.4444 * -0.5819 + -0.4444*-0.1455)

= acos(0.4284)

= 1.1281 rad = 64.64 degree

2.3

The plane equation is n dot (D-A) = 0

That is

<0.36,0.27,0.89> dot ((1.9 , 4.4 ,2.71) - (-1.0, 4.0, 4.0))

= <0.36,0.27,0.89> dot <2.9, 0.4, -1.29>

= (0.36*2.9 + 0.27*0.4+ 0.89*-1.29)

= -0.0039 $\approx$ 0

Therefore the point D is on the plane

Q3

3.1 The scaling about center of the cube, will consist of following steps:

i. Translate in -center (i.e. (-4,-3,-3))

ii. Scale 1.5

iii. Translate back to center (i.e. (4,3,3))

In matrix form, it will be

$$\begin{bmatrix}1&0&0&4\\0&1&0&3\\0&0&1&3\\0&0&0&1\end{bmatrix}\begin{bmatrix}1.5&0&0&0\\0&1.5&0&0\\0&0&1.5&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&-4\\0&1&0&-3\\0&0&1&-3\\0&0&0&1\end{bmatrix}\begin{bmatrix}5\\4\\4\\1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&4\\0&1&0&3\\0&0&1&3\\0&0&0&1\end{bmatrix}\begin{bmatrix}1.5&0&0&0\\0&1.5&0&0\\0&0&1.5&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0&4\\0&1&0&3\\0&0&1&3\\0&0&0&1\end{bmatrix}\begin{bmatrix}1.5\\1.5\\1.5\\1\end{bmatrix}$$

$$=\begin{bmatrix}5.5\\4.5\\4.5\\1\end{bmatrix}$$

So, vertex V will transform to (5.5,4.5,4.5)

3.2

Model matrix and view matrix.

Because all the vertices will be in coordinate system of the camera, the formation of projection matrix will be much simpler.

3.3

Using formula of similar triangles

We first consider in the x-z plane, and x_s is the projected x coordinate

$x_s = \frac{d}{z}x$

$= \frac{2.0}{10.0}6.0$

$= 1.2$

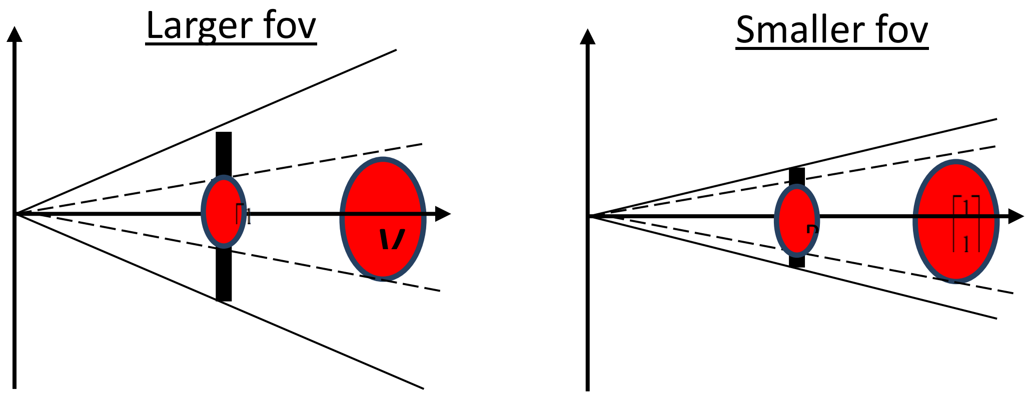
Similarly, for y-z plane, we have

$y_s = \frac{d}{z}y$

$= \frac{2.0}{10.0} 7.0$
=1.4

Therefore, the projected 2D coordinate is (1.2,1.4)

3.4
There will be problem if the scaling lets the cube enlarge to a degree that the camera goes inside of it. That will be totally different to the effect of zoom-in. Also, we need to enlarge all other objects in the scene to make the effect consistent.
A better solution to this is to adjust the field of view in the camera, so that a smaller FOV can emulate the zoom-in effect.



Q4

4.1
Spot light and point light are originated from a single point source, but spotlight shines in a particular direction and have an effective cutoff range.

4.2
L dot n = <0.577, 0.577, 0.577> dot < 0.707, 0.707, 0.0>
= (0.577* 0.707 + 0.577*0.707 + 0.577*0.0)
= 0.816
Red channel diffusion = Kd * Id * (L dot n)
= 0.5 * 1.0 * 0.816
= 0.408
Green channel diffusion = Kd * Id * (L dot n)
= 1.0 * 0.3 * 0.816
= 0.2448
Blue channel diffusion = Kd * Id * (L dot n)
= 0.4 * 0.3 * 0.816
= 0.09792

4.3
As flat shading is used in the shown sphere, and the lighting model is computed in a per-surface basis without interpolation. We can use Gouraud shading (or Phong shading) to improve quality of shading smoothness. Since Gouraud shading evaluates the lighting model in per-vertex manner, and interpolate within the surface, so the shading looks smooth. (Since Phong shading computes the lighting model in a per-fragment manner, so it is accurate and looks smooth).

4.4
Gouraud shading (or Phong shading) requires much more computations (in interpolation/on lighting model) than flat shading

4.5
Aliasing will appear when textured objects are far away and become small on screen, as there is not enough sampling.
[Figure refers to lecture notes]

Q4

4.1

The geometry of surface can be very smooth, especially when modeling curvy surfaces. A parametric surface is defined by a number of control points and the parameters u,v.
Tessellation or triangulation

4.2

If indexed triangular mesh is used, we store all vertex coordinates with no repetition, and each of them given an index e.g.

Vertex indices	3D coordinate
0	A
1	B
2	C
3	D
4	E
5	F
6	G

Then, each face is formed by the 3 vertices’ indices, e.g.

Face	Vertex indices
T1	0,1,6
T2	0,5,6
T3	5,4,6
T4	4,3,6
T5	3,2,6
T6	2,1,6

As a result, there are totally 3* 7 = 21 floating point numbers used, and 3*6 = 18 integers used to store this mesh structure.

4.3

It is possible to use key-frame animation for dancing animation; however, it will be very tedious to edit the motion of the cat statue frame by frame. Also, the dancing motion may not be realistic as human does. A better solution is to use motion capture which captures human dancer’s motion, and retarget onto the cat statue model afterwards.