5. Equity and Fixed Income Portfolio Management

5.1. Equity portfolio management

5.1.1. Passive management techniques

Passive equity portfolio management attempts to replicate the performance of a specific market index. According to many results from financial research, a broad-based market index (e.g. S&P 500) can outperform a majority of active portfolio managers annually. One explanation is that the market is efficient, and it is difficult for active strategies to achieve outperformance after taking into account of fees.

Passive management funds are popular in recent years, especially in the form of Exchange Traded Funds (ETF). Hence it is important to understand how the index is calculated. The following example is taken from Reilly and Brown (2003), p.154.

Stock	Share Price	Number of Shares	Market Value
December 31, 2002			
A	$10.00	1,000,000	$ 10,000,000
B	15.00	6,000,000	90,000,000
C	20.00	5,000,000	100,000,000
Total			$200,000,000
			Base Value Equal to an Index of 100
December 31, 2003			
A	$12.00	1,000,000	$ 12,000,000
B	10.00	12,000,000[a]	120,000,000
C	20.00	5,500,000[b]	110,000,000
Total			$242,000,000

$$\begin{aligned}\text{New Index Value} &= \frac{\text{Current Market Value}}{\text{Base Value}} \times \text{Beginning Index Value} \\ &= \frac{\$242{,}000{,}000}{\$200{,}000{,}000} \times 100 \\ &= 1.21 \times 100 \\ &= 121\end{aligned}$$

[a]Stock split two-for-one during the year.
[b]Company paid a 10 percent stock dividend during the year.

This example is self-explanatory. Basically, the index level reflects the change in the value of a portfolio which includes all the constituent stocks. However, we need to distinguish between two concepts with respect to the weighting of each stock in the index. In this example, stock B's *dollar weighting* was 90000000/200000000 = 45% as of Dec 31, 2002, and was 120000000 / 242000000 = 49.59% on Dec 31, 2003. It is evident that dollar weighting is not a constant; it changes everyday, depending on share prices.

Another related term is the *basket weight* for each stock used to replicate the index (in number of shares). This figure remains unchanged, unless there is a change in the number of shares outstanding due to technical adjustments. In this example, the number of shares changed because of the stock split, which is one of the events leading to an adjustment in the number of shares.

We can now look at a more realistic example. The table below shows a partial list of stocks required for a theoretical basket which is equivalent to 25 futures contract of the Hang Seng Index as of Feb 10, 2014.

stock code	name	shares outstandin (millions)	10-Feb close	board lot	shares required for 25 futures	rounded
5	HSBC	13,413	79.2	400	52,225	52,400
700	Tencent	1,117	533.5	100	4,350	4,400
941	China Mobile	6,034	73.55	500	23,493	23,500
939	CCB	84,146	5.18	1,000	327,632	328,000
1299	AIA Group	12,044	35.65	200	46,895	46,800
1398	ICBC	69,435	4.64	1,000	270,353	270,000
3988	Bank of China	79,441	3.18	1,000	309,313	309,000
883	CNOOC	17,859	12.1	1,000	69,536	70,000
13	Hutchison	2,132	96.45	1,000	8,300	8,000
27	Galaxy Entertainment	2,325	69.6	1,000	9,053	9,000
857	PetroChina	21,099	7.67	2,000	82,151	82,000
1	Cheung Kong	1,390	112.7	1,000	5,411	5,000
2628	China Life Insurance	7,441	20.55	1,000	28,973	29,000
386	Sinopec	25,513	5.78	2,000	99,339	100,000
1928	Sands China	2,419	58.2	400	9,418	9,600
388	Hong Kong Exchanges	1,103	121.1	100	4,296	4,300
16	Sun Hung Kai Prop	1,352	95.15	1,000	5,265	5,000
2318	Ping An Insurance	2,034	60.55	500	7,921	8,000
2	CLP	1,895	58.05	500	7,378	7,500
11	Hang Seng Bank	765	119	100	2,978	3,000

Hang Sang Index (HSI) calculation methodology

$$I_t = I_{t-1} \times \frac{\sum w_{i,t} \times S_{i,t} \times FAF_i \times CF_i}{\sum w_{i,t} \times S_{i,t-1} \times FAF_i \times CF_i}$$

Similar to the example used earlier, the formula above indicates that the index is calculated based on the ratio of portfolio value between the current date and the previous date. Here I_{t-1} is the index level at time $t-1$, $S_{i,t}$ is the price of stock i at time t, $w_{i,t}$ is the number of shares outstanding for company i. By definition, HSI started at 100 on its base date at July 31, 1964, but there were numerous adjustments in subsequent years, such as the regular review of constituent stocks and technical adjustments due to corporate events. One minor point to note is that the index is calculated based on current weights, as terms such as $w_{i,t-1}$ is not used in the formula above.

HSI calculation makes use of two additional terms: a free float adjustment factor FAF_i to take into account of the fact that some stocks are mostly held by the majority shareholder(s) and thus their liquidity may be limited; secondly, a capitalization adjustment factor CF_i is used to limit the weight of a stock to a maximum of 10% (effective from September 4, 2015, with a higher percentage in earlier dates; revised every quarter). The table below shows the FAF and CF for selected stocks (as of September 8, 2014).

Code 股份代號	Stock 股份		FAF	CF
0001.HK	Cheung Kong	長江實業	60.00%	100.00%
0002.HK	CLP Hldgs	中電控股	75.00%	100.00%
0003.HK	HK & China Gas	香港中華煤氣	60.00%	100.00%
0004.HK	Wharf (Hldgs)	九龍倉集團	50.00%	100.00%
0005.HK	HSBC Hldgs	滙豐控股	100.00%	71.24%
0006.HK	Power Assets	電能實業	65.00%	100.00%
0011.HK	Hang Seng Bank	恒生銀行	40.00%	100.00%
0012.HK	Henderson Land	恒基地產	35.00%	100.00%
0013.HK	Hutchison	和記黃埔	50.00%	100.00%
0016.HK	SHK Prop	新鴻基地產	45.00%	100.00%
0017.HK	New World Dev	新世界發展	60.00%	100.00%
0019.HK	Swire Pacific 'A'	太古股份公司'A'	70.00%	100.00%
0023.HK	Bank of E Asia	東亞銀行	60.00%	100.00%
0027.HK	Galaxy Ent	銀河娛樂	55.00%	100.00%
0066.HK	MTR Corporation	港鐵公司	25.00%	100.00%

Equity portfolio indexing techniques

In the HSI example above, the index just contains 50 stocks. If an index contains a comparatively small number of stocks and they are liquid, an index fund will usually attempt to engage a full replication of the index, i.e. every stock will be held according to its proportion as required in the index calculation. However, if some stocks are illiquid, it means that some stocks may not be held by the fund, and this could easily lead to significant tracking error. Alternatively, a sampling technique can be used, whereas the stocks with most weights will be held in full and those with less weights are sampled. It is also possible to use a quadratic optimization programme to match the characteristics of the index performance. Most index funds will have some kind of tracking error.

Period	Manager	Index	Difference (Δ)
1	2.3%	2.7%	–0.4%
2	–3.6	–4.6	1.0
3	11.2	10.1	1.1
4	1.2	2.2	–1.0
5	1.5	0.4	1.1
6	3.2	2.8	0.5
7	8.9	8.1	0.8
8	–0.8	0.6	–1.6

The above (taken from Reilly and Brown (2003), p.657) is an example of the calculation of tracking error. The data include the returns of the portfolio (in the column "Manager") and the index for eight consecutive quarters. The average tracking error is 0.2% and standard deviation (per quarter) is 1.0%. The annualized standard deviation of the tracking error = 1.0% x sqrt(4) = 2.0%.

5.1.2. Active equity portfolio management strategies

Although there has been much evidence in the superiority of passive management techniques, active management has its own appeal. Many fund managers aim to earn a return that exceeds the return of a passive benchmark portfolio, where the comparison is calculated net of transaction costs and on a risk-adjusted basis (c.f. topic 4). Two groups of strategies are often used in equity portfolio management. The first strategy is based on **stock valuation**. Analysts try to assess the intrinsic value of each stock, and thus identify stocks which are undervalued or overvalued. The second strategy, known as **technical analysis**, would not require any research of the fundamentals, but derives a prediction of future price trends based on historical asset prices alone.

Stock valuation approaches

There are two basic valuation methods. In the *accounting approach*, one can assess the book value, liquation value and/or the replacement cost of the company. Book value is the net worth of the company as shown in the balance sheet and annual report. Liquidation value is the amount of money that could be realized by selling all the assets and repaying all the debts. Replacement cost is the cost of replacing all the assets and liabilities of the company. Estimating these parameters are difficult if the operations of a company are complex. Furthermore, these methods tend to concentrate on "historical" performance.

A second approach is known as a *corporate finance approach*. This is more forward looking, as one needs to calculate the present value of projected future cash flows. While this method is by no means easier than the accounting approach, one can aim at adopting a consistent procedure in the estimation so as to give a reasonable prediction when there are changes in the input parameters.

Dividend discount model (DDM)

A popular basic model is the known as the DDM. The fair price of a stock today V_0 should equal the present value of all future dividends to be paid by the share:

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots$$

- D_i is the dividend to be received at the end of year i
- k is the required rate of return (i.e. the risky discount rate) – may come from CAPM

In practice it would be impossible to estimate all the future dividends. Analysts often focus on estimating an ultimate share price P at year H:

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \cdots + \frac{D_H + P_H}{(1+k)^H}$$

Even for a mature company, hopefully the dividends paid each year would not stay constant. A version of the model, known as the *Constant Growth DDM*, makes an assumption that the dividend would grow at constant rate g, such that

$$D_{i+1} = D_i(1+g)$$

We could then show that (using the formula of a summation of an infinite geometric series):

$$V_0 = \frac{D_1}{k-g}, \text{if } g < k$$

Example of Constant Growth DDM

Growth rate g	Rate of return k						
	8%	10%	12%	14%	16%	18%	20%
0%	12.50	10.00	8.33	7.14	6.25	5.56	5.00
1%	14.29	11.11	9.09	7.69	6.67	5.88	5.26
2%	16.67	12.50	10.00	8.33	7.14	6.25	5.56
4%	25.00	16.67	12.50	10.00	8.33	7.14	6.25
6%	50.00	25.00	16.67	12.50	10.00	8.33	7.14
8%	-	50.00	25.00	16.67	12.50	10.00	8.33
10%	-	-	50.00	25.00	16.67	12.50	10.00

Price of stock (V0) assuming initial dividend is $1

We could see that the stock price is very sensitive to the assumptions for k and g. Basically *almost any theoretical price could be achieved!*

A particularly convenient form of the model is known as the **Gordon equation** (also known as the discounted cash flow formula):

$$V_0 = \frac{D_1}{k-g} \Rightarrow k = \frac{D_1}{V_0} + g$$

In other words, the (risk adjusted) rate of return equals the dividend yield (defined as the current dividend divided by the share price) plus the dividend growth rate. If we know the dividend yield and dividend growth rate, we could estimate the rate of return. If the stock price is currently trading at its intrinsic value, then this rate of return is also the expected return of the stock.

While the current stock price is rarely trading at its intrinsic value, the above method is sometimes used in rate hearings for regulated utilities companies, in order to decide what should be an acceptable level of return of investment. This method is also used to estimate a competitive expected return that would be demanded by stock investors.

Multi-stage DDM

A slightly more flexible form can accommodate a non-constant dividend growth rate. Assume that the risky discount rate k is 15% and we have estimated the following dividend streams: year 1 = $1.40, year 2 = $1.95, year 3 = $2.55, and 10% growth from year 4 onwards. Using the earlier formula, we can derive the stock price at year 3 as:

$$P_3 = \frac{D_4}{k-g} = \frac{D_3(1+g)}{k-g} = \frac{2.55 \times (1+0.1)}{0.15-0.1} = \$56.1$$

Therefore the fair price today is:

$$P_0 = \frac{1.40}{1+0.15} + \frac{1.95}{(1+0.15)^2} + \frac{2.55+56.10}{(1+0.15)^3} = \$41.26$$

Using earnings in the DDM formula

Define b as the *plowback ratio*, which is the fraction of earnings E re-invested in the firm (E is known as the *earnings per share* (EPS)). By definition, dividend for the coming year is given by the total earning less the re-investment amount, i.e.:

$$D_1 = E(1-b)$$

Price of stock today is thus

$$V_0 = \frac{E(1-b)}{k-g} \Rightarrow \frac{V_0}{E} = \frac{1-b}{k-g}$$

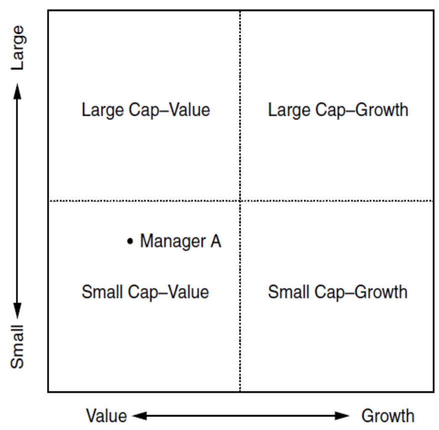
The ratio V_0/E is called the Price-earning multiple or *Price-earning* (P/E) ratio. It has been the most commonly used equity multiple (first introduced in the 1930s). More formally, P/E ratio is defined as the Price per share / Earnings per share. Equity research analysts would often include three numbers in their reports: the P/E of the past year, an estimate of the current year's P/E, and a forecast of next year's P/E.

The earnings estimation can be combined with the DDM approach, as illustrated in the following example. Assume that the risky discount rate k is 15% and we have estimated the following dividend streams: year 1 = \$1.40, year 2 = \$1.95, year 3 = \$2.55. Instead of estimating a terminal stock price or a dividend growth rate, the P/E ratio at the end of year 3 is forecast to be 13 and earnings per share (EPS) is forecast to be \$5.20. We can then derive the fair price at year 3: $P_3 = \text{P/E ratio} \times \text{EPS} = 13 \times 5.20 = \$67.60$.
Thus the fair price today is:

$$P_0 = \frac{1.40}{1+0.15} + \frac{1.95}{(1+0.15)^2} + \frac{2.55+67.60}{(1+0.15)^3} = \$48.82$$

To summarize, an analyst can assume a particular P/E multiple, and then try to estimate the future earnings and hence derive the fair price. Alternatively, the dividend, the dividend growth rate and the risky discount rate can be estimated, and the stock's fair price can be derived accordingly. However, note that in each case, strong assumptions are made and there is no guarantee that the fair price can be reachable.

Equity style analysis (Reilly and Brown (2003), p.675)

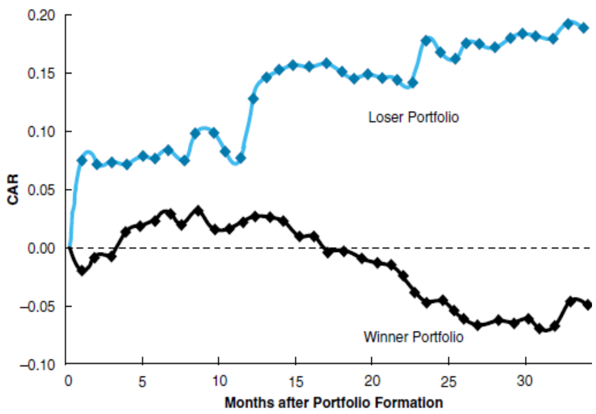


Utilizing the techniques that we have learnt above, financial analysts often attribute companies based on a *style analysis grid*. Some fund managers prefer the value style, where they look for stocks that are cheap in terms of earnings, low price/book ratio and low P/E. This strategy may incur a higher risk, and therefore it can potentially have higher return. For managers who prefer the growth style, the chosen companies are those that can hope to deliver future earnings growth. These companies usually have earnings momentum and higher valuation multiples. However, there is no consensus as to which style can achieve a better result in the long run.

Technical trading strategies

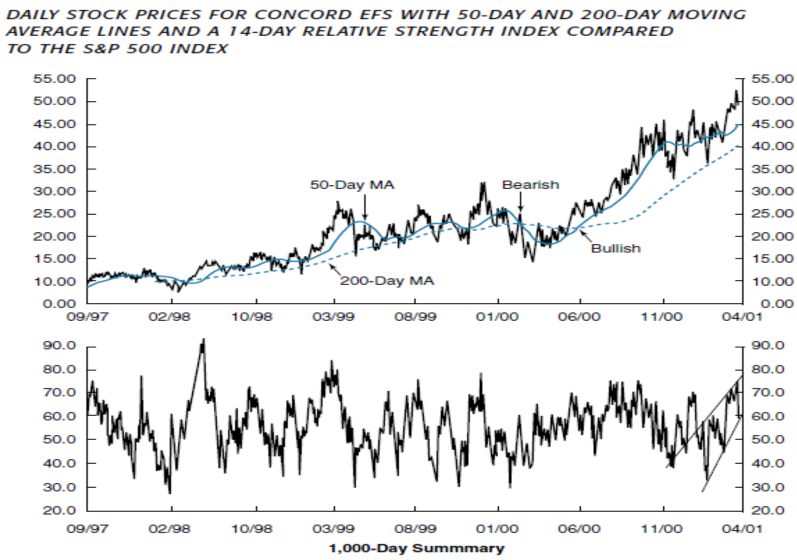
Technical strategies make use of historical data to predict future price trends. One class of strategies, known as price momentum, assumes that the recent price trend will continue. It would look for stocks that are going up in price because these stocks would probably go even higher. However, a contrarian strategy assumes the opposite: that recent price trend will reverse. So if the stock price is low now, it could be a good time to buy because it is more likely that the price would become higher. These strategies tend to contradict each other in its underlying assumptions, but there have been studies that support both theses, e.g. DeBondt and Thaler (1985), Chan, Jegadeesh and Lakonishok (1999) etc.

An example of a contrarian strategy (Reilly and Brown (2003), p.665)



In one such strategy, the performance of a portfolio of stocks that had the worst performance over the last 3 years (loser) is compared against a portfolio with best past performance over the last 3 years (winner). In this example, the loser portfolio achieves a significant outperformance in the 36 months after the portfolios are formed. (CAR represents the Cumulative Abnormal Return.)

Technical analysis example (Reilly and Brown (2003), p.640)

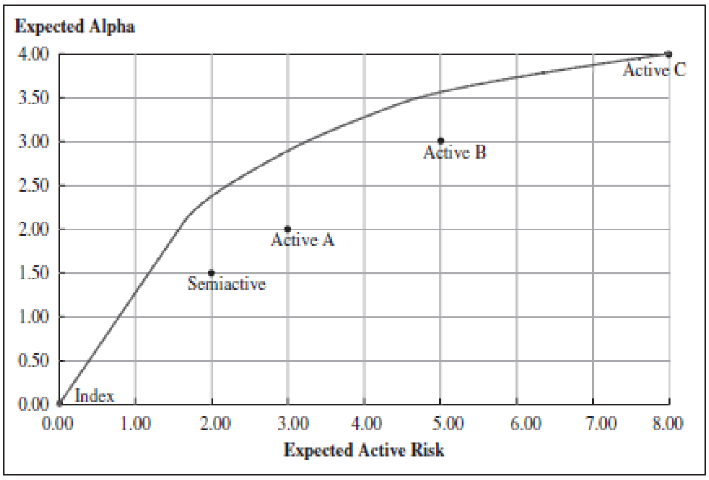


These graphs make use of moving averages (MA) and the Relative Strength Index (RSI) to generate trading signals. For example, when the 50-day MA moves below the 200-day MA, it is a bearish signal and thus it is a time to sell. When 50-day MA moves above 200-day MA, it is a good time to buy.

Challenges to technical trading rules

While technical analysis is popular especially among the non-professionals, it is important to realize that past patterns or relationships may not repeat in the future. However, when many people are using the same rules, the prices can start to behave as a self-fulfilling prophecy, although they will revert to their true equilibrium values after a short period of time. Also, we note that if the strategy becomes too successful, it will lead to many imitators, which will then make the rule obsolete. Finally, while the rule is supposed to be quantitative, a great deal of subjective judgment is still required (e.g. what constitutes a "wave"?)

5.2.3. Managing a portfolio of managers (Magnin et al. (2007), p.459)



Assume managers have 0 correlation with each other

	Return	Risk
Index	0.0%	0%
Semi-Active	1.5%	2%
Active A	2.0%	3%
Active B	3.0%	5%
Active C	4.0%	8%

The above diagram plots the expected alpha of an investment against the "active" risk. By definition, the reference index would not have any alpha, and its active risk is 0. A fund manager may allocate its assets into different funds, by using MPT to work out the efficient frontier. Effectively each fund is treated as if it is a single stock. If most of the assets are allocated to an index portfolio or an enhanced index portfolio, the arrangement is known as a core-satellite strategy.

5.2. Fixed Income portfolio management
5.2.1. Introduction

Modern portfolio theory has made less impact on bond portfolio management than in equity portfolio management, and separate techniques have been developed in this area. A major source of risk is the change in the interest rate yield curve, and there is a need to quantify this risk through some sensitivity measure.

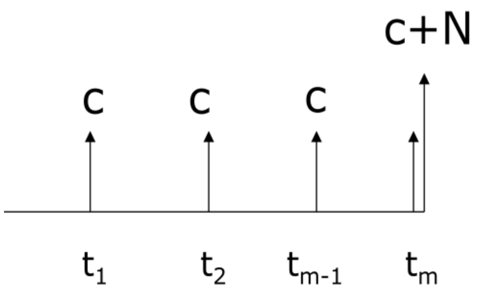
Another major area which is not covered in this notes concerns the change in the creditworthiness of the issuer. To facilitate discussion, we recall that issuing bonds is a means of raising money by the corporates (which is usually cheaper than equity financing). However, the coupon being paid by these bonds could be higher than comparable government bonds.

	MTRC	Hutchison	US Government
Issue Date	14/1/2004	19/11/2003	17/11/2003
Maturity	21/1/2014	24/1/2014	15/11/2013
Coupon	4.75%	6.25%	4.25%
Initial Price	99.121%	99.897%	99.076%
Spread	0.83%	2.05%	
Rating	A1/A+	A3/A-	AAA/Aaa

In the examples above, all the bonds were issued within 2 months. US government bond is usually considered as the benchmark. The Hutchison bond has a similar maturity, but it was paying a much higher interest rate. MTRC's bond paid a coupon slightly higher than the US government, but much lower than Hutchison. The reason of this difference was mostly caused by the market perception of the *probability of default* of each issuer. This aspect, known as credit trading, has been an important area of risk management.

5.2.2. Concept in bond mathematics

Bond price and yield-to-maturity



c is the bond coupon and N is the bond par amount

$$
\begin{aligned}
cf_i &= \text{cash flow at time } i \\
&= \text{c}, i \neq m \\
&= \text{c} + \text{N}, i = m \\
c &= \text{bond coupon rate} \times \tau \\
\tau &= \text{accrual factor,} \\
&\quad \text{e.g. 0.5 if coupon is semi-annual} \\
\text{f} &= \text{compounding frequency} \\
&\quad (1 \text{ if annual, } 2 \text{ if semi-annual})
\end{aligned}
$$

Price

$$P = \sum_{i=1}^{m} cf_i PV_i$$

$$PV_i = \frac{1}{(1+r_i)^{t_i}}$$

Yield: find y such that

$$P = \sum_{i=1}^{m} cf_i Y_i$$

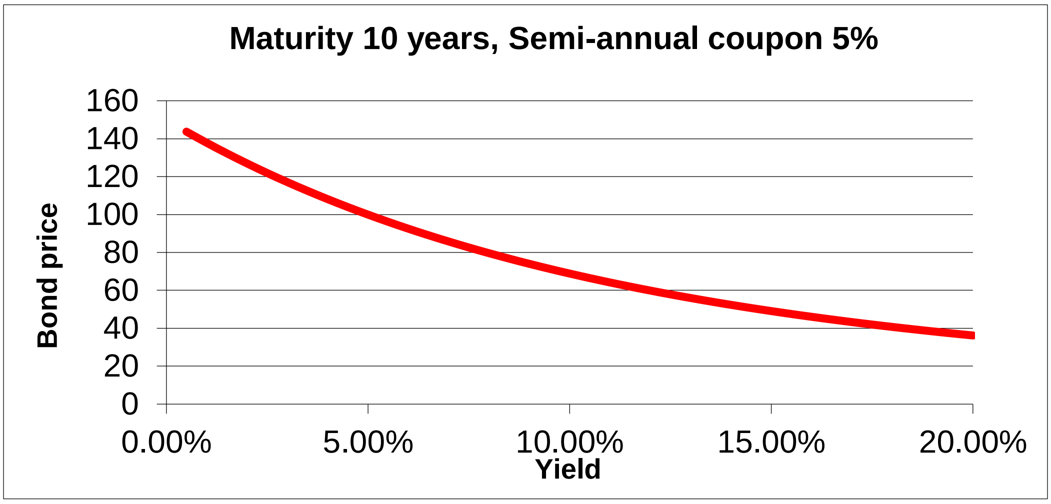
$$Y_i = \frac{1}{(1+y/f)^{t_i f}}$$

The most important parameters in describing a bond's value are the price and the yield. Bond yield is the discount rate that makes the present value of the cash flows of the bond equal to the market price of the bond. As an example, a 3-year bond pays an annual coupon of 6% per annum. To calculate the theoretical price of a bond, we are supposed to discount each cash flow at the appropriate discount rate for that maturity, which are obtained separately. On the other hand, suppose that the market price of the bond is given, which is equal to 97.392. The bond yield, which is a single discount rate applicable to all maturity, is obtained by solving the pricing equation:

$$6e^{-y\times 1.0} + 6e^{-y\times 2.0} + 106e^{-y\times 3.0} = 97.392$$

In this equation, e^{-yt} is the "discounting factor" at time t using y as the discounting rate. Solving this equation by iteration, we get the yield y=0.0676 or 6.76%.

Properties from the pricing formula



The price of a standard bond is plotted against its yield in the diagram above. It is apparent that the *price-yield relationship* is non-linear and inverse, i.e. the higher the yield, the lower the price. We also note that the slope of the tangent of this curve is always negative.

We can also make some general observations about the coupon, yield and price of the bond. In particular, if coupon is greater than yield, then the price is greater than 100%. For example, a 3 year bond with coupon = 4% and yield = 3% would have a theoretical price at 102.8486% (c.f. the pricing formula in the section above). On the other hand, if coupon is less than the yield, price is less than 100%. For example, the 3 year bond with coupon = 4% has a yield of 4.5%, and the theoretical price becomes 98.6114%. There is an extreme case where coupon = 0 (known as a *zero coupon bond*), and the price is always smaller than 100%, i.e. the bond is sold at a discount.[1]

Comparing bonds: duration

Different bonds will have different coupons and maturities, and it would be difficult to compare their performance when the interest rates change. One convenient benchmark for comparison is known as **bond duration**, which is roughly the weighted average time to maturity of a bond. The *modified duration* is defined by the following formula:

% change in bond price = –(modified duration) x (change in yield)

$$\frac{\Delta P}{P} = -D'\Delta y$$

e.g. let's say bond price = 102.4% and modified duration = 5.4; if yield increases by 2.5% to 2.55% (i.e. a move of +0.05%), the % change in bond price = –5.4 x 0.05 = –0.27%. Estimated bond price = 102.4% x (1 – 0.27%) = 102.12%.

[1] This statement is always true unless bond yield becomes negative, which is an abnormal situation. However, Germany's 2-year government bond yield has turned negative since 2012, so it is not impossible.

We note two points from this relationship. Firstly, there is a negative sign, which indicates that when yield moves in one direction (say it increases), the bond price would move in the other direction (decreases). Secondly, we could easily see that if a bond has a larger duration, its price would be more affected by changes in yield. Therefore bonds with longer maturities tend to be more sensitive to interest rate changes.

More formally, the duration comes from the following (assume annual compounding below, i.e. $f = 1$):

$$
\begin{aligned}
P &= \sum \frac{cf_i}{(1+y)^{t_i}}, \quad cf_i = \text{cashflow at time } t_i,\, y = \text{yield} \\
\frac{dP}{dy} &= -\frac{1}{1+y} \sum \frac{cf_i t_i}{(1+y)^{t_i}} \\
&= -\frac{PD}{1+y} \quad \text{(by definition)} \Rightarrow D = \frac{1}{P} \sum \frac{cf_i t_i}{(1+y)^{t_i}} \\
&= -PD' \quad \text{(by definition)} \Rightarrow D' = \frac{D}{1+y}
\end{aligned}
$$

D is the Macaulay duration, and D' is the modified duration

In discrete form, $\dfrac{\Delta P}{P} = -D'\Delta y$

In general, duration is affected by the maturity of the bond, its coupon, and the interest rate, and a bond with longer maturity tends to have greater duration. Furthermore, if other variables are held constant:

- Increase in coupon -> decrease duration;
- Increase in interest rate -> decrease duration.

The following table gives examples of bond duration where interest rate = 10% and the bond pays annual coupon):

Coupon \ maturity	3 yrs	5 yrs	10 yrs
4%	2.88	4.57	7.95
6%	2.82	4.41	7.42
8%	2.78	4.28	7.04
10%	2.74	4.17	6.76

Comparing bonds: convexity

Because the price/yield relationship is non-linear, duration is only a first-order approximation in calculating the percentage change in bond price when yield moves. A more accurate measure is to include the **convexity**, which represents the *curvature* of the price/yield relationship. Given that the shape of the relationship is convex, more convexity is good for bond holders: as yield moves, investor is better off – yield increases and the loss is smaller, yield decreases and the gain is larger when compared to a linear relationship (see diagram below).

The convexity can be calculated from the following formula:

$$\text{Convexity } X = \frac{1}{P}\frac{d^2P}{dy^2} = \frac{1}{P}\frac{1}{(1+y)^2}\sum\frac{cf_i t_i (t_i+1)}{(1+y)^{t_i}}$$

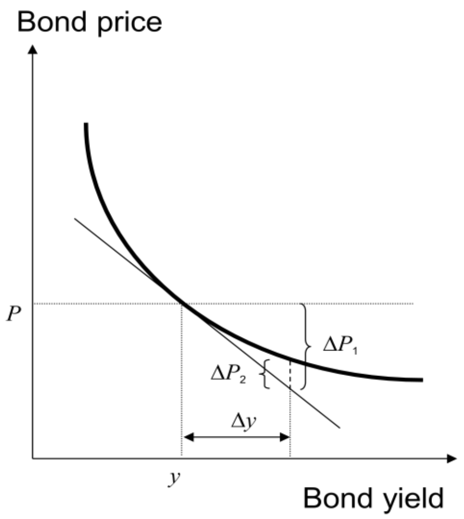
$$\Delta P \approx \Delta y \frac{\partial P}{\partial y} + \frac{(\Delta y)^2}{2}\frac{\partial^2 P}{\partial y^2},$$

$$\frac{\Delta P}{P} = -D'\Delta y + X\frac{(\Delta y)^2}{2}$$

Thus a more accurate representation is:

% change in bond price = –(modified duration) x (change in yield)
+ convexity x (change in yield)^2 / 2

(Note that in some textbooks, the ½ factor is included in the definition of convexity, so that the formula becomes: % change in bond price = –(modified duration) x (change in yield)
+ convexity x (change in yield)^2)



Duration and convexity adjustments are represented schematically in the diagram on the left. The real price/yield relationship is given by the dark line When yield changes by Δy, bond price would change by ΔP_1 if we use duration adjustment alone. The convexity term would compensate for the error, adding ΔP_2 to the bond price. Of course, the price/yield relationship is not a 2nd order function, so even after adjusting for convexity usually there would still be a small error.

As an example, let's say a 3-year bond pays coupon annually, with coupon rate = 5% and yield = 5.5%.

$$\text{Price } P = \frac{0.05}{1+0.055} + \frac{0.05}{(1+0.055)^2} + \frac{1.05}{(1+0.055)^3} = 98.65\%$$

$$\text{Duration } D = \frac{1}{0.9865}\left[\frac{0.05\times 1}{1+0.055} + \frac{0.05\times 2}{(1+0.055)^2} + \frac{1.05\times 3}{(1+0.055)^3}\right] = 2.858$$

$$\text{Modified Duration } D' = \frac{2.858}{1+0.055} = 2.709$$

$$\text{Convexity } X = \frac{1}{0.9865}\frac{1}{(1+0.055)^2}\left[\frac{0.05\times 1\times 2}{1+0.055} + \frac{0.05\times 2\times 3}{(1+0.055)^2} + \frac{1.05\times 3\times 4}{(1+0.055)^3}\right] = 10.104$$

We see that two bonds with the same duration can have different convexities, and thus they behave differently when interest rate moves. For example, a 12.75-year zero coupon bond has these parameters: yield = 8%, modified duration = 11.81, convexity = 150.3. A 30-year bond with 6% coupon has yield = 8%, duration = 11.79 and convexity = 231.2. When interest rate goes up, the bond with higher convexity will lose less in value. (You can try to calculate the percentage moves in these two cases as an exercise).

The duration and convexity of a portfolio of bonds would just be the weighted average of these parameters of the individual bonds in the portfolio. For example, there are two bonds in the portfolio:

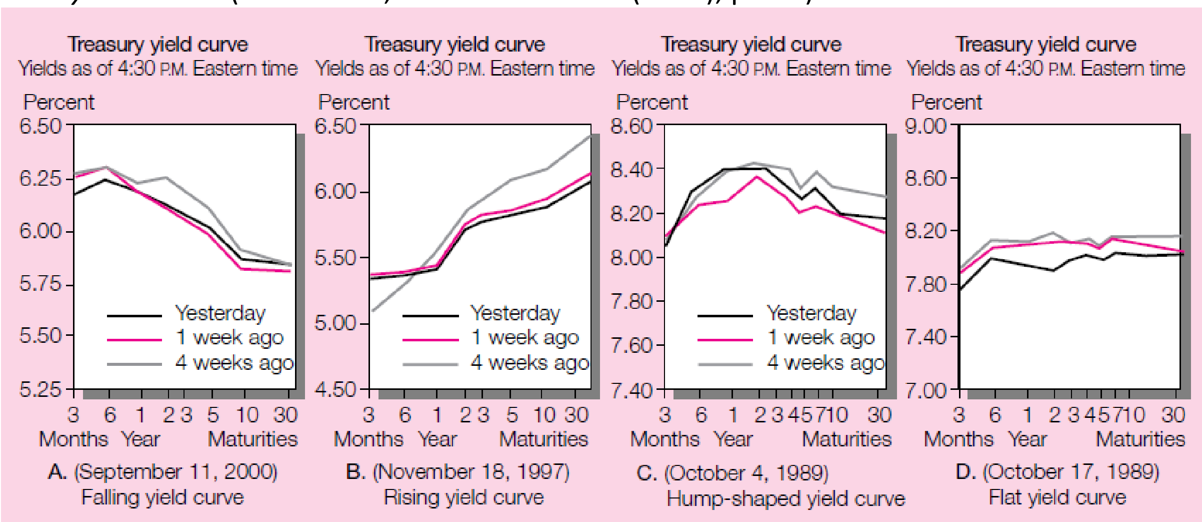
- Bond A: modified duration 2.709, current value: $7.4 million
- Bond B: modified duration 11.81, current value: $2.6 million

The duration of the portfolio is: (7.4 x 2.709 + 2.6 x 11.81) / (7.4+2.6) = 5.075. The portfolio's convexity can be calculated in the same manner (provided that the convexities of the bonds in the portfolio are given).

Finally, we note that duration and convexity measures are only appropriate when the whole interest rate yield curve moves in a parallel fashion (i.e. "parallel shift" in the yield curve). In practice this is not the case, and for more precise management, we have to calculate the bucket sensitivity, i.e. partial derivative with respective to each time bucket. Some of these topics are discussed in the section on key rate immunization below.

5.2.3. Techniques in bond portfolio management

Bond yield curves (from Bodie, Kane and Marcus (2003), p.455)

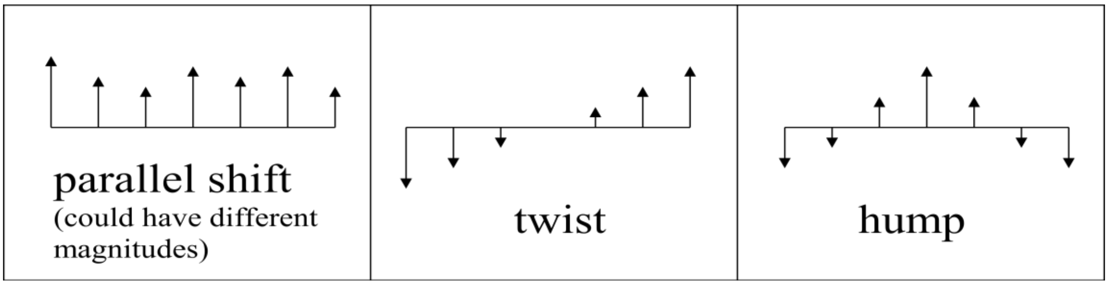


In the real markets, bonds with different maturities would have different yields. A plot of the yield against the time to maturity is called the yield curve. In many markets, it is historically true that most of the time the yield curves are upward sloping, with long term rates higher than short term rates. In some occasions, yield curves can be relatively flat, inverted (short term rates higher than long term rates), or exhibit a hump shape (medium term rates being the highest). Some examples from the US market are shown above.

Different theories have been proposed in order to explain the shapes of these curves. However, for portfolio management purposes, we are more interested in *how* it moves. Basically there are three main types of movements:

- *Parallel shift*: include flattening/steepening, but all rates move in one direction;

- *Twist*: long and short end may move in different directions;
- *Hump*: e.g. long and short end both move down, but mid-range move up.



Passive bond management procedures

We would like to reduce the risk of a bond portfolio when interest rate moves. Three methods are commonly used to manage these portfolios (note that they do not normally apply to investment portfolios as these are considered as hedging strategies):

- *Passive management through indexing*

Try to duplicate the performance of a portfolio based on a pre-defined bond market index.

- *Exact matching*

Try to find the lowest cost portfolio that produces cash flows which match exactly the obligations within the portfolio.

- *Immunization*

Basically this is a duration matching procedure, used to eliminate the changes in portfolio values due to a parallel yield curve shift. This is mostly used by insurance companies and pension funds.

Exact matching procedure

In the simplest case, all cash flow liabilities are matched exactly; alternatively, surplus in one period can be carried forward to pay off the liability in the next period.

	Period 1	**Period 2**	**Period 3**
Liability	$100	$1000	$2000
Portfolio A	$100	$1000	$2000
Portfolio B	$195	$900	$2000

In this example, portfolio A is an exact match to the liability cash flows and has no residual risk. In portfolio B, a surplus of $95 in period 1 is invested and carried forward to finance the shortfall of $100 in period 2.

More formally, we define:

- $L(t)$ as the liabilities in time t
- $C(t,i)$ as the cash flows in period t from a bond of type i
- $P(i)$ as the price of bond i
- $N(i)$ as the number of bond of type i purchased

The objective is to minimize the total cost of the bond subject to some constraints, i.e.

$$\text{minimize} \sum_i N(i)P(i)$$

subject to

$$\text{i)} \sum_i N(i)C(t,i) \geq L(t) \text{ for all } t$$

$$\text{ii)} \; N(i) \geq 0 \text{ for all } i$$

A disadvantage of this method is that there is a significant constraint imposed on bond selection, and thus a company may not be able to purchase significantly "underpriced" bonds and achieve superior returns. Also one needs to be concerned about pre-payment risks or the quality of the bonds selected.

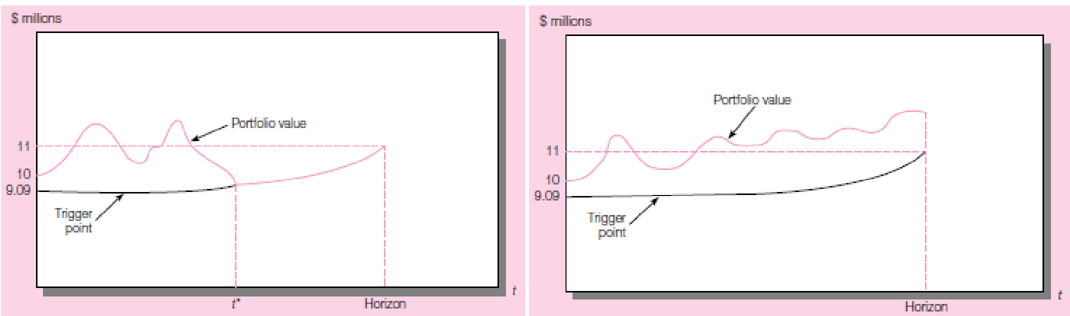
Immunization example

		Value as of Period 4		
Time	Cash Flow	11%	10%	12%
1	13.52	$13.52(1.11)^3$	$13.52(1.1)^3$	$13.52(1.12)^3$
2	13.52	$13.52(1.11)^2$	$13.52(1.1)^2$	$13.52(1.12)^2$
3	13.52	$13.52(1.11)^1$	$13.52(1.1)^1$	$13.52(1.12)^1$
4	13.52	13.52	13.52	13.52
5	113.52	$113.52(1.11)^{-1}$	$113.52(1.1)^{-1}$	$113.52(1.12)^{-1}$
		165.946	165.946	165.974

A 5-year bond with annual coupon 13.52% has a duration of 4 years, and we look at the cash flow at 4 years (i.e. we set the reference point to measure the cash flow at 4 years, so that the 4 year cash flow has no discount factor, the 5-year cash flow is discounted for 1 year, and so on). When interest rate changes from 11% to 10% or 12%, it is seen that the total value of the bond at period 4 does not change much. Therefore this bond can be effectively used to match a single liability at 4 years.

Compared to the exact matching procedures, coupon bonds are more common than zero coupon bonds, so that there are more choices in terms of the availability of the hedging instruments. However, as time progresses, the portfolio needs to be re-balanced as the change in the duration of the coupon bonds would not match the change of the liability, and thus the procedure will incur additional trading costs. Furthermore, as common to the use of duration and convexity, the procedure can only hedge the risks that come from parallel shifts of the yield curve.

Contingent immunization (from Bodie, Kane and Marcus (2003), p.513)



The fund manager may want to pursue active management in order to increase return. If contingent immunization is adopted, a minimum acceptable terminal value is decided, and the corresponding value required for immunization is calculated (known as the trigger point). Whenever the portfolio value drops below the trigger point, an immunization strategy is engaged, otherwise active management is continued.

Dealing with non-parallel shift

Finally, we discuss briefly how to deal with the case for a non-parallel shift in the yield curve. In a standard implementation, in order to calculate the key rate duration, all the interest rates are held constant along the yield curve, except one of the maturities. By changing just one spot yield rate, the change in the portfolio value can be calculated accordingly. The process is then repeated for other key points (e.g. 3 years, 7 years, 10 years, 15 years) and the sensitivities are measured. This is a useful technique to access the validity of portfolio strategies in handling non-parallel shifts in the yield curves, in which various kinds of scenario analysis can also be performed based on this method. The downside is that it is computationally intensive as we must compute the sensitivity under many different scenarios.

Additional References:

Bodie, Kane and Marcus, *Investments*, 9th edition (or earlier editions), McGraw Hill (2011).
Frank Reilly and Keith Brown, *Investment Analysis and Portfolio Management*, 10th edition (or earlier editions), South-Western Cengage Learning (2012).
Maginn, John L. et al. *Managing Investment Portfolios*, 3rd edition, Wiley (2007).