PHYS 8601 – Problem 7

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1 Introduction

We are going to study classical Heisenberg model in this work. Different from a quantum spin, the classical spin vector considered here can point to any direction in the space, so we can have infinite numbers of internal energy values. Three different lattice sizes are tested on various temperatures in this work.

2 Calculation Details

We use spherical coordinates to represents the classical spin vector of unit length $\vec{S} = (1, \theta, \phi)$ with $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. The Heisenberg Hamiltonian with nearest-neighbor interactions is written as

$$H = -J \sum_{(i,j)} \vec{S}_i \cdot \vec{S}_j, \tag{1}$$

where $\vec{S} = (S_x, S_y, S_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Because of the classical representation of spin, the magnetization per site now becomes

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}, \tag{2}$$

where $M_\alpha = \frac{1}{N}\sum_i S_{i\alpha}$. The specific heat and susceptibility are obtained by applying the fluctuation relation.

The code is built on previous version. Metropolis method is applied for the sampling. We enable 3 options in this code, namely quantum spin, classical spin and quantum like classical spin. The quantum like classical spin option only allow two values for θ (0 for spin up and π for spin down), so that the classical can behave like a quantum spin. We've been tested that the quantum like classical spin can get the same results as quantum spin option if the same random number stream is used. 22000 simulations are draw in each independent run with the first 2000 are thrown away.

3 Results and Discussion

The results are presented in figures. Because of enough simulations included, the error bars are very small even near the critical temperature. The internal energy curve of a larger lattice size reaches the largest slop faster than that of a smaller lattice size. In specific heat figure. The peak of L = 24 significant shifted to the left. By doing a little math, we extrapolate the data to infinite size to get the critical temperature 1.1983. The effects of finite size are more clear in the magnetization figure. At high temperatures, results with larger lattice size tend to give a magnetization closer to zero.

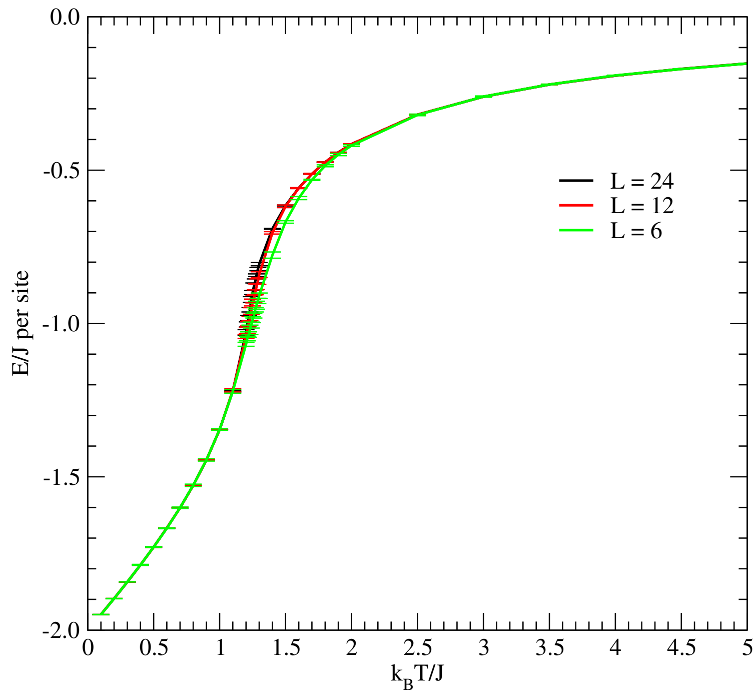

Figure 1: Temperature dependence of the internal energy for different lattice sizes.

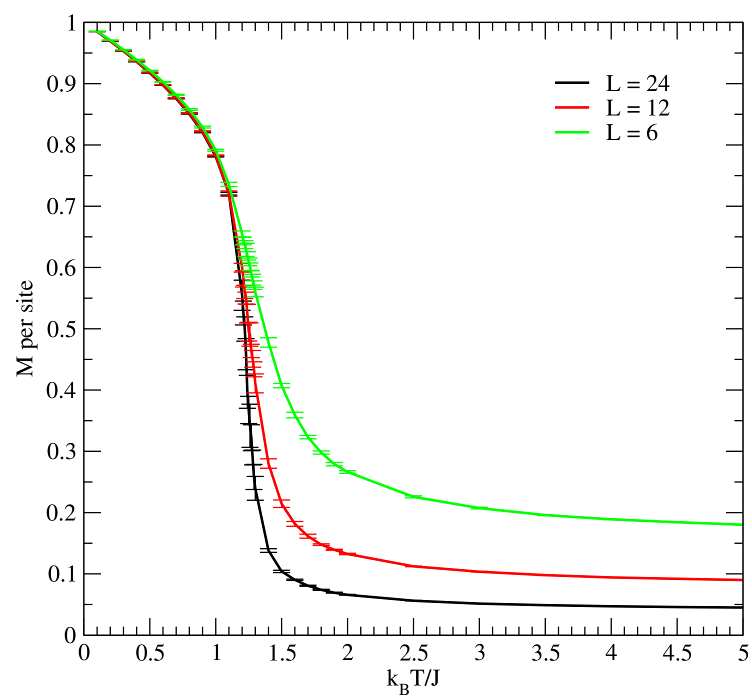

Figure 2: Temperature dependence of the magnetization for different lattice sizes.

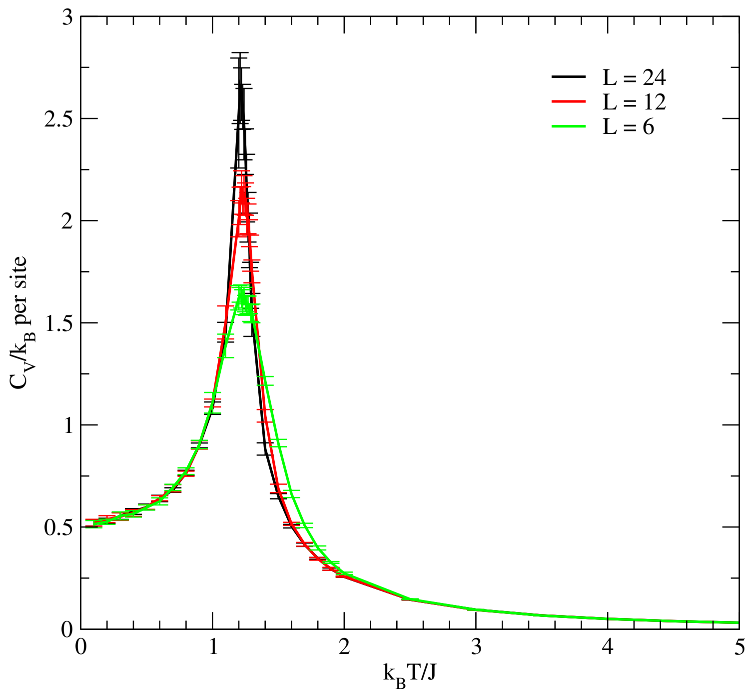

Figure 3: Temperature dependence of the specific heat for different lattice sizes.

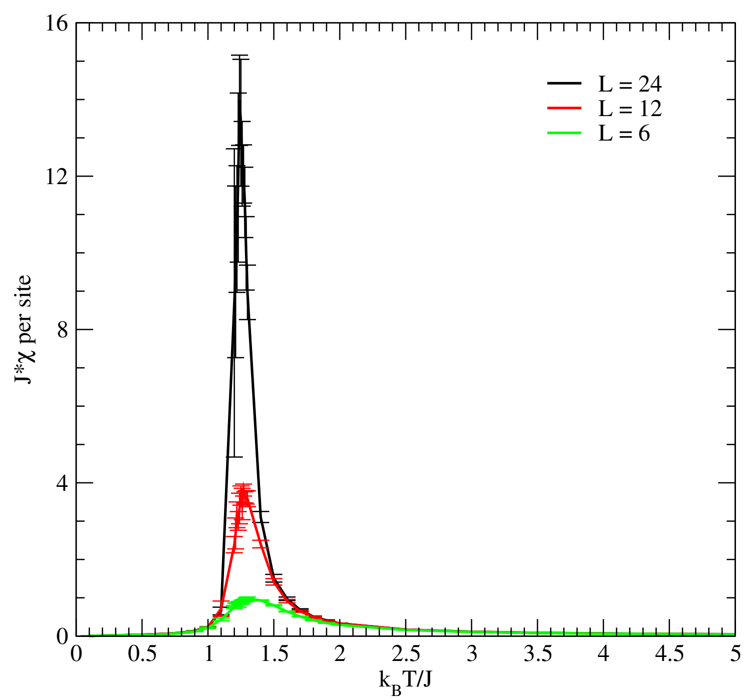

Figure 4: Temperature dependence of the susceptibility for different lattice sizes.