Rational Inattention Choices in Firms and Households *

Yifan Zhang†

University of Oxford

October 29, 2024

[Click here for the most recent version]

Abstract

Recent surveys indicate that households associate higher expected inflation with lower expected output growth, while firms and professionals often associate higher expected inflation with higher expected growth. Standard macroeconomic models struggle to explain this heterogeneity. This paper explains the asymmetric view by allowing households and firms to endogenously choose what they pay attention to, based on their respective incentives. Households find it optimal to pay more attention to supply shocks, while firms optimally pay more attention to demand shocks. Their beliefs then reflect their optimally chosen partial information. I develop a dynamic stochastic general equilibrium (DSGE) model with rationally inattentive households and firms and show that its predictions align with survey evidence. Attention choices influence the propagation of the shocks, affecting the slope of the Phillips curve. Furthermore, central bank communication may have unintended consequences in a rational inattention environment.

Keywords: Rational inattention; Expectation formation; Firms; Households;
JEL classification: D83, E31, E32, E71.

*I wish to thank Martin Ellison, Alexandre Kohlhas, Guido Ascari, Luigi Iovino, Andrea Ferrero, Bartosz Maćkowiak, Kerstin Westergren, and participants at the JME-SNB-Gerzensee Conference 2024, EEA-ESEM 2024, 53rd Annual Conference of the Money, Macro and Finance Society, RES 2024 Annual Conference; EPOC Doctoral Workshop, Conference on Expectations in Dynamic Macroeconomic Models, Norges Bank Workshop, Behavioral Finance Group, University of Oxford for valuable feedback. Financial support from the Departmental Doctoral Bursary and the David Walton Distinguished Doctoral Scholarship are greatly appreciated.

†Department of Economics, University of Oxford, Manor Road, Oxford OX1 3UQ, United Kingdom. E-mail address: yifan.zhang@economics.ox.ac.uk

1 Introduction

Expectations are a key driver of economic decisions. How households and firms revise their expectations in response to shocks is central to their consumption and pricing decisions, and therefore to the propagation of these shocks into aggregate output and prices. However, survey evidence reveals substantial heterogeneity in expectations across agents (Carroll, 2003; Mankiw et al., 2003; Candia et al., 2020, 2023; Link et al., 2023). One particularly striking heterogeneity concerns how they perceive the correlation between output and inflation in their expectations: households tend to associate higher expected inflation with lower output growth. In contrast, firms and professional forecasters tend to associate higher expected inflation with higher expected growth, although the correlation for firms is relatively weak.[1] The negative association is labeled as a supply-side view in Candia et al. (2020) as supply shocks such as productivity shocks are expansionary for output and reduce inflation, leading to the negative comovement between output and inflation. Similarly, the positive association is labeled as a demand-side view, as demand shocks such as monetary shocks are expansionary for output and inflation. This evidence raises a critical question for theory: how can we reconcile these contrasting views?

Standard macroeconomic models are not consistent with this evidence. In any full-information model, agents should hold the same belief about future inflation and output.[2] Recent advances in the theory of expectations that depart from rational expectations or full information also struggle to explain the systematic heterogeneity observed across agents (e.g., Evans and Honkapohja (2001); Woodford (2003); Gabaix and Laibson (2017); Bordalo et al. (2018)). In principle, existing expectation models could potentially explain these contrasting views by imposing different partial information or subjective models for different agents (Han, 2022; Andre et al., 2022). For example, one might assume that households observe mostly supply shocks, while firms observe more demand shocks. However, such assumptions lack theoretical basis on why these information differences arise. The contribution of this paper is to allow agents to endogenously choose their (partial) information sets and to show that they optimally acquire information in a way that generates such differing partial information, leading to the observed heterogeneity in their views.

This paper develops a dynamic general equilibrium model with rational inattentive firms and households. The assumption of rational inattention generates *endogenous* and *asymmetric* attention choices for households and firms. The endogeneity of attention choices stems from optimizing agents who pay more attention to the economic shocks that matter most for their objectives. The attention choices by firms and households are asymmetric as they have different objective functions. With standard utility and profit functions, this paper shows that it is optimal for households to pay more attention to supply shocks;

[1] This evidence is based on the U.S. data; see Section 2 for further details.

[2] Frictionless real-business-cycle (RBC) models generally predict a negative correlation, while the Keynesian paradigm with full information can reconcile one of the two views by adjusting the relative weight of demand and supply shocks – but not both simultaneously.

while firm optimally allocate slightly more attention to demand shocks. These asymmetric attention choices are sufficient on their own to explain the supply-side view of households and demand-side view firms in the survey, as they would base their expectations on their respective partial information sets. As professional forecasters' expectations do not affect economic outcomes, I do not introduce them explicitly but assume they have full information. With reasonable calibration, the model can quantitatively match the survey expectations for households, firms and professional forecasters.

Furthermore, rich interactions between attention allocations arise in the general equilibrium model where both agents are rationally inattentive. More generally, attention allocation choices of firms and households are substitutional for demand shocks (households pay less attention if firms pay more attention), while complementary for supply shocks (households pay less attention if firms pay less).[3] The strategic complementarity in the case of productivity shocks can create a downward spiral of inattention, dampening the importance of productivity shocks in driving business-cycle fluctuations.

I use measured survey beliefs to quantify the inattention of households study the implications for business cycles. Rational inattention increases the relative importance of demand shocks in driving business-cycle fluctuations and results in a weakly positive Phillips curve, which would otherwise be negative under full information. Moreover, the slope of Phillips curve are endogenous to the conduct of monetary policy. Specifically, a more hawkish monetary policy reduces firms' attention, making prices less sensitive to output changes. It also shifts households' attention from supply shocks to demand shocks – amplifying the volatility in output gaps. Both forces helps explain the documented flattening of the Phillips curve over the past few decades (see, for example, Coibion and Gorodnichenko (2015), Blanchard (2016), Bullard (2018), Hooper et al. (2020)).

The rational inattention model offers several insights for central bank communication beyond standard models. First, standard theory predicts that an exogenous rise in expected inflation would increase households' spending today before prices rise, a key mechanism of forward guidance. However, with rational inattention, households are more likely to interpret the communicated higher inflation as originating from a contractionary productivity shock, leading them to lower growth expectations and reduce spending.[4] Second, central bank may commit to a lower interest rate path during periods of economic slack to stimulate demand. However, inattentive firms unaware of the slack may misinterpret the lower rate as a originating from an expansionary demand shock and raise prices, which may reduce demand further.

[3]The strategic interactions in information acquisition have been studied in several studies, Maćkowiak and Wiederholt (2009) and Hellwig and Veldkamp (2009), among others, which argue complementarity (substitutability) in information choices arises from the complementarity (substitutability) in actions. Here I demonstrate that complementarity (substitutability) can also arise through the value of information in a general equilibrium model with multiple inattentive agents.

[4]This model prediction aligns with evidence from randomized control trials (RCTs) on households by Coibion et al. (2023), who find that when households lower their inflation expectations after receiving information treatments, they also anticipate higher real incomes and increase spending.

Related Literature. This study contributes to the research agenda that seeks to develop a data-consistent model of expectation formation. Three closely related studies are presented by Kamdar et al. (2018), Michelacci and Paciello (2024), and Han (2022). Kamdar et al. (2018) proposes a partial equilibrium inattentive consumer model that attributes the supply-side view by consumers to information compression. By contrast, this paper develops a general equilibrium model with rational inattention, and shows that households' supply-side view arises from the optimal responses of firms.[5] Michelacci and Paciello (2024) proposes an ambiguity averse model and show households' expectations are tilted toward the contingencies that households dislike more. Han (2022) explains observed heterogeneity by assuming different partial information for different agents exogenously. In contrast, this paper argues that agents' partial information is optimally chosen based on their respective objectives.

This paper broadly relates to the rational inattention literature following Sims (2003). The core premise of this literature is that incentives drive attention, implying that agents pay more attention to certain components or in certain circumstances than others (e.g., Maćkowiak and Wiederholt (2009); Kohlhas and Walther (2021); Flynn and Sastry (2024)). In this regard, this paper contributes by showing that agents' attention to particular shocks can be higher than their attention to others. Another contribution of this paper is that it solves a DSGE model with rational inattention for both firms and households, which is less investigated due to computational challenges. While Maćkowiak and Wiederholt (2015) also features rational inattentive households and firms, this paper extends the analysis by studying further expectation-related moments, which were not incorporated in previous work.

This paper also connects to a vast literature in macroeconomics on the role of imperfect information in business cycle dynamics and central bank communication. A partial list of prominent contributions in this strand includes Lucas (1972); Woodford (2001); Eusepi and Preston (2010); Blanchard et al. (2013); Angeletos and La'o (2013); Chahrour and Ulbricht (2023). The contribution of this paper, in this context, is to highlight the macroeconomic consequences when agents endogenously choose different partial information, and offer new insights on the central bank communication in a rational inattention environment.

Layout. The paper is organized as follows. In Section 2, I document the heterogeneous views by different agents and provide some supplementary evidence on the role of attention. In Section 3, I provide a closed-form characterization of households' and firms' attention choices under rational inattention. In Section 4, I extend the simple model to

[5]This study also differs from Kamdar et al. (2018) in several critical aspects. First, due to the reliance on information compression, in Kamdar et al. (2018), agents' belief does not approach the true data-generating process as information costs decrease. In contrast, this model converges to the full-information equilibrium as information costs approach zero. Second, this paper focuses on the correlation of expectations rather than posterior beliefs, making the results directly relevant to survey evidence where questions pertain to agents' expectations rather than their posterior beliefs. Third, while Kamdar et al. (2018) examines static attention choices, this model incorporates dynamic attention choices.

a full dynamic general equilibrium model, where I calibrate the model and analyze the impact on macroeconomic dynamics. In Section 5, I discuss the implications for central bank communication. Section 6 concludes.

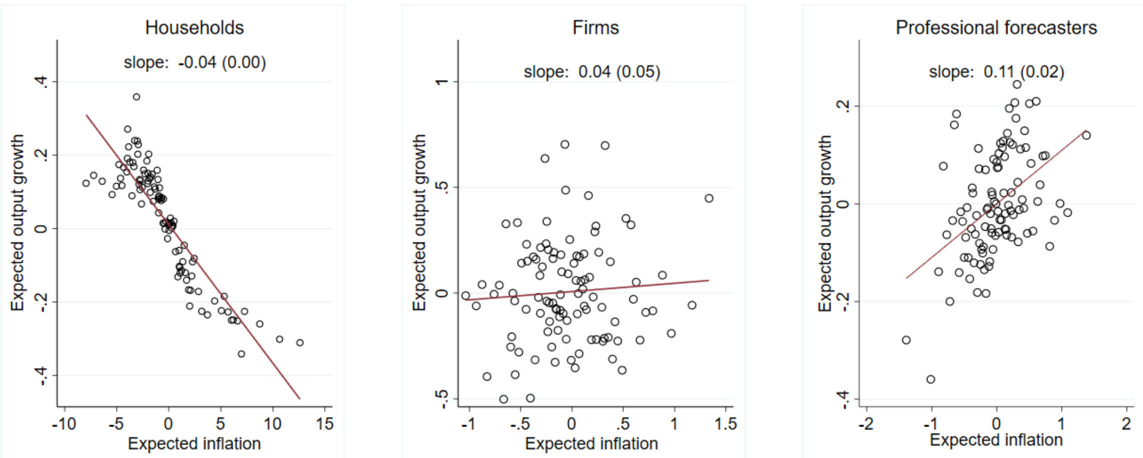
2 Stylized Facts on Expectations and Attention Choices

In Section 2.1, I provide detailed evidence on the perceived relationship between inflation and output growth among various agents. I then present stylized facts on households' and firms' attention choices in Section 2.2 and show their attention choices matter for their views in Section 2.3.

2.1 Perceived Relationship between Inflation and Real Economy

In this section, I investigate the perceived relationship between inflation and the real side of the economy among households, firms and professional forecasters, following the regression proposed by Candia et al. (2020). Figure 1 plots the joint distributions of expected inflation and expected output growth for different economic agents in the United States. The data used are from Michigan Survey of Consumers, the Livingston Survey (non-financial business) and the Survey of Professional Forecasters.[6]

Figure 1: Correlation between expected inflation and expected output



Notes: Each panel plots a bin-scatter for the joint distribution of expectations for output growth rate and inflation in the next year across different economic agents in the United States. For each variable, I take out the time fixed effect so that all variables are mean zero. Table A.1 provides a summary of the associated regression statistics.

Data Sources: Michigan Survey of Consumers; The Livingston Survey; The Survey of Professional Forecasters.

[6]While in the Michigan Survey of Consumers, respondents are not asked to provide a quantitative forecast for output growth, they are asked about whether they expect business conditions in the next year to improve, stay the same or deteriorate. Following Candia et al. (2020), I assign point values to each answer ranging from 1 (improve) to -1 (deteriorate). I then plot the households' responses to this question and their quantitative inflation forecasts, taking out time fixed effects.

Households tend to associate higher expected inflation with lower expected growth, the pattern is labeled as a supply-side view. Firms and professional forecasters, on the other hand, tend to associate higher expected inflation with higher expected growth, labeled as a demand side view. I also plot the joint distribution of expected inflation and expected unemployment rates (see Figure A.1), which conveys a similar message. Such patterns are consistently observed across different time periods and various countries.[7] Notably, the negative correlation for households persisted even during the Great Moderation, a period when inflation was primarily demand-driven. Similar results are also found in randomized control trials. For instance, Coibion et al. (2023) find that when Dutch households' inflation expectations are exogenously increased, they revise their expectation about future growth downward. And Coibion et al. (2021) finds similar weak correlation for New Zealand firms.

Despite the consistent evidence, a potential concern arises from the fact that the plots are essentially cross-sectional. Consequently, the negative slope observed for households might reflect across-household variation. For instance, certain households might consistently forecast low output growth and high inflation, while others consistently predict the opposite, irrespective of prevailing economic conditions. This possibility would undermine the claims that households hold a supply-side view.

To address this concern, I leverage the panel dimension of the surveys. Focusing on respondents that appear at least three times in the Michigan Survey of Consumers (MSC), I conduct individual regressions – mirroring the approach above – for each respondents.[8] The results explicitly characterize how households' beliefs about the inflation and output growth expectations evolve jointly. Of the 4,276 respondents interviewed at least three times, 75.3% demonstrated a negative slope. The firms pool in the Livingston Survey is relatively stable, with firms being asked between 1 and 38 times in the survey over time. I focus on the firms that have at least 5 observations and run the regression for each firm. Around 54.3% of the firms show a positive slope and 45.7% show a negative slope. The Survey of Professional Forecasters is also panel with relatively longer time span, and I focus on forecasters with at least 10 observations, of which 73.7% have a positive slope.

2.2 Stylized Facts about Agents' Attention Choices

One explanation for the contrasting views among different agents is their different information sets, shaped by their different information choices. This section presents stylized facts on the information choices of households and firms, drawing insights from survey data.

[7] Candia et al. (2020) examine how the beliefs about joint expectations of inflation and output growth for different agents in Belgium, France, Germany, Italy, the Netherlands and Spain. The findings are strikingly consistent with those observed for the U.S.

[8] The survey features a rotating panel sample design. Typically, any given survey sample from the MSC comprises two-thirds new respondents and one-third being interviewed for the second time. This setup creates a short panel where each cross-sectional unit appears in the survey more than once.

For households, I use additional data from Michigan Survey of Consumers in which respondents were asked to report the news related to business conditions that they heard of during the last few months while making their predictions about inflation and output growth in the next year.[9] This question captures the information that households have *internalized* and therefore reflects what information they pay attention to. The answers to this question are categorized into arbitrary but well-defined groups. Figure 2 shows spike plots for news heard that is price-related or employment-related over time. The news households have internalized is predominantly employment-related, while news about prices stands out only in particular time periods, indicating a consistent high level attention to the real side of the economy among households.

(a) News about employment (b) News about prices

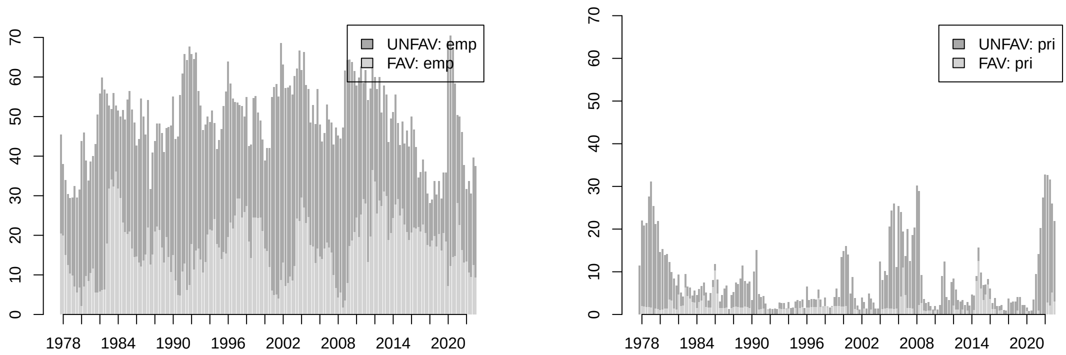


Figure 2: Spike Plots of News Heard Categories

Note: These plots show the fraction of survey respondents having heard news in each category in the relevant quarter. Each category further distinguishes between favorable (depicted in light gray) and unfavorable news (shown in dark gray).

While the Livingston Survey doesn't offer analogous questions for firms, the Business Inflation Expectations (BIE) Survey – which also targets U.S. firms – provides insight into their pricing considerations. This survey reveals that firms are well aware of future nominal marginal costs, and these play a significant role in their price-setting strategies. Specifically, from 2011 to 2023, 69% of respondents in the BIE survey indicated that labor costs would affect the prices of their products and/or services in the upcoming 12 months. Although this is not direct evidence on firms' attention choices, it at least suggests that firms have incentives to pay attention to variations in their nominal marginal costs.

In summary, households and firms appear to pay attention to different variables: households tend to pay more attention to the real labor market conditions, firms have incentives to pay attention to their nominal marginal costs.

[9]A detailed description of the question, along with a comprehensive list of categories, is available on the Michigan Survey of Consumers.

2.3 Attention Choices Matter for Expectations

Does households' attention to labor market conditions influence their perception of the relationship between expected inflation and expected growth? To test the this, I run the following regression for households

$$\mathbb{E}_t^i[Growth] = \beta_0 + \beta_1 \mathbb{E}_t^i[Inflation] + \gamma_1 \mathbb{E}_t^i[Inflation] \times News_{i,t}^{labor} + \gamma_2 \mathbb{E}_t^i[Inflation] \times News_{i,t}^{price} + \alpha_1 News_{i,t}^{labor} + \alpha_2 News_{i,t}^{price} + \alpha_t + u_{i,t} \tag{2.1}$$

Here the labor news $News_{i,t}^{labor}$ is a binary variable, taking a value of 1 if a respondent i reports having heard news about labor market conditions recently, and 0 otherwise. Similarly, the price news variable $News_{i,t}^{price}$ is set to 1 if the respondent i has recently heard news related to prices, and 0 otherwise. A supply-side view corresponds to a negative β_1. If the coefficient of the cross term is negative $\gamma_1 < 0$ or $\gamma_2 < 0$, attention to that news contributes to a supply-side view. Conversely, a positive coefficient $\gamma_1 > 0$ or $\gamma_2 > 0$ suggests that paying attention to this news contributes to a more demand-side view.

Table 1 reports the results of this regression and finds $\gamma_1 < 0$ and significant – households who pay attention to labor market news hold an even stronger supply-side view compared to those who don't. Conversely, attention to price news seems to contribute to a demand-side view $\gamma_2 > 0$, though its impact is relatively weak.

One potential concern is that if labor news is more likely to be bad news, the coefficient might reflect pessimism – households who hear bad news could become more pessimistic, leading to lower growth forecasts. To address this, I further divide labor news into positive and negative categories and re-run the regression. This division does not change the sign of γ_1 (see column 2 and 3 in Table 1). Thus, regardless of whether the news is positive or negative, attention to this information reinforces the negative correlation households perceive between inflation and growth predictions. These results also rule out pessimism as the sole explanation for the negative correlation between inflation and output.[10]

[10] Bhandari et al. (2022) finds that increased pessimism generates upward bias in unemployment and inflation forecasts, contributing to the negative correlation between inflation and real activity. However, the results in this paper suggest that pessimism is only part of the explanation behind households' supply-side view.

Table 1: Perceived Relationship between Inflation and Growth: Households

	Growth Forecasts		
	All	Labor news (+)	Labor news (-)
Inflation Forecasts	-0.047^{***}	-0.047^{***}	-0.047^{***}
	(0.001)	(0.001)	(0.001)
Inflation Forecasts $\times$ **Labor news**	-0.0186^{**}	-0.019^{*}	-0.013^{*}
	(0.007)	(0.010)	(0.008)
Inflation Forecasts $\times$ **Price news**	0.006	0.006	0.006
	(0.027)	(0.027)	(0.027)
Labor news	-0.091^{***}	0.150^{***}	-0.237^{***}
	(0.025)	(0.025)	(0.022)
Price news	0.061	0.061	0.061
	(0.073)	(0.073)	(0.073)
Intercept	0.019	0.019	0.020
	(0.002)	(0.002)	(0.002)

Note: The table reports the results of regression (2.1). Column 1 reports the results for the full sample. Column 2 and 3 show that the results are robust despite dividing into favorable/unfavorable labor news.

Taken together, households pay more attention to labor market conditions compared to prices, which reinforces their supply-side view. Firms have incentives to pay more attention to their nominal marginal costs. However, the survey data available for firms does not allow for an examination of whether their attention choices affect their joint expectations about output growth and inflation.

3 Attention Choices in Firms and Households

In this section, I present a simple model with rational inattention to illustrate the asymmetry in the attention choices of households and firms.

3.1 Environment

Households. There is a continuum of hand-to-mouth households indexed by $i \in [0,1]$. Household i in each period chooses consumption $C_{i,t}$ to maximize its expected utility and supplies labor $L_{i,t}$ such that the budget constraint binds. Household i's period utility at time t is

$$U\left(C_{i,t}, L_{i,t}\right) = \left[\frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{L_{i,t}^{1+\eta}}{1+\eta}\right] \tag{3.1}$$

$$s.t.\ P_t C_{i,t} = W_t L_{i,t}, \quad C_{i,t} = \left[\int_0^1 C_{i,j,t}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{3.2}$$

where β denotes the time discount factor, $C_{i,j,t}$ is household i's demand for variety j given its price $P_{j,t}$ and $C_{i,t}$ is the final consumption good aggregated with a constant elasticity of substitution $\theta > 1$ across varieties. W_t is the nominal wage, and $P_t = [\int_0^1 P_{j,t}^{1/(\theta-1)} dj]^{\theta-1}$

is the aggregate price index. The parameter $\gamma > 1$ is the risk aversion coefficient and the parameter η is the inverse of Frisch elasticity of labor supply.

Firms. There is a continuum of firms producing differentiated goods, each indexed by $j \in [0, 1]$. Each firm j is a monopoly producer of its own variety and faces a demand curve $Y_{j,t} = (P_{j,t}/P_t)^{-\theta} Y_t$, where $Y_t = \int_0^1 Y_{j,t} dj$ is the aggregate output. They hire labor $L_{j,t}$, pay wages W_t per worker, and produce with a linear technology

$$Y_{j,t} = A_t L_{j,t} \tag{3.3}$$

where A_t is the aggregate productivity.

In each period, firm j sets the price $P_{j,t}$ for its own product to maximize its expected profit and produces a sufficient quantity of goods to meet the demand $Y_{j,t}$. The profit of firm j at time t, discounted by the household's marginal utility of consumption, is expressed as

$$\Pi_{j,t}(P_{j,t}, L_{j,t}, Y_{j,t}) = \frac{1}{P_t C_t^{\gamma}} \left[P_{j,t} Y_{j,t} - (1 - \theta^{-1}) W_t L_{j,t} \right] \tag{3.4}$$

where $(1 - \theta^{-1}) W_t$ denotes the subsidized wage rate, with the subsidy θ^{-1} paid to eliminate steady-state distortions introduced by monopolistic competition.

Central Bank. For analytical tractability, I assume that central bank controls nominal aggregate demand $Q_t \equiv P_t Y_t$. This assumption is a popular framework in the rational inattention literature to study the effects of monetary policy on pricing.[11] I further assume that the central bank has full information and interpret it as the model counterpart of the professional forecasters in the survey.

Shocks. The economy is subject to both demand and supply shocks. I model the demand shock as shocks to the nominal aggregate demand ($q_t \equiv \log Q_t$), and the supply shock as shocks to all firms' productivity levels ($a_t \equiv \log A_t$). The two exogenous processes follow Gaussian white noise distributions with variance $\sigma_q^2 > 0$ and $\sigma_a^2 > 0$, and are mutually independent.

3.2 Attention Costs and Information Structure

Costly Attention. In this environment, agents must pay attention in order to be aware of the economic conditions. While the cost of attention can, in principle, take many different forms (see e.g., Hébert and Woodford (2018)), I follow Sims (2003) and model the attention costs as linear in Shannon's mutual information $\mu \mathcal{I}\left(X; S^t | S^{t-1}\right)$, where μ is the marginal cost of attention. Specifically, $S_t \in \mathcal{S}^t$ denotes the signals at time t, and $\mathcal{S}^t$ is the set of

[11]See for example Mankiw et al. (2003); Woodford (2003); Maćkowiak and Wiederholt (2009); Paciello (2012); Afrouzi and Yang (2021) among others.

available signals. The history of signals up to time t is denoted by $S^t = S^{t-1} \cup S_t$. Mutual information is defined as

$$\mathcal{I}\left(X; S^t | S^{t-1}\right) \equiv h(X|\mathcal{S}^{t-1}) - \mathbb{E}\left[h(X|\mathcal{S}^t)|\mathcal{S}^{t-1}\right]$$

This measures the reduction in entropy of the object X due to information of gained from signal S^t conditional on the history of signals S^{t-1}.

This formulation assumes that the agents do not forget information over time, and thus the information chosen today can have a continuation value. In the simple model presented in this section, this condition does not matter as shocks are i.i.d, so the knowledge about the shocks today does not affect future priors. However, in the full model presented in Section 4, where shock processes are more complex and intertemporal decisions are involved, past information becomes useful for agents.

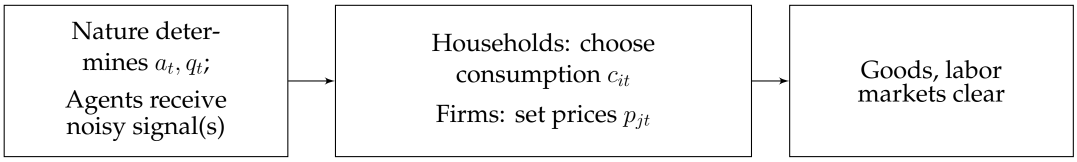
Information Structure. It is necessary to specify the information structure, i.e., the available signal set $\mathcal{S}^t$. I consider two popular approaches in the literature. One approach, optimal signal design, explored by Sims (2003) and Maćkowiak et al. (2018), allows agents full flexibility when designing the conditional distribution of their signals given the state of the economy. An alternative approach, constrained information structure, restricts agents to acquiring N separate, conditionally independent signals about N different components in their optimal action. In the current context, I partition the signal into one subvector that contains only information on demand shock q_t and another subvector that contains only information on the productivity shock a_t.

The choice of information structure typically depends on the problem at hand. In this context, optimal signal design is more realistic than restricting agents to separate signals for different shocks.[12] However, for analytical tractability and interpretability, in Section 3.4 and 3.5, I solve the attention problem under a constrained information structure. In Section 3.5, I compare the predictions of each approach and find that the choice of information structure does not significantly affect the results. In other sections, including the quantitative model in Section 4, I adopt optimal signal design to better capture how households and firms acquire information.

Timing. In the initial period $t = 0$, households and firms make their ex ante attention choices, which we can think of determining the form and precision of the associated signals. In each subsequent period $t > 0$, shocks (q_t, a_t) realize. The economy proceeds through three stages: (i) Depending on their respective attention choices, households and firms receive different forms of signals with different precision levels; (ii) Upon receiving the signals, households choose their consumption and firms set their prices for their own

[12]The model implied optimal signals aligns with the survey evidence on agents' attention choices. See 2 for details.

varieties. (iii) After their choices are committed, households supply labor to cover their consumption and firms produce sufficient goods to meet the demand. Finally, the real wage adjusts to clear the labor market.



Once the attention choices have been made, the problem is straightforward, so the key is to understand how agents make their attention choices.

3.3 Attention Problems of Households and Firms

Households. For tractability, I simplify the households' utility function (3.1) with quadratic approximations (derivation see Appendix A.1). After the approximation, household i's objective (3.1) at time t can be expressed as the utility loss from deviating from the optimal consumption level $c^*_{i,t}$ – the consumption level that households would choose under full information[13]

$$\left[-\frac{(\gamma+\eta)}{2}\left(c_{i,t}-c^*_{i,t}\right)^2\right] + \text{terms independent of } \{c_{i,t}\} \tag{3.5}$$

Here, lowercase letters denote the logs of the corresponding variables. $c_{i,t}$ is the actual consumption choice made by household i. When the household deviates from its optimal choice, the utility loss is proportional to the risk aversion coefficient γ and the inverse of Frisch elasticity of labor supply η. Households that are more risk averse and less elastic in labor supply lose more utility by choosing a suboptimal consumption level.

The optimal consumption level is obtained by equating the marginal rate of substitution between consumption and leisure to real wage[14]

$$c^*_{i,t} = \frac{1+\eta}{(\gamma+\eta)}\left(w_t - p_t\right) \tag{3.6}$$

The equation states that optimal consumption is a function of real wage. If households know real wage, they can achieve optimal consumption level, which also implies that households want to learn about real wages to guide their consumption decisions. This aligns with survey evidence that households pay more attention to real labor market developments (see Section 2).

Substituting the optimal consumption from Equation (3.6) into the utility function (3.5), and add the attention cost term, household $i \in [0,1]$ attention problem is formally

[13] The first-order term of this approximation drops out due to the envelope theorem: there are no first-order costs of deviating from c^*_{it}. Full derivation see A.1.

[14] The optimal consumption is derived by substituting $l_{i,t}$ using the budget constraint $p_t + c_{i,t} = w_t + l_{i,t}$ into the Intra-temporal Euler Equation $\gamma c_{i,t} + \eta l_{i,t} = w_t - p_t$.

defined as

$$\max_{\{s_{i,t}\in\mathcal{S}_h^t\}} \mathbb{E}_t^h\left[-\frac{(\gamma+\eta)}{2}\left(c_{i,t}-\frac{1+\eta}{\gamma+\eta}(w_t-p_t)\right)^2-\mu^h\mathcal{I}(a_t,q_t;s_{i,t})\right] \tag{3.7}$$

The first term in Equation (3.7) captures the benefits of attention, as $c_{i,t}$ gets closer to the optimal level, which is a function of the real wage. The second term reflects the cost of attention, measured by the marginal cost of attention $\mu^h > 0$ times the expected entropy reduction after observing signal $s_{i,t} \in \mathcal{S}_h^t$, where S_h^t is the set of all available signals for households at time t.

Firms. I simplify the firms' profit function (3.4) with quadratic approximations (derivation see Appendix A.2), yields

$$\left[-\frac{\theta-1}{2}\left(p_{j,t}-p_{j,t}^*\right)^2\right]+\text{terms independent of }\{p_{j,t}\} \tag{3.8}$$

where lowercase letters denote the logs of the corresponding variables. Equation (3.8) states that firm j experiences a profit loss from setting a price $p_{j,t}$ that deviates from its optimal price level $p_{j,t}^*$. In this simple setup, firm's optimal price is just its nominal marginal costs

$$p_{j,t}^* = w_t - a_t \tag{3.9}$$

Moreover, the magnitude of profit losses is proportional to firm's demand elasticity $(\theta - 1)$. In other words, firms with more elastic demand experience larger profit losses when charging a suboptimal price.

Substituting the optimal price using Equation (3.9) into the profit function (3.4), and add the attention costs, firm j's attention problem is formally defined as

$$\max_{\{s_{j,t}\in\mathcal{S}_f^t\}} \mathbb{E}_t^f\left[-\frac{\theta-1}{2}(p_{j,t}-(w_t-a_t))^2-\mu^f\mathcal{I}(q_t,a_t;s_{j,t})\right] \tag{3.10}$$

The first term captures the benefit of paying attention, that the firm's price $p_{j,t}$ gets closer to the optimal level, i.e., firm j's nominal marginal cost. The second term is the cost of attention, measured by firm's marginal cost of attention $\mu^f > 0$ times the expected entropy reduction about the optimal price $p_{j,t}^*$ after observing $s_{j,t} \in \mathcal{S}_f^t$.

The equilibrium of the model is defined as in Definition 1.

Definition 1 (Equilibrium). *Given the processes for the productivity and monetary policy shocks $\{q_t, a_t\}_{t\geq 0}$, a general equilibrium of this economy is an allocation for every household $i \in [0,1]$, $\Omega_i \equiv \{s_{i,t} \in \mathcal{S}_{i,t}, C_{i,t}, L_{i,t}\}_{t=0}^{\infty}$, given their initial set of signals; an allocation for every firm $j \in [0,1]$, $\Omega_j \equiv \{s_{j,t} \in \mathcal{S}_{j,t}, P_{j,t}, L_{j,t}, Y_{j,t}\}_{t=0}^{\infty}$ given their initial set of signals; a set of prices $\{P_t, W_t\}_{t=0}^{\infty}$*

1. *Given the processes for* $\{P_t, W_t\}_{t=0}^{\infty}$ *and all firms' decisions* $\{\Omega_j\}_{j\in[0,1]}$*, every household i's allocation solves the attention problem (3.7);*

2. *Given the processes for* $\{P_t, W_t\}_{t=0}^{\infty}$ *and all households' allocations* $\{\Omega_i\}_{i\in[0,1]}$*, every firm j's allocation solves the attention problem (3.10);*

3. *The equilibrium processes* $\{P_t, W_t\}_{t=0}^{\infty}$ *are consistent with households' and firms' allocation,* $\{\Omega_i\}_{i\in[0,1]}$ *and* $\{\Omega_j\}_{j\in[0,1]}$*.*

Solving for the equilibrium where both households and firms are subject to rational inattention is challenging, as their attention and decisions would depend on on endogenous variables as well as each other's attention choices. To provide intuition on the attention choices of households and firms, I simplify the general equilibrium model by first considering a case where only households are subject to rational inattention while firms have full information (Section 3.4). Next, I examine the case where only firms are rationally inattentive while households have full information (Section 3.5). Finally, in Section 3.7, I explore the rich interactions between the attention choices of households and firms.

3.4 Households' Attention Choices

I begin by analyzing the case where households are subject to rational inattention while firms have full information. In this case, firms set prices at their optimal level according to Equation (3.9), which implies that the real wage is fully determined by productivity

$$w_t - p_t = a_t \tag{3.11}$$

From Equation (3.11), the real wage is not affected by demand shocks q_t, this is due to firms' optimizing behavior – following a demand shock, nominal wages rise, fully informed firms increase prices one-to-one to nominal wage, and the real wage is thus unaffected. This follows the classical dichotomy.

To develop an intuition about households' attention choices, imagine that a measure of zero of households have no information, while all others have full information. Since all other households have full information, the utility-maximizing consumption remains $\frac{1+\eta}{\gamma+\eta}(w_t - p_t) = \frac{1+\eta}{\gamma+\eta} a_t$. However, households with no information fail to adjust their consumption (i.e., $c_{i,t} = 0$), resulting in an expected utility loss proportional to

$$\mathbb{E}_{i,t}\left[-\left(c_{i,t} - \frac{1+\eta}{\gamma+\eta}(w_t - p_t)\right)^2\right] = \mathbb{E}_{i,t}\left[-\left(0 - \frac{1+\eta}{\gamma+\eta} a_t\right)^2\right] = -\left(\frac{1+\eta}{\gamma+\eta}\right)^2 \sigma_a^2$$

As long as firms have full information and adjust their prices to fully track changes in the nominal marginal costs, there is no utility loss for households from misinformation about demand shocks for households, even if they pay no attention to those shocks. The expected utility loss arises solely from misinformation about supply shocks. Furthermore, this loss is higher when (i) optimal consumption is more responsive to productivity shock (i.e., high γ or η) (ii) shocks are more volatile (i.e., high σ_a^2). Figure 3a illustrates this with a contour plot showing utility loss when a_t and q_t are misperceived. The plot consists of horizontal lines, indicating no loss from not attending and responding to q_t.

Under constrained information structure, households can obtain N separate, conditionally independent signals about real wage $w_t - p_t$. In this context, households can obtain one signal about the demand shock and another signal about the productivity shock[15], i.e.,

$$s_{i,t} = \{s_{i,q,t}, s_{i,a,t}\} \tag{3.12}$$

where

$$s_{i,q,t} = q_t + e_{i,q,t} \quad \text{and} \quad s_{i,a,t} = a_t + e_{i,a,t} \tag{3.13}$$

and $\{s_{i,q,t}, q_t\}$ and $\{s_{i,a,t}, a_t\}$ are independent. The signals follow stationary Gaussian processes, and all noises are idiosyncratic.

Upon receiving these signals, consumption $c_{i,t} = \lambda_{h,a}\,\mathbb{E}[a_t|s_{i,a,t}]$ maximizes the expected utility for any given posterior belief, with $\lambda_{h,a} = \frac{1+\eta}{\gamma+\eta}$. Define $\sigma^2_{a|s}$ as the posterior uncertainty about a_t. Substituting $c_{i,t}$ and real wage (3.11) into Equation (3.7) yields

$$\begin{aligned} &\max_{\{s_{i,t}\in\mathcal{S}_i^t\}} \mathbb{E}_t^i\left[-\frac{\gamma+\eta}{2}\left(\lambda_{h,a}\,\mathbb{E}[a_t|s_{i,a,t}] - \lambda_{h,a}a_t\right)^2 - \mu^h\mathcal{I}\left(a_t, q_t; s_{i,t}\right)\right] \\ &= \frac{1}{2}\max_{\sigma^2_{a|s}\le\sigma^2_a}\left[-(\gamma+\eta){\lambda_{h,a}}^2\sigma^2_{a|s} - \mu^h \ln\frac{\sigma_a^2}{\sigma^2_{a|s}}\right] \end{aligned} \tag{3.14}$$

Solving this problem characterizes households' attention choices, as summarized in Proposition 1.

Proposition 1. *Households optimally allocate more attention towards supply shocks*

1. *When firms have full information, and households can obtain a signal vector of the form $s_{i,t} = \{s_{i,q,t}, s_{i,a,t}\}$, households only attend to signal about supply shocks*

$$s_{i,a,t} = a_t + e_{i,a,t}$$

2. *Household's consumption evolves according to*

$$c_{i,t} = \lambda_{h,a}\,\mathbb{E}[a_t|s_{i,a,t}] = \lambda_{h,a}\xi_{h,a}\left(c^*_{i,t} + e_{i,t}\right)$$

where the attention weight on supply shocks (the Kalman-gain) is

$$\xi_{h,a} = \max\left(0, 1 - \frac{\mu^h}{(\gamma+\eta)\lambda^2_{h,a}\sigma^2_a}\right)$$

[15] In the households' attention problem, both the constrained and flexible information structures yield the same signal form since optimal consumption depends solely on productivity shocks.

and the attention weight on demand shock is $\xi_{h,q} = 0$.

Proof. See Appendix A.3.

The first part of Proposition 1 shows that households never pay attention to demand shocks, as such information has no value for them. As long as firms are fully attentive and set prices to offset changes in q_t, optimal consumption is unaffected by the demand shock – the classical dichotomy holds. Therefore, when information is costly, households would not choose to acquire such information. The second part shows that households pay more attention to supply shocks if (i) the information generates a higher payoff (reflected as higher $\lambda_{h,a}$, γ, or η), and (ii) households are sufficiently uncertain about it (i.e., higher prior uncertainty σ_a^2), and (iii) attention costs are relatively low (i.e., low μ^h).

The Proposition 1 shows information on demand shocks q_t has no value for households when firms have full information. However, if firms are also inattentive, they under-react due to incomplete information, and prices adjust only gradually to demand shocks, no longer fully offsetting changes in q_t. As a result, demand shocks have a real impact. Then, information about demand shocks becomes valuable for households – but only secondarily. The finding is summarized in Corollary 1, while detailed derivation can be found in Section 3.7.

Corollary 1. *When firms are inattentive and price adjustments are sub-optimal, households have an incentive to pay attention to demand shocks.*

3.5 Firms' Attention Choices

I analyze the case where firms are subject to rational inattention while households have full information.[16] When households have full information, all households equate the marginal rate of substitution between consumption and labor to real wage, i.e., $\gamma c_{i,t} + \eta l_{i,t} = w_t - p_t$ and the budget constraint holds as $p_t + c_{i,t} = w_t + l_{i,t}$, $\forall i$. The nominal marginal cost in this case can be expressed as a function of exogenous shocks only

$$w_t - a_t = q_t - \frac{1+\eta}{\gamma+\eta} a_t \tag{3.15}$$

To develop an intuition about firms' attention choices, imagine that a measure of zero of firms have no information while all other firms have full information. Since all other firms have full information, the profit-maximizing price remains $p_{j,t}^* = q_t - \frac{1+\eta}{\gamma+\eta} a_t$. However, firms without information fail to adjust their prices (i.e., $p_{j,t} = 0$), resulting in expected profit losses proportional to

$$\mathbb{E}_{j,t}\left[-\left(p_{j,t}^* - p_{j,t}\right)^2\right] = \mathbb{E}_{j,t}\left[-\left(q_t - \frac{1+\eta}{\gamma+\eta} a_t - 0\right)^2\right] = \sigma_q^2 + \left(\frac{1+\eta}{\gamma+\eta}\right)^2 \sigma_a^2 \tag{3.16}$$

[16] For tractability, I further assume that no general equilibrium feedback through strategic complementarity in price setting. However, this feedback effect is included in the full model.

As shown in Equation (3.16), misinformation about both shocks results in profit losses. This is illustrated in Figure 3b, where losses arise from misinformation about a_t and q_t. The magnitude of profit losses due to misinformation about a particular shock depends on i) the volatility of each shock (i.e., σ_a^2 versus σ_q^2), with more volatile shocks causing greater losses from misinformation; ii) the responsiveness of optimal price to each shock. In some cases, misinformation about demand shocks can result in greater profit losses than misinformation about supply shocks, especially when agents have a higher γ. Under standard parameter values, misinformation about demand shocks would incur larger profit losses for firms (see Section 4 for detailed parameterization).

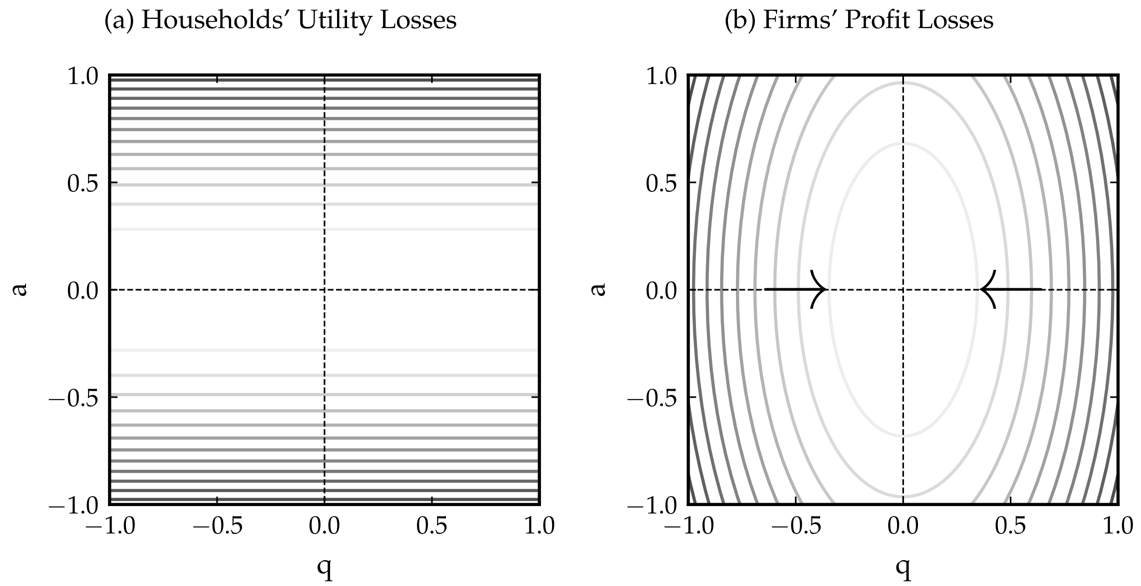


Figure 3: Losses from Misperceptions of $(q\ ,a)$

*Notes:*Figure 3a shows a contour plot of households' utility losses when q and a are misperceived. It shows that the losses occur only along a varying a, which is thus the only component for households to pay attention to. Figure 3b shows a contour plot of firms' profit losses when unit shocks q and a are misperceived. It shows that the descent of losses is steeper in the case of demand shocks q, which is thus the more important component for firms to pay attention to.

Suppose firms can possibly obtain separate, conditionally independent signals about q_t and a_t, as defined in Equation (3.12) and (3.13). For ease of notation, let $\lambda_{f,q} \equiv 1$ and $\lambda_{f,a} \equiv -\frac{1+\eta}{\gamma+\eta}$. Under this notation, the nominal marginal cost is given by $w_t - a_t = \lambda_{f,q} q_t + \lambda_{f,a} a_t$.

Firms' attention choices are characterized by the following Proposition 2.

Proposition 2. *Firms optimally allocate attention towards both shocks*

1. *When households have full information, and firms can obtain a signal vector of the form $s_{j,t} = (s_{j,q,t}, s_{j,a,t})$, firms attend to both signals*

$$s_{j,q,t} = q_t + e_{j,q,t}, \quad \textit{and} \quad s_{j,a,t} = a_t + e_{j,a,t},$$

2. *Firms' prices evolve according to*

$$p_{j,t} = \lambda_{f,q}\xi_{f,q}[q_t + e_{j,q,t}] + \lambda_{f,a}\xi_{f,a}[a_t + e_{j,a,t}] \tag{3.17}$$

where the attention weights (Kalman gain) on each signal are given by

$$\xi_{f,q} = \max\left(0, 1 - \frac{\mu^f}{(\theta - 1)\lambda_{f,q}^2 \sigma_q^2}\right), \tag{3.18a}$$

$$\xi_{f,a} = \max\left(0, 1 - \frac{\mu^f}{(\theta - 1)\lambda_{f,a}^2 \sigma_a^2}\right). \tag{3.18b}$$

Proof. See Appendix A.4.

The first part of the Proposition 2 shows that the allocation of attention to q_t and a_t is independent. The second part implies that firms have incentives to pay attention to both shocks, and they choose to pay more attention to a particular shock, if (i) the shock is particularly volatile (σ_q^2 or σ_a^2 large), (ii) the optimal price is particularly responsive to that shock ($\lambda_{f,q}$ or $\lambda_{f,a}$ large). In particular, for relatively high values of γ, the attention weight may be slightly higher for demand shocks, i.e., $\xi_{f,q} \gtrapprox \xi_{f,a}$, in which cases firms firms find it optimal to pay attention to both shocks with slightly more attention to demand shocks. The intuition is that, following a positive productivity shock, the optimal price should decrease on impact $p_{j,t}^* = w_t - a_t$. This reduction in price leads to a surge in demand c_t. For large values of γ, the income effect dominates, this results in a decrease in labor supply, which in turn causes wages to rise. This offsets the initial downward pressure on prices, so $p_{j,t}^*$ is less affected by supply shocks when γ is large.

Comparison to Optimal Signal Design. Proposition 2 characterizes the solution to the constrained information choice problem. Alternatively, firms can freely design their optimal signal and will thus optimally obtain a single signal of their optimal action, i.e., the nominal marginal costs. Define the prior uncertainty about the optimal price is $\sigma_p^2 \equiv \lambda_{f,q}^2\sigma_q^2 + \lambda_{f,a}^2\sigma_a^2$, the solution to the firms' attention problem is characterized in Proposition 3 below.

Proposition 3. *When households have full information and firms can freely design their optimal signal, firms would pay more attention to demand shocks. Formally,*

1. *The optimal signal for firms skews towards demand shocks as $|\lambda_{f,q}| > |\lambda_{f,a}|$*

$$s_{j,t} = p_{j,t}^* + e_{j,t} = \lambda_{f,q}q_t + \lambda_{f,a}a_t + e_{j,t}$$

where $e_{j,t}$ is the idiosyncratic noise in the signal;

2. *Firm's price evolves according to*

$$p_{j,t} = \xi_f \left(p^*_{j,t} + e_{j.t} \right) = \xi_f \lambda_{f,q} q_t + \xi_f \lambda_{f,a} a_t + \epsilon_{j,t}$$

where the Kalman-gain of the firm's signal under optimal information structure is

$$\xi_f = \max \left(0, 1 - \frac{\mu^f}{(\theta - 1)\left(\lambda^2_{f,q} \sigma^2_q + \lambda^2_{f,a} \sigma^2_a \right)} \right)$$

Proof: See Appendix A.5.

From the first part of Proposition 3, the optimal signal is skewed towards q_t, i.e., $|\lambda_{f,q}| = 1$ is greater than $|\lambda_{f,a}| = \frac{1+\eta}{\gamma+\eta}$ when $\gamma > 1$. As a result, more attention is allocated to demand shocks. The results relate to Kohlhas and Walther (2021), that the asymmetry of attention under optimal signal design depends on the weight $\lambda_{f,q}$ and $\lambda_{f,a}$ in agents' optimal action through their influence on $p^*_{j,t}$. The second part of Proposition 3 shows that firms attention is higher if (i) either shock is more volatile (high σ^2_q or σ^2_a); (ii) the loss from misinformation is high (i.e., high θ or $\lambda_{f,q}$ or $\lambda_{f,a}$); and (iii) the marginal cost of firms μ^f is relatively low.

The key difference between optimal signal design and the constrained information structure is evident from Proposition 2 and Corollary 3. With optimal signal design, the higher attention weight for one shock over the other is driven by the fact that the optimal signal is skewed toward that shock. As a result, the volatility of the shocks does not influence relative attention; instead, relative attention depends solely on the relative responsiveness of price to the shock, i.e., $\lambda_{f,q}/\lambda_{f,a}$. In contrast, under the constrained information structure, higher attention weight is given to the shock either because price is more responsive to it or because the shock is particularly volatile. Therefore, in this case, relative attention depends on both $\lambda_{f,q}/\lambda_{f,a}$ and σ^2_q/σ^2_a.

In summary, the attention choices of households and firms differ significantly. Households tend to allocate substantially more attention to supply shocks than to demand shocks, while firms have an incentive to attend to both shocks, with slightly higher attention toward demand shocks. So far, I have solved the attention problem for households assuming firms are fully informed, and for firms assuming households are fully informed. Before addressing the case where both households and firms are subject to rational inattention, I first demonstrate how attention choices explain the supply-side view by households and demand-side view by firms.

3.6 Implications of Attention Choices on Beliefs

Sections 3.4 and 3.5 show that households optimally allocate most of their attention to supply shocks, whereas firms pay attention to both shocks, with slightly more attention toward demand shocks. This section investigates how attention choices shape beliefs about the covariance between expected inflation and expected growth.

Suppose the actual data generating processes are characterized by

$$y_t = \Psi_{y,q} q_t + \Psi_{y,a} a_t, \tag{3.19}$$

$$p_t = \Psi_{p,q} q_t - \Psi_{p,a} a_t. \tag{3.20}$$

Here, Ψs denotes the responses of aggregate output y_t and aggregate price p_t to demand and supply shocks. Their specific values would depend on the model parameters as well as the equilibrium attention choices and decisions made by firms and households, which are not central to the discussion in this section. Nonetheless, consistent with conventional wisdom in the New Keynesian literature, a positive demand shock is typically expansionary and inflationary (i.e., $\Psi_{y,q} > 0$ and $\Psi_{p,q} > 0$), while a positive supply shock tends to increase output but decrease prices (i.e., $\Psi_{y,a} > 0$ and $\Psi_{p,a} < 0$).

Let's define the expected output growth of agent k as $\mathbb{E}^k(y_{t+1} - y_t)$ and expected inflation as $\mathbb{E}^k(\pi_{t+1}) = \mathbb{E}^k(p_{t+1} - p_t)$, where $k = \{h, f, cb\}$ represents the expectations of households, firms and central bank (the model counterpart of professional forecasters). With these definitions in place, I can derive the unconditional covariance between expected output growth and expected inflation

$$Cov\left(\mathbb{E}^k(y_{t+1} - y_t), \mathbb{E}^k(\pi_{t+1})\right) = \Psi_{y,q}\Psi_{p,q}\xi_{k,q}^2\sigma_q^2 - \Psi_{y,a}\Psi_{p,a}\xi_{k,a}^2\sigma_a^2 \tag{3.21}$$

Here, $\xi_{k,q}$ is the attention weight that agent k assigns to demand shocks, while $\xi_{k,a}$ is the attention weight on supply shocks. Both $\xi_{k,q}$ and $\xi_{k,a}$ range between 0 and 1, where a value of 1 indicates full attention (thus agents have full information) and 0 indicates that agents receive no information. The covariance is the sum of two components: the first component is positive, indicating that conditional on demand shocks, the covariance is positive; the second component is negative, indicating that conditional on supply shocks, the covariance is negative. The unconditional covariance is the sum of these two components.

Full Information Benchmark. If all the agents have full information, then the attention weights for all agents k on both shocks equal 1. The covariance is thus uniform across all agents

$$Cov\left(\mathbb{E}(y_{t+1} - y_t), \mathbb{E}(\pi_{t+1})\right) = \Psi_{y,q}\Psi_{p,q}\sigma_q^2 - \Psi_{y,a}\Psi_{p,a}\sigma_a^2 \tag{3.22}$$

The covariance (3.22) is the same across all agents, and can be either positive or negative depending on the parameterization, which contradicts the survey evidence showing that agents hold different views.

Rational Inattention Framework. In the current model, rationally inattentive households have little incentive to pay attention to demand shocks, i.e., $\xi_{h,q} \ll \xi_{h,a}$. As a result, the second component in Equation (3.21) dominates, leading to a negative covariance between expected output growth and expected inflation, i.e., a supply-side view. Firms allocate attention to both shocks, with slightly more attention toward demand shocks $\xi_{f,q} \gtrapprox \xi_{f,a}$, resulting in a weak positive covariance. The central bank is assumed to have full information (i.e., fully attentive $\xi_{cb,q} = \xi_{cb,a} = 1$), so their view is determined by the actual output and price responses in Equation (3.22). Formally, the findings are summarized in Proposition 4.

Proposition 4. *The asymmetric attention choice is sufficient on their own to explain the contrasting views by different agents. In particular*

1. *Households optimally pay more attention to supply shocks, and thereby form a negative correlation between output growth and inflation in their expectations;*

2. *Firms find it optimal to pay attention to both shocks, with slightly more attention toward demand shocks, and thus form a weak-positive correlation between output growth and inflation in their expectations;*

3. *Central Bank has full information and its view reflects the correlation between output and inflation in equilibrium (Equation 3.22).*

Using the simple model, I analytically show that the proposed mechanism can potentially match survey expectations. To quantitatively evaluate the model and determine the numerical values of the covariance, I extend the simple model into a more plausible setting and solve it numerically in Section 4.

3.7 Strategic Interactions in Attention Allocation

This section solves for the equilibrium where both households and firms are subject to rational inattention, and discusses the strategic interactions in attention allocation between households and firms. As described in Section 3.3, when both agents are subject to rational inattention, their optimal actions depend on the exogenous shocks, endogenous variables as well as each other's attention choices. And the equilibrium is characterized by a fixed-point problem (see Definition 1).

For illustrative purpose, I solve the model separately for demand shock and supply shock, and discusses the strategic interactions in attention allocation between households and firms in each case.

Substitutability in Attention Allocation in Demand Shocks. I begin by guessing that in equilibrium, the nominal wage is a linear function of the demand shock, i.e., $w_t =$

$H_{w,q}q_t$ (this guess will be verified). Given this, the rational inattention problem of firm j (3.10) becomes[17]

$$\max_{\{s_{j,t}\in\mathcal{S}_f^t\}} \mathbb{E}_t^f\left[-\frac{\theta-1}{2}\left(p_{j,t}-(w_t-a_t)\right)^2-\mu^f\mathcal{I}\left(q_t,a_t;s_{j,t}\right)\right]$$
$$=-\frac{1}{2}\max_{\sigma^2_{f,q|s}\geq\sigma^2_q}\left[(\theta-1)\,H^2_{w,q}\sigma^2_{f,q|s}+\mu^f\ln\frac{\sigma^2_q}{\sigma^2_{f,q|s}}\right]$$

where $\sigma^2_{f,q|s}$ denotes the posterior uncertainty about q_t by firms. Solve the first order condition gives

$$p_{j,t}=\xi_{f,q}\left(w_t+e_{j,t}\right),\qquad \xi_{f,q}\equiv\max\left(0,1-\frac{\mu^f}{(\theta-1)\,H^2_{w,q}\sigma^2_q}\right)$$

where $e_{j,t}$ is firm j's rational inattention error, assumed to be mean-zero and independently distributed across firms. Note that firms' attention $\xi_{f,q}$ increases if the equilibrium nominal wage is very responsive to demand shocks q_t, as indicated by a higher value of $H_{w,q}$.

As firms have the same prior and attention choices, and their rational inattention errors are independently distributed, I can aggregate over firms. Aggregating over j gives the price level

$$p_t\equiv\int_0^1 p_{j,t}dj=\xi_{f,q}w_t=\xi_{f,q}H_{w,q}q_t \tag{3.23}$$

The attention weight $\xi_{f,q}$ governs how responsive the aggregate price level is to changes in the nominal wage. In particular, if $\xi_{f,q}=1$, all firms are fully attentive, and the price will move one-to-one with equilibrium nominal wage $p_t=w_t$, in which case the real wage is unaffected; if $\xi_{f,q}=0$, firms pay no attention and do not respond to q_t. When $\xi_{f,q}\in(0,1)$, the price level rises less than optimal, that is, firms make pricing mistakes due to inattention and set the price too low, i.e., $p_t<w_t$.

Substituting the aggregate price level (3.23) and the guess $w_t=H_{w,q}q_t$ into households' objective (3.7) yields the following

$$\max_{\{s_{i,t}\in\mathcal{S}_h^t\}_{t\geq0}} \mathbb{E}\left[-\frac{(\gamma+\eta)}{2}\left(c_{i,t}-\frac{1+\eta}{\gamma+\eta}(w_t-p_t)\right)^2-\mu^h\mathcal{I}\left(q_t;s_{i,t}\right)\right]$$
$$=-\frac{1}{2}\max_{\sigma^2_{h,q|s}\geq\sigma^2_q}\left[(\gamma+\eta)\left[\frac{1+\eta}{\gamma+\eta}(1-\xi_{f,q})H_{w,q}\right]^2\sigma^2_{h,q|s}+\mu^h\ln\frac{\sigma^2_q}{\sigma^2_{h,q|s}}\right]$$

The first term in the equation represents the benefit of paying attention, and it decreases with firms' attention $\xi_{f,q}$, or, in other words, increases as firms make pricing mistakes due to inattention. When firms pay full attention, i.e., $\xi_{f,q}=1$ and $p_t=w_t$, households

[17] The derivation follows the same steps as in Section 3.5.

receive no benefit from paying attention (this is discussed extensively in Section 3.4). This is because any fluctuation in the nominal wage is exactly offset by an equivalent change in the price level, leaving the real wage and optimal consumption level unchanged ($w_t - p_t = 0, c^*_{i,t} = 0$). In this case, when attention is costly, households do not pay attention. However, when firms pay less attention and set the price below the optimal level, i.e., $p_t = \xi_{f,q} w_t$ with $\xi_{f,q} < 1$, it becomes beneficial for households to pay attention. The benefit increases as firms make larger pricing mistakes. Therefore, in the case of demand shocks, the attention choices by households and firms are substitutional – if firms pay less attention to demand shocks, households will more attention.

Solving the problem in steady state, the consumption choice by household i is given by

$$c_{i,t} = \xi_{h,q} \left[\frac{1+\eta}{\gamma+\eta} \left(1 - \xi_{f,q}\right) w_t + e_{i,t} \right]$$

with

$$\xi_{h,q} \equiv \max \left(0, 1 - \frac{\mu^h}{(\gamma+\eta) \left[\frac{1+\eta}{\gamma+\eta} (1 - \xi_{f,q}) H_{w,q} \right]^2 \sigma_q^2} \right)$$

where $e_{i,t}$ is the idiosyncratic noise in the signal, which is assumed to be mean-zero and independently distributed across households. Note that households have an incentive to pay attention to demand shocks only when firms are sufficiently inattentive, indicated by sufficiently low $\xi_{f,q}$. Formally, the attention allocated by households is inversely related to the attention allocated by firms, i.e., $\partial \xi_{h,q} / \partial \xi_{f,q} < 0$. In this sense, the attention choices made by households and firms are substitutable, as illustrated in the Figure 4a.

When firms are sufficiently inattentive to demand shocks, demand shocks have real impact, aggregate consumption can respond to demand shocks

$$c_t \equiv \int_0^1 c_{i,t} di = \xi_{h,q} \left[\frac{1+\eta}{\gamma+\eta} \left(1 - \xi_{f,q}\right) H_{w,q} \right] q_t$$

The equilibrium nominal wage $w_t = H_{w,q} q_t$ is determined in equilibrium and must be consistent with the attention choices and actions of firms and households.

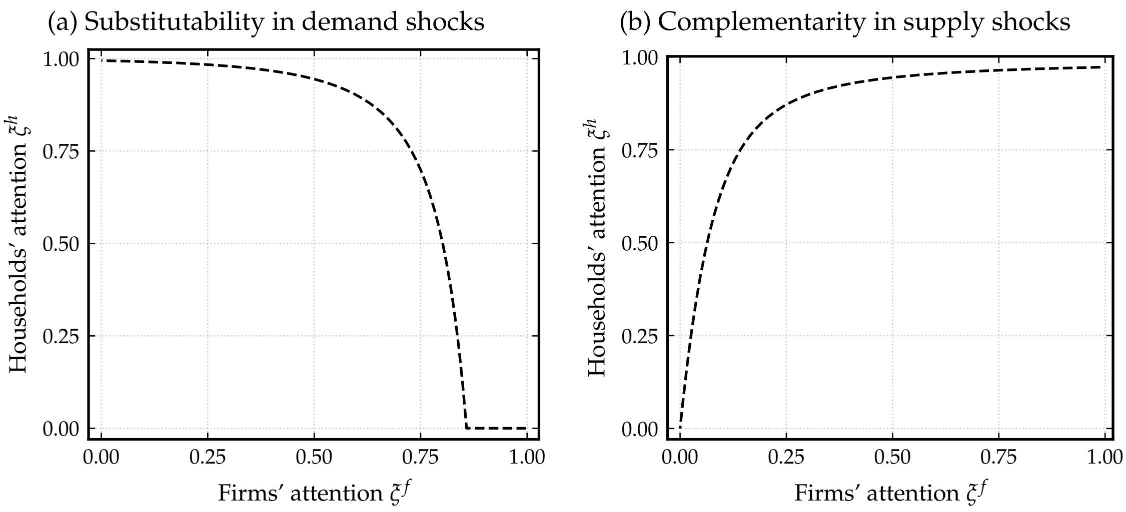


Figure 4: Strategic Interactions in Attention Allocation

Notes: The figure plots the attention (Kalman gain) for households and firms for different costs of firms' information. As the cost of firms' information decreases (μ_f declines), the firms pay more attention. In the limit, firms learn the exact optimal price. The households' attention varies with firm's attention.

Complementarity in attention allocation in productivity shocks. In the case of productivity shocks, the optimal price $p^*_{j,t} = w_t - a_t$ is a function of both endogenous and exogenous variables.[18] While solving for the equilibrium follows the same guess-and-verify method as before, the intuition in the case of productivity shocks is less straightforward. To gain insight into the strategic interactions involved in attention allocation, imagine for a moment that the labor supply is *perfectly* elastic ($\eta \to \infty$). Since labor is perfectly elastic, the wage does not change following a productivity shock ($w_t = 0$), and the optimal price decision simplifies to $p_{j,t} = -a_t$. Intuitively, when firms pay full attention, the price drop is the most significant. This, in turn, suggests that optimal consumption will experience the most substantial increase, incentivising households to pay more attention. Thus, in the case of a productivity shock, attention choices made by households and firms exhibit strategic complementarity.

Generalizing to the case where labor supply is not perfectly elastic, I first guess that in equilibrium nominal wage is a linear function of the productivity shock, i.e., $w_t = H_{w,a} a_t$. Given this guess, the rational inattention problem of firm j (3.10) becomes

$$\max_{\sigma^2_{a|f,a|s} \geq \sigma^2_a} -\frac{1}{2} \left[(\theta - 1) \left(H_{w,a} - 1 \right)^2 \sigma^2_{f,a|s} + \mu^f \ln \frac{\sigma^2_a}{\sigma^2_{f,a|s}} \right]$$

where $\sigma^2_{f,a|s}$ denotes the posterior uncertainty about a_t by firms. Solve the attention prob-

[18] This contrasts with the case of demand shocks, where the optimal price is solely a function of endogenous variables, i.e., $p^*_{j,t} = w_t$.

lem gives

$$p_{j,t} = \xi_{f,a}\left(w_t - a_t + e_{j,t}\right), \qquad \xi_{f,a} \equiv \max\left(0, 1 - \frac{\mu^f}{(\theta-1)\left(H_{w,a}-1\right)^2\sigma_a^2}\right)$$

where $e_{j,t}$ is the firm j's idiosyncratic noise, with zero mean and independently distributed across firms. Aggregating over j gives

$$p_t \equiv \int_0^1 p_{j,t}dj = \xi_{f,a}\left(w_t - a_t\right) = \xi_{f,a}\left(H_{w,a}-1\right)a_t \tag{3.24}$$

The aggregate price depends on the equilibrium wage, productivity shock, and firms' attention choice. Substituting the aggregate price (3.23) and the guess $w_t = H_{w,a}a_t$ into household i's rational inattention problem yields

$$\max_{\sigma^2_{h,a|s}\geq\sigma^2_a} -\frac{1}{2}\left[(\gamma+\eta)\left[\frac{1+\eta}{\gamma+\eta}\left(H_{w,a}-\xi_{f,a}\left(H_{w,a}-1\right)\right)\right]^2\sigma^2_{h,a|s} + \mu^h \ln\frac{\sigma^2_a}{\sigma^2_{h,a|s}}\right]$$

and the solution is characterized by

$$c_{i,t} = \xi_{h,a}\left[\frac{1+\eta}{\gamma+\eta}\left(w_t - \xi_{f,a}\left(w_t-a_t\right)\right) + e_{i,t}\right],$$

$$\text{with} \quad \xi_{h,a} \equiv \max\left(0, 1 - \frac{\mu^h}{(\gamma+\eta)\left[\frac{1+\eta}{\gamma+\eta}\left(H_{w,a}-\xi_{f,a}\left(H_{w,a}-1\right)\right)\right]^2\sigma_a^2}\right)$$

The solution implies that $\partial\xi_{h,a}/\partial\xi_{f,a} > 0$, meaning that as firms allocate more attention to supply shocks (high $\xi_{f,a}$), households tend to allocate more attention as well (high $\xi_{h,a}$), and vice versa. Consequently, in the case of a productivity shock, attention choices made by households and firms exhibit strategic complementarity, as illustrated in the right panel of Figure 4b.

4 Quantitative Model

In this section, I extend the simple model in Section 3 to a dynamic general equilibrium model and incorporate the role for monetary policy. The objective is to (i) assess whether the proposed mechanism can generate quantitative plausible results in a dynamic framework; (ii) quantifying the consequences of asymmetric attention by households and firms on business cycles.

4.1 Extended model

I extend the simple model in three dimensions. First, I relax the assumption of hand-to-mouth behavior and allow households to engage in intertemporal substitution through trading nominal bonds. Second, I allow for strategic complementarities in pricing by assuming a segmented labor market, which matters quantitatively for the price dynamics;[19] Third, I assume the central bank sets the interest rate following a standard Taylor rule, which reflects a more plausible monetary policy framework. As before, the central bank has full information and is the model counterpart of professional forecasters in the survey.

Households. There is a continuum of households, indexed by $i \in [0,1]$. Each period, household i chooses the consumption level $C_{i,t}$ and bond holdings $B_{i,t}$ based on their information set $s_i^t = \{s_{i,\tau}\}_{\tau=0}^{\tau=t}$. After deciding on consumption and bond holdings, household i supplies labor $L_{i,t}$ at given wage W_t such that the budget constraint holds. Formally, the household i's expected present value of utility is given by

$$\mathbb{E}^i \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{L_{i,t}^{1+\eta}}{1+\eta} \right) \right] \tag{4.1}$$

$$s.t.\ P_t C_{i,t} + B_{i,t} = W_t L_{i,t} + R_{t-1} B_{i,t-1} + D_t + T_t, \quad C_{i,t} = \left[\int_0^1 C_{i,j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{4.2}$$

less the cost of attention. Here B_t is the demand for nominal bonds at t that yield a nominal return of R_t at $t+1$, D_t is the aggregated profits of firms, and T_t is the net lump-sum transfers (or taxes if negative). Household i takes $\{W_t, D_t, T_t\}$ as given.

Firms. There is a continuum of firms producing differentiated goods, indexed by $j \in [0,1]$. Firm j faces a demand curve given by $Y_{j,t} = (P_{j,t}/P_t)^{-\theta} Y_t$. Firm j take the wage $W_{j,t}$ and demand for its good as given. In each period, firm j sets price for its own variety $P_{j,t}$ based on its information, and then hires sufficient labor $L_{j,t}$ to produce to meet its demand according to production function $Y_{j,t} = A_t L_{j,t}$. Formally, firm j's expected present value of profit discounted by households' marginal utility of consumption is given by

$$\mathbb{E}^j \left[\sum_{t=0}^{\infty} C_t^{-\gamma} \left[P_{j,t} Y_{j,t} - (1-\theta^{-1}) \frac{W_{j,t}}{A_t} \left(\frac{P_{j,t}}{P_t} \right)^{-\theta} Y_t \right] \right] \tag{4.3}$$

less the cost of attention. Here A_t is an aggregate productivity shock, with $a_t \equiv \log(A_t)$ follows a AR(1) process: $a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$. Other variables are defined similarly as in Section 3.

[19] This is a popular approach in this literature to generate pricing complementarity.

Central Bank. I assume the central bank has full information – it knows the shocks, households' and firms' actions, and the equilibrium outcomes. Monetary policy is specified as the following standard Taylor rule with interest smoothing

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho} \left[\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_y}\right]^{1-\rho} e^{-\sigma_u u_t} \tag{4.4}$$

where R_t is the nominal interest rate, $\bar{R}$ is the steady state nominal rate, $Y_t \equiv C_t$ is aggregate output, Y_t^n is natural level of output in the economy with no frictions, and $u_t \sim N(0,1)$ is a monetary policy shock. I specify the rule such that a positive u_t shock corresponds to an expansionary monetary policy shock. Denote $i_t \equiv \log(R_t)$, the log-linearized Taylor rule is

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_x x_t + \phi_{dy} \Delta y_t\right) - u_t \tag{4.5}$$

I interpret the central bank in the model as the counterpart of professional forecasters in the survey.

Fiscal authority. The government has to finance maturing nominal government bonds and the wage subsidy. The government can collect lump-sum taxes or issue new bonds. The government's budget constraint is

$$\frac{B_t}{P_t} = \frac{R_{t-1}}{\Pi_t}\frac{B_{t-1}}{P_{t-1}} + \theta^{-1}\frac{W_t L_t}{P_t} + \frac{T_t}{P_t}$$

The fiscal responses matter a great deal for the overall macroeconomic impact of economic shocks in this model. Here I consider two assumptions about how the government satisfies its intertemporal budget constraint: (i) government debt is held constant, and transfers adjust in every instant; (ii) let government debt absorb the majority of the fiscal imbalance in the short run, and adjust the path of lump-sum tax to satisfy long-run solvency. In particular, raises taxes to repay all the interest payments and repay a portion $\bar{\tau}$ of existing debts.

$$-\frac{T_t}{P_t} = \frac{R_{t-1}}{\Pi_t}\frac{B_{t-1}}{P_{t-1}} + \bar{\tau}\left(\frac{B_{t-1}}{P_{t-1}} - \frac{\bar{B}}{\bar{P}}\right)$$

Following common practice in the New Keynesian literature, I restrict the value for τ such that monetary policy is active and fiscal policy is passive in the sense of Leeper (1991).

Timing. The timing is specified similarly to Section 3. In the initial period, each household and firm first chooses their attention allocation (what information to pay attention to and how much attention to pay); In each subsequent period, shocks are realized. The economy proceeds as: (i) based on household i's attention choice, they receive a vector of signal $s_{i,t} \in \mathcal{S}_t^h$ at time t, and their information set is then the current signal and the history of the past signals up to $t-1$, i.e., $s_i^t \equiv \{s_{i,t} \cup s_i^{t-1}\}$; firm j receives a signal based on its attention choices $s_{j,t} \in \mathcal{S}_t^j$, and firm j's information set is the current signal plus the history of the past signals, i.e., $s_j^t \equiv \{s_{j,t} \cup s_j^{t-1}\}$ (ii) based on the information set s_i^t, household i chooses consumption and bond holdings; based on firm j's information set s_j^t, firm j sets it price. (iii) once those decisions are sunk, in the final period, household i supplies sufficient labor such that budget constraint binds; firm j hires labor and produces sufficient goods to meet its demand. And markets clear.

4.2 Households' Attention Problem

Analogous to Section **??**, I derive an expression for the expected discounted sum of utility losses when actions of household i deviate from the utility-maximizing actions. Household chooses real bond holdings, $\tilde{b}_{i,t}$, and consumption level, $c_{i,t}$, in each period t. This is equivalent to directly choosing the vector x_t in Equation (4.7) if the household knows their own past actions. Formally, household i's rational inattention problem is (for detailed derivation see Appendix B.1)

$$\max_{s_{i,t}\in\mathcal{S}_h^t} \sum_{t=0}^{\infty} \beta^t \,\mathbb{E}\left[\frac{1}{2}\left(x_{i,t}-x_{i,t}^*\right)' \Theta \left(x_{i,t}-x_{i,t}^*\right) - \mu^h \mathcal{I}\left(\left\{x_{i,t-j}^*\right\}_{j=0}^{\infty}; s_{i,t}|s_i^{t-1}\right)|s_i^{-1}\right] \tag{4.6}$$

Here s_i^{t-1} denotes the history of signals up to time $t-1$. And the choice vector is

$$x_{i,t} = \begin{pmatrix} \omega_B\left(\tilde{b}_{i,t}-\tilde{b}_{i,t-1}\right) \\ -\omega_B\left(\frac{1}{\beta}\tilde{b}_{i,t-1}-\tilde{b}_{i,t}\right)+\left(\gamma\frac{\omega_W}{\eta}+1\right)c_{i,t} \end{pmatrix} \tag{4.7}$$

and

$$\Theta = -\bar{C}^{1-\gamma}\begin{bmatrix} \left(\gamma-\frac{\gamma^2\omega_W}{\gamma\omega_W+\eta}\right)\frac{1}{\beta} & 0 \\ 0 & \frac{\omega_W}{\gamma\omega_W+\eta} \end{bmatrix} \tag{4.8}$$

Moreover, $x_{i,t}^*$ is the optimal choice vector for household i, which is given by

$$x_{i,t}^* = \begin{pmatrix} z_t-(1-\beta)\sum_{s=t}^{\infty}\beta^{s-t}\mathbb{E}_t\left[z_s\right]+\frac{\beta}{\gamma}\left(1+\omega_W\frac{\gamma}{\eta}\right)\sum_{s=t}^{\infty}\beta^{s-t}\mathbb{E}_t\left(i_s-\pi_{s+1}\right) \\ \omega_W\left(\frac{1}{\eta}+1\right)\tilde{w}_t+\left[\frac{1}{\beta}\omega_B\left(i_{t-1}-\pi_t\right)+\omega_D\tilde{d}_t+\omega_T\tilde{\tau}_t\right] \end{pmatrix} \tag{4.9}$$

The lowercase variables denote the log deviations of the corresponding variables. And variable with a tilde implies it is a real variable. Moreover, $z_t \equiv \omega_W\left(1+1/\eta\right)\tilde{w}_t+\frac{1}{\beta}\omega_B\left(i_{t-1}-\pi_t\right)+\omega_D\tilde{d}_t+\omega_T\tilde{\tau}_t$. And the $(\omega_B,\omega_W,\omega_D,\omega_T)$ denote the steady-state ratios of $\left(\frac{\bar{B}}{\bar{C}\bar{P}},\frac{\bar{W}\bar{L}}{\bar{C}\bar{P}},\frac{\bar{D}}{\bar{C}\bar{P}},\frac{\bar{T}}{\bar{C}\bar{P}}\right)$.

The first element in the choice vector $x_{i,t}$ is change in bond holdings, and the second element of $x_{i,t}$ is the component of the marginal rate of substitution between consumption and leisure. These two element are directly chosen by household through the choice of real bond holdings $\tilde{b}_{i,t}$ and $c_{i,t}$. The formulation of the optimal choice vector (4.9) implies that: (i) it is optimal to increase bond holdings when income is high relative to permanent income or when the return on bond is high; (ii) it is optimal to equate the marginal rate of substitution between consumption and leisure to the real wage. When the household deviates from these optimal choices, the household loses an amount of utility determined by the matrix Θ. This matrix is diagonal, because a suboptimal marginal rate of substitution between consumption and leisure does not affect the optimal change in bond holdings, and a suboptimal change in bond holdings does not affect the optimal marginal rate of substitution between consumption and leisure.

4.3 Firms' Attention Problem

After a log-quadratic approximation, I derive the firm j's expected profit loss

$$\max_{s_{j,t}\in\mathcal{S}_f^t} \sum_{t=0}^{\infty} \beta^t \, \mathbb{E}\left[-\frac{\theta-1}{2}\left(p_{j,t}-p_{j,t}^*\right)^2 - \mu^f \mathcal{I}\left(p_{j,t}^*; s_{j,t}|s_j^{t-1}\right)|s_j^{-1}\right] \tag{4.10}$$

where

$$p_{j,t}^* = w_{j,t} - a_t = p_t + \alpha\left[y_t - \frac{1+\eta}{\eta+\gamma}a_t\right] \tag{4.11}$$

where $\alpha = \frac{(\eta+\gamma)}{(1+\theta\eta)}$ is the pricing complementarity. Equation (4.11) implies it is optimal for firm j to increase its price if its nominal marginal costs increases, and vice versa.

4.4 Definition of Equilibrium

Given exogenous processes for productivity and monetary policy shocks $\{a_t, u_t\}$ and initial sets of signals for households and firms, a general equilibrium for this economy is an allocation for every household $i \in [0,1]$, $\Omega_i^h \equiv \{s_{i,t} \in \mathcal{S}_{i,t}^h, C_{i,t}, B_{i,t}, L_{i,t}\}_{t=0}^{\infty}$, an allocation for every firm $j \in [0,1]$, $\Omega_j^f \equiv \{s_{j,t} \in \mathcal{S}_{j,t}^f, P_{j,t}, L_{j,t}, Y_{j,t}\}_{t=0}^{\infty}$, a set of prices $\{P_t, R_t, W_t\}$. Aggregate variables are obtained by aggregating the individual actions, such that

1. Given the set of prices and $\{\Omega_j^f\}_{j\in[0,1]}$, the households' allocation solves the problem in Equation (4.6)
2. Given the set of prices and $\{\Omega_i^h\}_{i\in[0,1]}$, the firms' allocation solves the problem in Equation (4.10)
3. Central bank sets the nominal interest rate according to the rule in Equation (4.5)
4. Good market clears, labor market clears, and bond market clears

4.5 Computing the Equilibrium

I solve a dynamic stochastic general equilibrium model in which both agents are rationally inattentive, a non-trivial task. As defined in Section 4.4, the equilibrium is characterized by a fixed-point problem. Specifically, given the processes for the optimal actions of households and firms, $(x^*_{i,t}, p^*_{j,t})$, I can solve their respective attention problems. However, the processes $(x^*_{i,t}, p^*_{j,t})$ are endogenous to the equilibrium decisions of households and firms. In equilibrium, these two processes must be consistent with each other.

I start by guessing the MA representation of the optimal actions $(x^*_{i,t}, p^*_{j,t})$ as functions of the productivity (ε_t) and monetary policy (u_t) shocks. I then approximate the processes with truncated $MA(200)$ processes.[20] Using the truncated MA processes, I solve the problem numerically based on the algorithm for dynamic rational inattention problems (DRIPs) developed in Afrouzi and Yang (2021). I then solve the implied state-space representations of other variables in the model, based on which I update the guess for the MA representation of the optimal actions $(x^*_{i,t}, p^*_{j,t})$, until the model converges. Appendix B.2 provides a detailed description of the implementation.

4.6 Calibration

Non-Rational Inattention Parameters. The model is calibrated at a quarterly frequency. Table 2 summarizes the assigned values for the non-rational-inattention parameters, which are estimated outside the model, as well as the calibrated values for the marginal attention costs of households and firms.

Table 2: Parameters Values

Parameter	Value	Source / Moment Matched
Panel A. Assigned parameters		
Time discount factor (β)	0.99	Quarterly frequency
Elasticity of substitution across firms (θ)	10	Firms' average markup
Risk aversion coefficient (γ)	3.5	Households' risk aversion level
Inverse of Frisch elasticity (η)	2.5	Aruoba et al. (2017)
Taylor rule: smoothing (ρ)	0.936	Estimates 1985-2017
Taylor rule: response to inflation (ϕ_π)	1.62	Estimates 1985-2017
Taylor rule: response to output gap (ϕ_x)	0.225	Estimates 1985-2017
Persistence of productivity shocks (ρ_a)	0.93	Estimates 1981-2022 based on Fernald (2014)
S.D of productivity shocks (σ_a)	0.86×10^{-2}	Estimates 1981-2022 based on Fernald (2014)
S.D of monetary shocks (σ_u)	0.41×10^{-2}	Estimates 1985-2017
Panel B. Calibrated parameters		
Attention cost of households (μ^h)	0.0106	Professional forecasters slope
Attention cost of firms (μ^f)	0.0095	Professional forecasters slope

I assign values for the non-rational inattention parameters based on the literature. I assume the inverse of the Frisch elasticity (η) to be 2.5 and the risk aversion coefficient to be 3.5, which are standard values in business cycle models. Following Afrouzi and Yang

[20]With a length of 200, I can get arbitrarily close to the true $MA(\infty)$ processes. Increasing the length does not significantly change the results.

(2021), I set the elasticity of substitution across firms (θ) to 10, corresponding to a markup of 11 percent.

I estimate the Taylor rule using real-time U.S. data. Specifically, I use the federal funds rate as a measure of the nominal interest rate, and the Tealbook forecast of inflation and output gap. I employ quarterly data from 1985:1 to 2017:4. The point estimates suggest a smoothing factor of approximately 0.936, with responses to inflation and the output gap of 1.62 and 0.225, respectively.[21] I then compute the model-consistent measure of the monetary policy shock u_t from the data, rewriting the monetary policy rule (4.5) as $u_t = i_t - \rho i_{t-1} - (1-\rho)[\phi_\pi \pi_t + \phi_x(y_t - y_t^n)]$. The standard deviation of u_t is estimated to be 0.41×10^{-2}.

To calibrate the parameters of the stochastic process for aggregate technology I use data on total factor productivity (TFP) reported by Fernald (2014), from 1981:1 to 2022:4. I regress the log of TFP on a constant and a time trend. I then regress the residual on its own lag. Based on the point estimates from this regression, I set the autocorrelation of aggregate technology to 0.93 and the standard deviation of the aggregate technology shock ε_t equal to 0.86×10^{-2}.

Rational Inattention Parameters. Holding the non-rational-inattention parameters constant at the selected values, I solve for the model for a grid of values of the attention parameters μ^h and μ^f. Changes in the attention costs affect the equilibrium output and inflation, and their correlation. In other words, a change in the attention cost will affect the relative importance of supply shocks and demand shocks and thus the slope of the expected output to expected inflation under full information. Thus I calibrate the values for μ^h and μ^f to match the slope coefficient of the professional forecasters in the survey. Solving over a grid of values of the attention costs, I find that $\mu^h = 0.0106$ and $\mu^f = 0.0095$ could generate data-consistent slope coefficient for professional forecasters.

4.7 Results

I simulate the model under the calibrated values for the cost of attention (Table 2). Table 3 reports the moments for expected inflation and output growth regressions – including the slope coefficients, their associated p-values, and the R-squared values across all agents. Column 2 reports the data moments. Note that the magnitude of slope coefficient of households' expectations does not have a meaningful quantitative interpretation; only the sign matters. This is because households do not provide quantitative forecasts for expected growth; I assign values to their growth expectations following Candia et al. (2020), as discussed in Section 2.1. However, the slope coefficients of firms and professional forecasters do have a meaningful quantitative meaning. Column 3 reports the model results.

[21] Because empirical Taylor rules are estimated using annualized rates while the Taylor rule in the model is expressed in quarterly rates, I rescale the coefficient on the output gap in the model, yielding $\phi_x = 0.9/4 = 0.225$.

I simulate the model 200 times, in each simulation, the time horizon is 200 quarters, consistent with the survey data. And the number of households and firms in the simulation are consistent with the survey sample size. I then report the median value of the 100 simulations in Column 3, and the 90 percent confidence interval in Column 4. Moreover, the slope coefficient of the professional forecasters is the targeted moments, and the rests are non-targeted moments.

Table 3: Moments in the Model and the Data

Moment	Data	Model	90% conf. interval
Slope coef. of HHs' expectations	-0.038	-0.047	[-0.056, -0.038]
Slope coef. of Firms' expectations	0.039	0.004	[-0.001,0.012]
Slope coef. of CB's expectations	0.109	0.100	[0.072, 0.137]
R-squared value of HH's expectations	0.022	0.045	[0.028, 0.063]
R-squared value of Firms's expectations	0.002	0.001	[0.000, 0.004]
R-squared value of CB's expectations	0.016	0.181	[0.145, 0.354]
P-value of HH's expectations	0.000***	0.000	[0.000,0.000]
P-value of Firm's expectations	0.428	0.443	[0.164, 0.874]
P-value of CB's expectations	0.000***	0.000	[0.000,0.000]

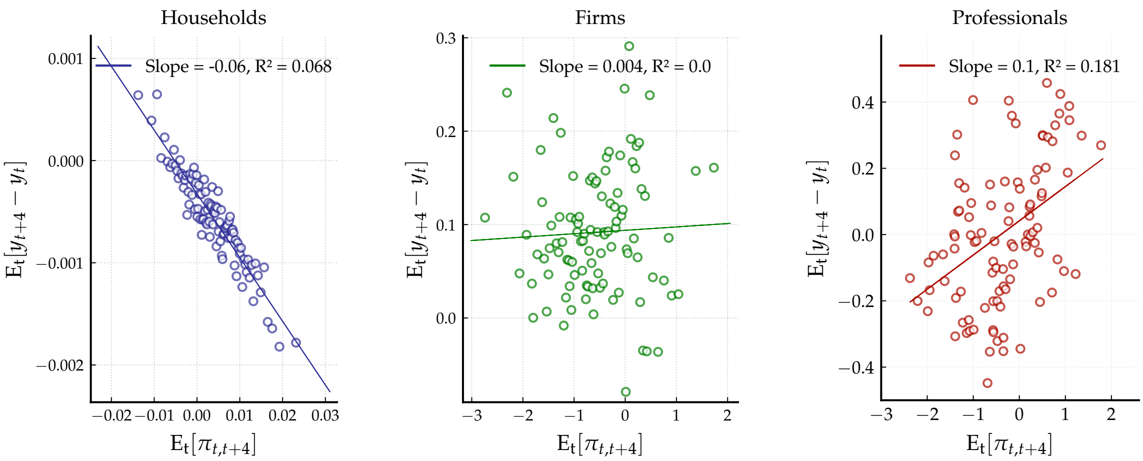
Notes: The table presents the data moments and model moments under calibration Table 2. I simulate the model of fifty years (200 quarters), a time horizon consistent with the survey data. The number of simulated households and the number of simulated firms for each period are consistent with the survey sample size. I simulate 200 times and report the median of the results.

The model matches the targeted moments – slope of the Central Bank's (professional forecasters') expectations. In the model, I assume the central bank has full information regarding fundamental shocks and the reactions of households and firms. Consequently, their beliefs are the correct beliefs about dynamics of future inflation and output growth. And a positive slope implies that demand shocks are the main driver of business cycles in the economy, despite that supply shocks are more volatile (Table 2). More discussion on the relative importance of the shocks and the comparison between rational inattention and full information can be found in Section 4.9.

The model performs reasonably well in matching the moments for households and firms – the non-targeted moments. First, by emphasizing the attention mechanism, the model successfully replicates the negative slope seen in households' expectations as well as the weakly positive slope observed in firms' expectations. This alignment can be attributed to the attention mechanism, wherein households pay more attention to supply shocks, whereas firms take into account both demand and supply shocks, with a slightly greater emphasis on demand shocks. In accordance with this dual attention to both types of shocks by firms, the p-value of the slope coefficient for firms is not statistically significant, in line with the survey evidence.

By simulating the model, I generate a counterpart to Figure 1, as illustrated in Figure 5. These two figures exhibit striking similarities, providing substantial support for the model's validity in explaining the contrasting views held by different economic agents in the survey. It's worth noting that the survey data displays a wider dispersion than the model, potentially stemming from inherent noise in the beliefs held by households and firms. Nevertheless, this specific aspect falls beyond the scope of the current model, which primarily focus on the correlation between expected inflation and expected growth. For comprehensive investigations into belief noises, I recommend referring to the literature on this topic, for example Juodis and Kučinskas (2023).

Figure 5: Simulated expected inflation and expected output

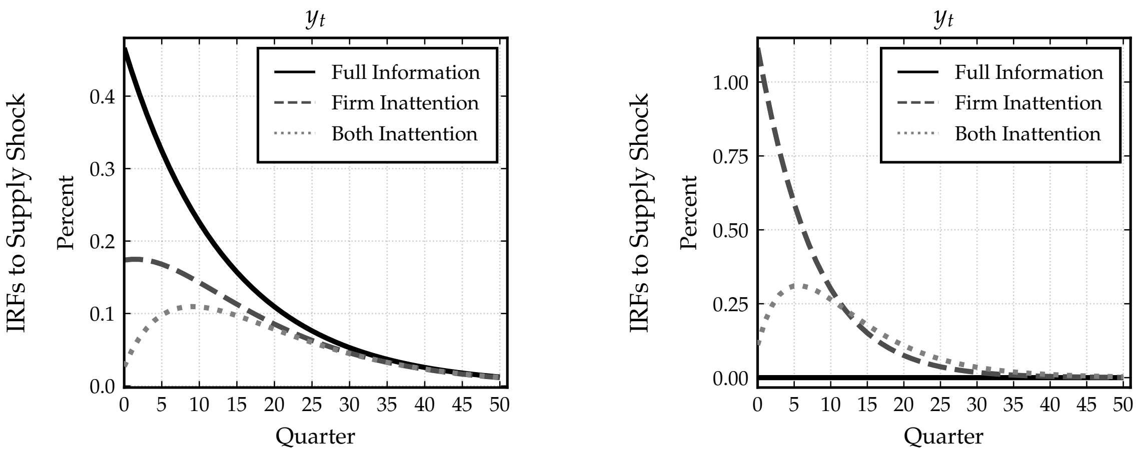


Notes: The figure plots the simulated expected inflation and expected output growth for households, firms, and professional forecasters. The parameterization values are from Table 2.

4.8 Quantifying the Consequences of Inattention

Inattention by households and firms significantly affects the output response to shocks. In particular, households' inattention always dampens the real effects of shocks. In contrast, firms' inattention dampens the real output response to supply shocks but amplifies the response to demand shocks. Demand shocks are amplified by firms inattention, when firms have full information and adjust price optimally, demand shocks have little real impact. However, when firms are inattentive, they do not raise prices as much as they would with full information. This leads to a dampened inflation response, allowing households to enjoy a relatively low price and low interest rate environment and increase spending, therefore, output increases. Moreover, aggregate output response to demand shocks increases with firms' inattention level.

Figure 6a plots the response of aggregate output to supply shocks under (i) both households and firms have full information; (ii) firms are inattentive but households have full information; (iii) both households and firms are inattentive. Compared to the full information results, firms' inattention dampens the output response to the supply shock (dashed versus solid line). The response of aggregate output is even lower when both households and firms are inattentive (dotted line). This is due to two factors: firstly, as households are also inattentive, they have incomplete information about the economics conditions and thus will under-react to the supply shock; secondly, attention levels by households and firms are complementary in the case of supply shocks, which implies that when introducing inattention in households, firms will choose an even lower attention level. As a result, inattention dampens the real impact of supply shocks.



(a) Supply Shocks

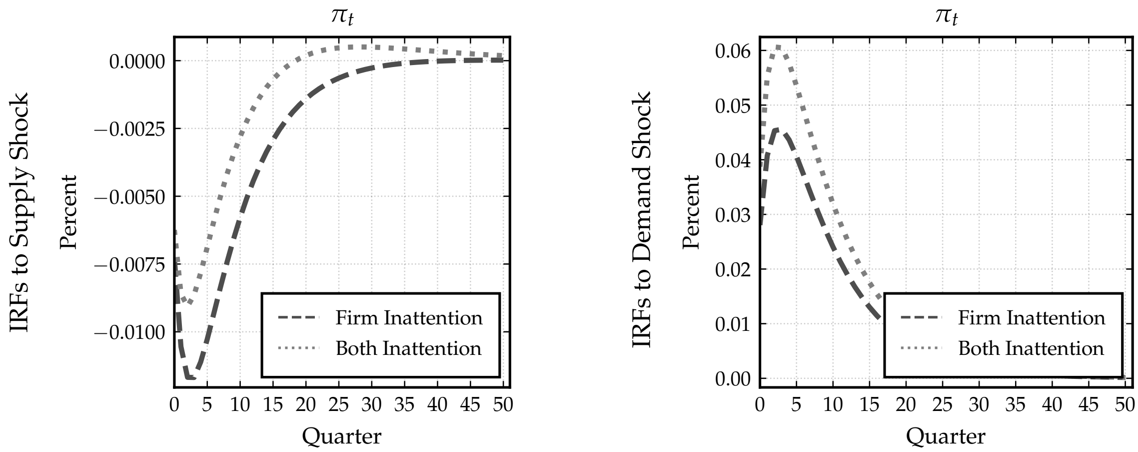
(b) Demand Shocks

Figure 6: Real Consequences of Inattention

Notes: This figure plots impulse responses of aggregate output to a one standard deviation supply shock and to a one standard deviation demand shock. Solid black lines are the responses in the model under perfect information, while dashed grey lines are the responses in the model with rational inattentive firms and fully informed households, dotted grey lines are the responses in the model with both rational inattentive households and rational inattentive firms. Parameterization values see Table 2.

Figure 6b plots the response of aggregate output to demand shocks under the same three scenarios. When firms are full informed, the demand shock does not have a real impact on output, this is driven by the optimality behavior of firms. When firms are inattentive, the demand shocks now have real impacts (dashed versus solid). This implies firms' inattention amplifies the real effects of demand shocks. Introducing inattentive households, the response in output is lower as households will under-react.

The strategic complementarity and substitutability between households' attention level and firms' attention level is more evident in the dynamics of inflation. Figure 7 consider two scenarios: (i) only firms are inattentive and households have full information; (ii) both households and firms are inattentive. From Figure 7a, when adding inattentive households, the response of inflation is smaller. As when households are inattentive, firms have even less incentive to pay attention to supply shocks – reflects the strategic complementarity in their attention allocation. In contrast, in Figure 7b, when households are also inattentive, the inflation response is stronger than the case where households have full information. This reflects the strategic substitutability between households and firms attention allocation – when households pay less attention to demand shocks, firms will pay more attention to demand shock and thus respond more strongly to that shock, therefore the price adjustment is more pronounced.



(a) Supply Shocks

(b) Demand Shocks

Figure 7: Responses of Inflation under Rational Inattention
Notes: This figure plots impulse responses of aggregate output to a one standard deviation supply shock and to a one standard deviation demand shock. Solid black lines are the responses in the model under perfect information, while dashed grey lines are the responses in the model with rational inattentive firms and fully informed households, dotted grey lines are the responses in the model with both rational inattentive households and rational inattentive firms. Parameterization values see Table 2.

4.9 The Relative Importance of Supply and Demand Shocks

Macroeconomists themselves have long argued about the importance of the Phillips curve and whether supply or demand shocks are more important. This section compares the implications under rational inattention and full information case. Formally, I simulate the model under different information assumption for $20,000$ periods, and estimate the following hybrid Phillips curve

$$\pi_t = \alpha + \varphi \mathbb{E}_t[\pi_{t+1}] + (1-\varphi)\pi_{t-1} + \kappa x_t + \varepsilon_t$$

As a benchmark, I simulate the model under full information, and the estimated slope is negative at -0.24, with a degree of "forwardness" of almost 1.[22] The negative slope is driven by the fact that, under full information, supply shocks are the dominant driver of business-cycle fluctuations in output and inflation.[23] Rational inattention significantly changes the relative importance of supply and demand shocks, as the estimated slope is around 0.004.[24] This is in line the literature estimates[25]. The positive slope implies

[22] Under full information, the output gap is always zero, therefore I run the regression using output instead of output gap.

[23] In the full information case, as price is fully flexible, demand shocks do not affect output. If price is assumed to be exogenously sticky (assume prices adjust on average every four quarters), under this specification, the implied slope coefficient is around 0.0093, and the coefficient of the expected inflation is almost 1.

[24] However, it worth noting that the Phillips curve is misspecified from this model's perspective, and provide biased estimates due to an omitted variables bias issue. However, they constitute a fair comparison to the evidence on the Phillips curve slope.

[25] Del Negro et al. (2020) estimates the slope to range between 0 and 0.01.

that, despite that under current calibration that supply shocks are as twice as volatile as demand shocks, rational inattention significantly amplifies the impact of demand shocks on the economy, while dampens the impact of supply shocks. Moreover, the estimated coefficient of the expected inflation is around 0.68. This is because the price only gradually adjust in response to shocks as firms learn about the economic conditions.

Importantly, a key feature of the model is that decision-makers' attention allocation is endogenous to the model parameters. Therefore, changes in parameter values lead to outcomes that differ significantly from those in a New Keynesian model with full or exogenous imperfect information, as decision-makers reallocate their attention. This is particularly relevant in the post-Volcker period, when monetary policy is more aggressive in inflation stabilization. In the next section, I demonstrate how this policy change can explain both the decline in macroeconomic volatility and the flattening of the Phillips curve in the post-Volcker period.[26]

4.9.1 The Flattening of the Phillips Curve

In this model, households and firms' incentives in information acquisition are endogenous to the conduct of monetary policy. Therefore, a change in the Taylor rule parameter will alter households and firms' attention choices and decisions, which in aggregate, will affect the dynamics of output and inflation. In this section, I study the implications of the change in the rule of monetary policy in the post-Volcker period. This exercise is similar in the spirit of Afrouzi and Yang (2021), where the authors develop a model with rational inattentive firms and show that the a more hawkish monetary policy induces firms pay less attention to changes in their input costs, which leads to a flatter Phillips curve in the post-Volcker period. The main difference is that in this model households are also rationally inattentive, this is important in explaining the output dynamics.

I use the calibration for monetary policy rules in Afrouzi and Yang (2021)'s paper, and re-run the model under the Section 4's specification. And then I compute the model implied slope of Phillips curve same as Afrouzi and Yang (2021)'s. The model predicts that the slope of the Phillips curve declined from 0.01 in the pre-Volcker era to 0.0057 in the post-Volcker period – a 44% decline. The volatility of output falls from 0.093 to 0.053, and the volatility of inflation falls from 0.017 to 0.015.

The fall in volatility in prices in the post-Volcker is straightforward. First, when monetary policy is more aggressive in inflation stabilization, price will be more stable. Second, rational inattention also suggests the firms will put less attention, and thus price changes are even less pronounced in response to shocks.

Change in monetary policy rules have mixed effects on output volatility. Fix for a moment the allocation of attention of households and firms. Furthermore, consider the

[26] A growing literature documents that the slope of the Phillips curve has flattened during the last few decades, see e.g. Coibion and Gorodnichenko (2015); Blanchard (2016); Bullard (2018); Hooper et al. (2020).

case of productivity shock. When monetary policy is more aggressive in inflation stabilization, the price becomes more stable. As a result, the nominal interest rate mimics will be closely the efficient level. In the case of productivity shock, this effects decreases deviations of output from efficient output, and thus output gap volatility falls and output volatility rises. However, when monetary policy is more aggressive in inflation stabilization. The firms will put less attention to variation in their input costs, and thus price changes are less pronounced in response to productivity shocks. Meanwhile, as households' and firms' attention allocation is complementary in the case of productivity shock, households pay less attention and under-react to the shock, this increases the deviations of output from efficient output, and thus the volatility of the output gap rises and the volatility of output falls. This second attention channel is absent from a New Keynesian model or models with exogenous imperfect information, and that's why those models predict higher output volatility. Under rational inattention, the second effect dominates and thus the volatility of output falls and the volatility of output gap rises.

Consider the case of monetary policy shock. When monetary policy is more aggressive in inflation stabilization, firms become more inattentive toward those shocks, and thus monetary policy shocks have larger real impacts on the economy, which incentives households to shift some attention towards demand shocks, creating a larger output gap.

Both the increase in output gap volatility and the less sensitivity of price to output changes discussed contribute to a flatter Phillips curve.

4.10 Additional Results

Information Frictions of Different Agents. With estimates of information frictions, a natural question is who has the most accurate inflation expectations. This section compares both the unconditional and conditional accuracy of inflation nowcast across households and firms. I choose the root mean squared error (RMSE) as the unconditional measure of nowcast accuracy. The RMSE measures the square root of the average of the squared errors. I define the RMSE as

$$RMSE(a) = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\pi_t - \bar{\mathbb{E}}_t^a \pi_t\right)^2} \tag{4.12}$$

where π_t is ex post realized inflation and $\bar{E}_t^a$ is the average expectation of agent type $a \in$ {Households, Firms}. As I assume central bank has full information, the RMSE of central bank is zero ($\pi_t = \mathbb{E}_t^{FI}\pi_t$). Simulating the model, the RMSE of households is around 1.3 times the RMSE of the firms.

Externality Validation. One of the model predictions is that households pay less attention to demand shocks. If this is the case, they are more likely to make large forecast errors during periods dominated by demand shocks. To test this, I use the supply shocks and demand shocks identified by Eickmeier and Hofmann (2022)[27] The forecast error for one-year-ahead inflation is measured as the absolute difference between the median forecast from the MSC and the realized inflation for the corresponding period. I find that forecast errors during periods dominated by demand shocks are about 1.6 times larger than during periods dominated by supply shocks.

5 Implications for Central Bank Communication

Rational inattention also has several implications for central bank communication. First, ordinary people are most likely pay little attention to even simple policy announcements (Sims, 2010), due to a lack of incentives. Based on the model, I formally show how the misalignment of interests limits the 'getting-through' of the policy announcements in Section 5.1. Second, even if the message reaches the public, it is still up to the public to interpret the information. In Section 5.2 and 5.3, I examine two cases where communication about higher inflation expectations or lower interest rates – policies that would stimulate the economy under full information – could backfire under rational inattention.

5.1 The Veil of Inattention

For communication to be effective, the receiver must also be *able* and *willing* to absorb, process and utilize the information. This section studies the central bank communication where the audience (households and firms) is rationally inattentive and provides a rationale of why the central bank communication fails to reach the general public. To fix ideas, consider communicating about monetary policy actions to public, i.e.,

$$S_{p,t} = i_t + \nu_t, \nu_t \sim N(0, \sigma_\nu^2)$$

However, whether households and firms have incentives to absorb the this information depends on how relevant they believe the signal is for their decisions. Formally, the benefit of absorbing the central bank signal is proportional to

$$\mathbb{E}[x_t^*|S_{k,t}, S_{p,t}] - \mathbb{E}[x_t^*|S_{k,t}] \propto \underbrace{\frac{\Sigma_0}{\Sigma_0 + \Delta_{k,p}\sigma_\nu^2}}_{\text{signal-to-noise ratio}} \times \underbrace{\Delta_{k,p}}_{\text{relevance of signal}} \times \underbrace{\left(S_p - \mathbb{E}[S_{p,t}|S_k^t]\right)}_{\text{marginal new info from } S_{p,t}} \quad (5.1)$$

where $\Delta_{p,k}$, $k = \{h, f\}$ reflects how relevant the central bank's signal is to households' or firms' objective, i.e., how much $S_{p,t}$ matters for households' optimal consumption and

[27] I use their identified shocks because their empirical analysis adopts the same definition of supply and demand shocks as in this paper: supply shocks move inflation and output in opposite directions, while demand shocks move both variables in the same direction. They estimated structural demand and supply factors for the period 1970Q1–2022Q2.

bond holdings, or how much it affects firms' pricing decision. The benefit of the signal about i_t is discounted by the term $\Delta_{k,p}$ because the signal is not of direct relevance to households' and firms' interest. Therefore, if it requires some cognitive costs or effort to process the central bank information, households and firms may choose not to pay attention to the signal.[28]

Nonetheless, if the content of the communication are more aligned with audience's interests, they will pay more attention to the signal. This analysis relates to (Angeletos and Sastry, 2021) – that should communications aim at anchoring expectations of the policy instrument (interest rate path) or of the targeted outcome (aggregate output/price). This paper focus on the incentive of rationally inattentive agents in learning about the central bank communication. They will pay more attention to the communication if the content is of direct relevance for their decisions (in an extreme case the central bank gives signal on the optimal actions of households and firms). In this sense, communicating targeted outcomes have better chances of reaching to the public than communicating policy instruments.

5.2 Communication about Higher Inflation

Even if the message does get through, it is still up to the public to interpret the information. Under rational inattention, agents may incorrectly attribute the origins of the shock, leading to unintended consequences from the communication.

One of the key mechanisms behind the effectiveness of quantitative easing and forward guidance is that when inflation expectations rise while nominal interest rates remain at zero, real interest rates fall. This lower real rate encourages households to spend more, and firms to invest, produce, and hire additional workers. Central bank sees this as a crucial channel for stimulating the economy through communication. However, as pointed by Candia et al. (2020), providing information about higher inflation expectations to households and firms can backfire, depending on how they interpret it. Since households tend to associate higher inflation with a worsening economy, they might reduce their spending when they expect inflation to rise.

This section formally shows that communicating higher inflation could lead to unintended consequences for households' beliefs about future growth, and, consequently, their actual spending. Using the quantitative model from Section 4, I conduct a policy experiment in which the central bank communicates a higher future inflation. Formally, I assume that households are at the steady state of the rational inattention problem, and they receive a one-time, perfectly informative signal about future inflation.

The left panel in Figure 8 shows that following the communication about higher future inflation path, households raise their inflation expectations (from prior to posterior).

[28]These cognitive costs explain why households are generally inattentive to policy but, when provided with information in randomized control trials (RCTs), adjust their expectations to some extent.

Follows this, households revise their beliefs about future growth downwards, further deviating the path under full information, as depicted in the middle panel. And consequently, they would reduce their spending (right panel). This unintended consequence arises because rationally inattentive households misinterpret the communication. In their beliefs, supply shocks are more important, therefore, they tend to associate higher inflation with lower output growth. When they receive information about higher future inflation, they attribute the increase in inflation to supply shocks, and thus revise downwards their growth expectations. This suggests that monetary policies aimed at stimulating the economy by raising inflation expectations can have unintended consequences on households beliefs about other economic variables. These effects can weaken, or even reverse, the intended impact, making the policy potentially counterproductive.

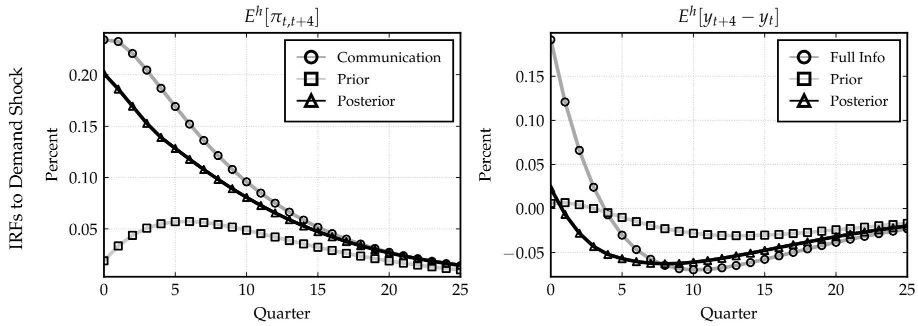


Figure 8: Communication of Higher Future Inflation

Notes: The figure plots the impulse responses of expected inflation and expected growth by households in response to the communication of a higher trajectory of future inflation. Households revise upwards their inflation expectations following the communication but at the same time revise downwards their growth expectations.

5.3 Communication about Lower Interest Rate Path

Central banks can influence the public's expectations about future interest rates by communicating the expected interest rate path. This section formally shows that communicating a lower future interest rates can have unintended consequences on firms' beliefs about economic conditions, which in turn affect their pricing decisions. Consider the case where, following a positive productivity shock, as consumption does not immediately adjust, there is a temporary negative output gap. Central bank would cut the interest rate in response to this economic slack.

When the central bank communicates a lower interest rate path to firms, as shown in the left panel in Figure 9, firms mistakenly interpret the lower interest rate as indicating economic expansion rather than responding to economic slack. As a result, they revise their inflation expectations upwards (middle panel) and raise prices (right panel) – the opposite of what they would do if they had full information. This is because firms believe that demand shocks are more important and thus they are more likely to interpret the lower interest rate as origins from an expansionary demand shock. As firms raise prices, aggregate demand falls further, worsening the economic slack.

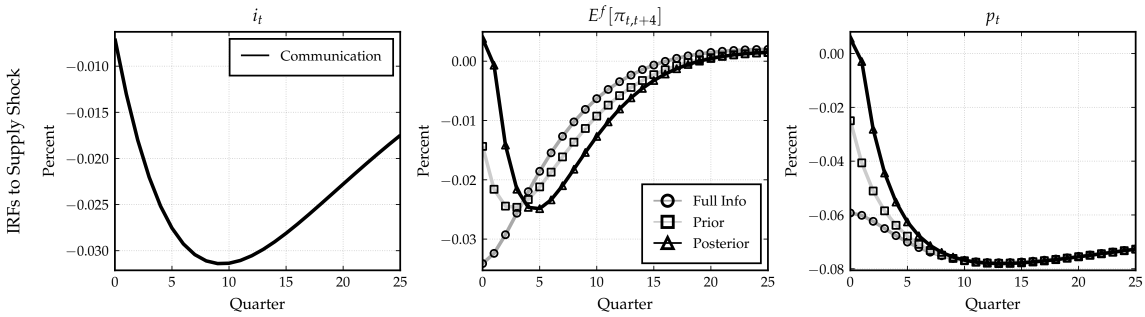


Figure 9: Communication of Lower Interest Rate

Notes: The figure plots the impulse responses of expected inflation and expected growth by firms in response to the communication of a lower trajectory of future interest rate. Firms revise their inflation expectations in a wrong direction following the communication and thus the price adjustment are even more sub-optimal.

In Section 5.2 to 5.3, central bank communication can lead to unintended consequences due to rational inattention. When the central bank communicates higher inflation expectations, households, who believe supply shocks are more important, may misinterpret the signal as a sign of worsening economic conditions. As a result, they lower their growth expectations and reduce spending, counteracting the central bank's intent to stimulate the economy. Similarly, when the central bank communicates a lower interest rate path, firms may misinterpret the signal as indicating economic expansion rather than responding to economic slack. As a result, they raise prices, further reducing aggregate demand and worsening the economic slack, which again undermines the central bank's objectives.

To minimize these unintended consequences, central banks should clearly explain the underlying causes of these policy decisions and their expected effects on the real economy, such as output and employment. By providing context and clarifying the reasons behind their actions, central banks can help households and firms better interpret the signals, reducing the likelihood of misinterpretation and enhancing the effectiveness of their policies.

6 Conclusions

This paper studies the role of rational inattention in shaping the expectations of households and firms, and its implications for business cycle fluctuations and monetary policy.

First, by allowing agents to endogenously choose their information, the model shows that households optimally pay more attention to supply shocks, while firms tend to allocate slightly more attention to demand shocks. Their asymmetric attention choices can help explain their different views observed in the survey. Second, as this is a general equilibrium model with both households and firms being rationally inattentive, there are lots of rich interactions between their attention allocation. I solve the model to quantify the impact of these asymmetric attention choices and their interactions on business cycles, showing that attention choices significantly affect how shocks transmit to output and inflation. Finally, the paper conducts several policy experiments, demonstrating that central bank communication can have unintended consequences when agents are rationally inattentive.

This paper contributes to the growing literature on expectation formation and information frictions by highlighting the endogenous and asymmetric nature of attention allocation across economic agents. While this paper compares households and firms, there is significant variation within these groups, driven by differences in characteristics such as income levels, education, or firm size. A natural extension of this work would be to examine the heterogeneity within each group. Moreover, this paper focuses on how households and firms allocate attention to aggregate economic conditions. However, there are also household- or firm-level factors that capture their attention. A valuable extension would be to explore how agents allocate attention between aggregate economic shocks and idiosyncratic, individual-level factors. Understanding how they allocate attention between these two dimensions could offer new insights into the decision-making processes of households and firms and their responses to broader economic policies.

References

Afrouzi, H. and C. Yang (2021): "Dynamic rational inattention and the Phillips curve," *Available at SSRN 3770462*. 9, 29, 35, 52

Andre, P., C. Pizzinelli, C. Roth, and J. Wohlfart (2022): "Subjective models of the macroeconomy: Evidence from experts and representative samples," *The Review of Economic Studies*, 89, 2958–2991. 1

Angeletos, G.-M. and J. La'o (2013): "Sentiments," *Econometrica*, 81, 739–779. 3

Angeletos, G.-M. and K. A. Sastry (2021): "Managing expectations: Instruments versus targets," *The Quarterly Journal of Economics*, 136, 2467–2532. 38

Aruoba, S. B., L. Bocola, and F. Schorfheide (2017): "Assessing DSGE model nonlinearities," *Journal of Economic Dynamics and Control*, 83, 34–54. 29

Bhandari, A., J. Borovička, and P. Ho (2022): "Survey data and subjective beliefs in business cycle models," *Available at SSRN 2763942*. 7

Blanchard, O. (2016): "The Phillips curve: back to the'60s?" *American Economic Review*, 106, 31–34. 2, 35

Blanchard, O. J., J.-P. L'Huillier, and G. Lorenzoni (2013): "News, noise, and fluctuations: An empirical exploration," *American Economic Review*, 103, 3045–3070. 3

Bordalo, P., N. Gennaioli, and A. Shleifer (2018): "Diagnostic expectations and credit cycles," *The Journal of Finance*, 73, 199–227. 1

Bullard, J. (2018): "The case of the disappearing Phillips curve," *Price and wage-setting in advanced economies*, 164. 2, 35

Candia, B., O. Coibion, and Y. Gorodnichenko (2020): "Communication and the beliefs of economic agents," Tech. rep., National Bureau of Economic Research. 1, 4, 5, 30, 38

——— (2023): "The macroeconomic expectations of firms," in *Handbook of Economic Expectations*, Elsevier, 321–353. 1

Carroll, C. D. (2003): "Macroeconomic expectations of households and professional forecasters," *the Quarterly Journal of economics*, 118, 269–298. 1

Chahrour, R. and R. Ulbricht (2023): "Robust predictions for DSGE models with incomplete information," *American Economic Journal: Macroeconomics*, 15, 173–208. 3

Coibion, O., D. Georgarakos, Y. Gorodnichenko, and M. Van Rooij (2023): "How does consumption respond to news about inflation? Field evidence from a randomized control trial," *American Economic Journal: Macroeconomics*, 15, 109–152. 2, 5

Coibion, O. and Y. Gorodnichenko (2015): "Is the Phillips curve alive and well after all? Inflation expectations and the missing disinflation," *American Economic Journal: Macroeconomics*, 7, 197–232. 2, 35

Coibion, O., Y. Gorodnichenko, S. Kumar, and J. Ryngaert (2021): "Do you know that I know that you know...? Higher-order beliefs in survey data," *The Quarterly Journal of Economics*, 136, 1387–1446. 5

Del Negro, M., M. Lenza, G. E. Primiceri, and A. Tambalotti (2020): "What's up with the Phillips Curve?" Tech. rep., National Bureau of Economic Research. 34

Eickmeier, S. and B. Hofmann (2022): "What drives inflation? Disentangling demand and supply factors," . 37

Eusepi, S. and B. Preston (2010): "Central bank communication and expectations stabilization," *American Economic Journal: Macroeconomics*, 2, 235–271. 3

Evans, G. W. and S. Honkapohja (2001): *Learning and expectations in macroeconomics*, Princeton University Press. 1

Fernald, J. (2014): "A quarterly, utilization-adjusted series on total factor productivity," in *A quarterly, utilization-adjusted series on total factor productivity*, Federal Reserve Bank of San Francisco. 29, 30

Flynn, J. P. and K. Sastry (2024): "Attention cycles," Tech. rep., National Bureau of Economic Research. 3

Gabaix, X. and D. Laibson (2017): "Myopia and discounting," Tech. rep., National bureau of economic research. 1

Han, Z. (2022): "Asymmetric Information and Misaligned Inflation Expectations," *Available at SSRN 4187209*. 1, 3

Hébert, B. and M. Woodford (2018): "Information costs and sequential information sampling," Tech. rep., National Bureau of Economic Research. 9

Hellwig, C. and L. Veldkamp (2009): "Knowing what others know: Coordination motives in information acquisition," *The Review of Economic Studies*, 76, 223–251. 2

Hooper, P., F. S. Mishkin, and A. Sufi (2020): "Prospects for inflation in a high pressure economy: Is the Phillips curve dead or is it just hibernating?" *Research in Economics*, 74, 26–62. 2, 35

Juodis, A. and S. Kučinskas (2023): "Quantifying noise in survey expectations," *Quantitative Economics*, 14, 609–650. 32

KAMDAR, R. ET AL. (2018): "The inattentive consumer: Sentiment and expectations," *Manuscript. https://rupalkamdar. github. io/pdfs/Inattentive Consumer. pdf*. 3

KOHLHAS, A. N. AND A. WALTHER (2021): "Asymmetric attention," *American Economic Review*, 111, 2879–2925. 3, 18

LEEPER, E. M. (1991): "Equilibria under 'active'and 'passive'monetary and fiscal policies," *Journal of monetary Economics*, 27, 129–147. 26

LINK, S., A. PEICHL, C. ROTH, AND J. WOHLFART (2023): "Information frictions among firms and households," *Journal of Monetary Economics*, 135, 99–115. 1

LUCAS, R. E. (1972): "Expectations and the Neutrality of Money," *Journal of economic theory*, 4, 103–124. 3

MAĆKOWIAK, B., F. MATĚJKA, AND M. WIEDERHOLT (2018): "Dynamic rational inattention: Analytical results," *Journal of Economic Theory*, 176, 650–692. 10

MAĆKOWIAK, B. AND M. WIEDERHOLT (2009): "Optimal sticky prices under rational inattention," *American Economic Review*, 99, 769–803. 2, 3, 9

——— (2015): "Business cycle dynamics under rational inattention," *The Review of Economic Studies*, 82, 1502–1532. 3

——— (2023): "Rational inattention and the business cycle effects of productivity and news shocks," . 51

MANKIW, N. G., R. REIS, AND J. WOLFERS (2003): "Disagreement about inflation expectations," *NBER macroeconomics annual*, 18, 209–248. 1, 9

MICHELACCI, C. AND L. PACIELLO (2024): "Ambiguity Aversion and Heterogeneity in Households' Beliefs," *American Economic Journal: Macroeconomics*, 16, 95–126. 3

PACIELLO, L. (2012): "Monetary policy and price responsiveness to aggregate shocks under rational inattention," *Journal of Money, Credit and Banking*, 44, 1375–1399. 9

SIMS, C. A. (2003): "Implications of rational inattention," *Journal of monetary Economics*, 50, 665–690. 3, 9, 10

——— (2010): "Chapter 4 - Rational Inattention and Monetary Economics," Elsevier, vol. 3 of *Handbook of Monetary Economics*, 155–181. 37

WOODFORD, M. (2001): "Imperfect common knowledge and the effects of monetary policy," . 3

——— (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, NJ: Princeton University Press. 1, 9

A Proofs for Section 3

A.1 Approximation of household's utility function

Household i's per period utility at time t is given by:

$$U\left(C_{i,t}, L_{i,t}\right) = \frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{L_{i,t}^{1+\eta}}{1+\eta}$$

As households are hand-to-mouth, labor supply can be substituted using the budget constraint $L_{it} = (P_t C_t)/W_t$. The utility then becomes

$$U\left(C_{i,t}, L_{i,t}\right) = \frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{\left(\frac{P_t C_{i,t}}{W_t}\right)^{1+\eta}}{1+\eta}$$

Households take wages and prices as given, meaning the only choice variable is consumption C_{it}. Expressing the per-period utility function in terms of log-deviations from the non-stochastic steady state yields

$$\hat{u}\left(c_{i,t}, p_t, w_t\right) = \left[\frac{\left(\bar{C}e^{c_{i,t}}\right)^{1-\gamma}}{1-\gamma} - \frac{\left(\frac{\bar{P}e^{p_t}\bar{C}e^{c_{i,t}}}{\bar{W}e^{w_t}}\right)^{1+\eta}}{1+\eta}\right]$$

The period utility of household i depends on choice variable $c_{i,t}$ and variables that the household takes as given, namely $\{w_t, p_t\}$. For any given $\{w_t, p_t\}$, the utility maximizing consumption level is

$$c_{i,t}^* = \arg\max_{c_{i,t}} \hat{u}\left(c_{i,t}, p_t, w_t\right) \Leftrightarrow \hat{u}_1\left(c_{i,t}^*, p_t, w_t\right) = 0$$

Taking a second-order approximation of the utility function $L\left(c_{i,t}, p_t, w_t\right) \equiv \hat{u}\left(c_{i,t}, p_t, w_t\right) - \hat{u}(c_{i,t}^*, p_t, w_t)$ around the steady state yields

$$\begin{aligned} L\left(c_{i,t}, p_t, w_t\right) = {} & \frac{1}{2}\hat{u}_{11}\left(c_{i,t}^2 - {c_{i,t}^*}^2\right) + \hat{u}_{12}p_t\left(c_{i,t} - c_{i,t}^*\right) \\ & + \hat{u}_{12}w_t\left(c_{i,t} - c_{i,t}^*\right) + \mathcal{O}\left(\|c_{i,t}, p_t, w_t\|^3\right) \end{aligned} \tag{A1}$$

where $\hat{u}_{1,n}, n \in \{1,2,3\}$ denotes the second-order derivatives of the utility function with respect to $c_{i,t}$, $c_{i,t}$ and p_t, and $c_{i,t}$ and w_t around the approximation point. Since $c_{i,t}^*$ maximizes the utility function for any p_t and w_t,

$$\hat{u}_1(c_{i,t}^*, p_t, w_t) = 0 \Rightarrow \hat{u}_{11}c_{i,t}^* + \hat{u}_{12}p_t + \hat{u}_{13}w_t + \mathcal{O}\left(\|p_t, w_t\|^2\right) = 0$$

Combining this with Equation (A1) I obtain

$$\begin{aligned}\hat{u}\left(c_{i,t}, p_t, w_t\right) &= L\left(c_{i,t}, p_t, w_t\right) + \hat{u}\left(c^*_{i,t}, p_t, w_t\right) \\ &= \frac{1}{2}\hat{u}_{11}\left(c_{i,t} - c^*_{i,t}\right)^2 + \mathcal{O}\left(\|c_{i,t}, p_t, w_t\|^3\right) + \text{terms independent of } c_{i,t}\end{aligned}$$

Given the specific utility function, $\hat{u}_{11} = -(\gamma + \eta)$ in the steady state. Moreover,

$$c^*_{i,t} = \frac{1+\eta}{\gamma+\eta}\left(w_t - p_t\right)$$

Hence, the household i's objective (3.1) is approximated by

$$\left[-\frac{(\gamma+\eta)}{2}\left(c_{i,t} - c^*_{i,t}\right)^2\right] + \text{terms independent of } \{c_{i,t}\}_{t\geq 0}$$

A.2 Approximation of firm's profit function

First, substituting the production function and demand function into firm j's per-period profit function

$$\Pi\left(P_{j,t}, W_t, X_t\right) = \frac{1}{P_t C_t}\left[P_{j,t}\left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t - (1-\theta^{-1})\frac{W_t}{A_t}\left(\frac{P_{j,t}}{P_t}\right)^{-\theta} Y_t\right]$$

The per-period profit function can be rewritten in terms of log-deviations from the non-stochastic steady state

$$\hat{\pi}\left(p_{jt}, w_t, a_t, x_t\right) = \bar{C} e^{c_t} e^{-\theta(p_{jt}-p_t)-p_t}\left[e^{p_{jt}} - \left(1-\theta^{-1}\right) e^{w_t - a_t}\right]$$

where the small letters denote the log-deviations of the corresponding variable. For any given $\{w_t, p_t, y_t, a_t\}$,

$$p^*_{jt} = \arg\max_{p_{jt}} \hat{\pi}\left(p_{jt}, p_t, w_t, y_t, a_t\right) \Leftrightarrow \hat{\pi}_1\left(p_{jt}, p_t, w_t, y_t, a_t\right) = 0$$

Define function $L\left(p_{jt}, p_t, w_t, y_t, a_t\right) \equiv \hat{\pi}\left(p_{jt}, p_t, w_t, y_t, a_t\right) - \hat{\pi}\left(p^*_{jt}, p_t, w_t, y_t, a_t\right)$, and take a second-order approximation around the steady state

$$\begin{aligned}L\left(p_{jt}, p_t, w_t, y_t, a_t\right) &= \frac{1}{2}\hat{\pi}_{11}\left(p_{jt}^2 - {p^*_{jt}}^2\right) + \hat{\pi}_{12} p_t\left(p_{jt} - p^*_{jt}\right) + \hat{\pi}_{13} w_t\left(p_{jt} - p^*_{jt}\right) \\ &\quad + \hat{\pi}_{14} y_t\left(p_{jt} - p^*_{jt}\right) + \hat{\pi}_{15} a_t\left(p_{jt} - p^*_{jt}\right) + \mathcal{O}\left(\|p_{jt}, p_t, w_t, y_t, a_t\|^3\right) \quad \text{(A2)}\end{aligned}$$

where $\hat{p}_{1,n}, n \in \{1,2,3,4,5\}$ denotes the second-order derivatives of the profit function with respect to p_{jt}, p_{jt} and p_t, p_{jt} and w_t, p_{jt} and y_t, and p_{jt} and a_t around the approximation point. Note also that since p^*_{jt} maximizes the profit function for any given

$\{w_t, p_t, y_t, a_t\}$,

$$\hat{\pi}\left(p_{jt}^*, p_t, w_t, y_t, a_t\right) = 0 \Rightarrow \hat{\pi}_{11} p_{jt}^* + \hat{\pi}_{12} p_t + \hat{\pi}_{13} w_t + \hat{\pi}_{13} y_t + \hat{\pi}_{13} a_t + \mathcal{O}\left(\|p_t, w_t, a_t, y_t\|^2\right) = 0$$

Combining this with Equation (A2) I obtain

$$\begin{aligned}\hat{\pi}\left(p_{jt}, p_t, w_t, y_t, a_t\right) &= L\left(p_{jt}, p_t, w_t, y_t, a_t\right) + \hat{\pi}\left(p_{jt}^*, p_t, w_t, y_t, a_t\right) \\ &= \frac{1}{2}\hat{\pi}_{11}\left(p_{jt} - p_{jt}^*\right)^2 + \mathcal{O}\left(\|p_{jt}, p_t, w_t, y_t, a_t\|^3\right) + \text{terms independent of } p_{jt}\end{aligned}$$

Given the particular profit function, $\hat{\pi}_{11} = -(\theta - 1)$ in the steady state. And the optimal price

$$p_{jt}^* = w_t - a_t$$

Hence, the firm j's objective (3.4) is approximated by

$$\sum_{t=0}^{\infty} \beta^t \, \mathbb{E}^j \left[-\frac{\theta - 1}{2} \left(p_{jt} - p_{jt}^*\right)^2 \right] + \text{terms independent of } \{p_{jt}\}_{t \geq 0}$$

A.3 Proof of Proposition 1

Upon reception of a signal $s_{i,a,t} = a_t + e_{i,a,t}$, the consumption $c_{i,t} = \lambda_{h,a} \, \mathbb{E}[a_t | s_{i,a,t}]$ maximizes the expected utility (3.14) for any given posterior belief. Bayesian updating with Gaussian prior uncertainty and signals delivers

$$\mathbb{E}\left[a_t | s_{i,a,t}\right] = \xi_{h,a}[a_t + e_{i,a,t}]$$

where $\xi_{h,a} \equiv (1 - \sigma_{a|s}^2 / \sigma_a^2) \in [0, 1]$, and $\xi_{h,a}$ is the Kalman-gain on the signal. Now rewrite the problem (3.14) in terms of choice variable $\xi_{h,a}$

$$\max_{\xi_{h,a} \in [0,1]} \left[-\left(\gamma + \eta\right) \lambda_{h,a}^2 \left(1 - \xi_{h,a}\right) \sigma_a^2 - \mu^h \ln \frac{1}{1 - \xi_{h,a}} \right]$$

Solving the first order condition, the solution is

$$\xi_{h,a} = \max\left(0, 1 - \frac{\mu^h}{(\gamma + \eta)\lambda_{h,a}^2 \sigma_a^2}\right)$$

A.4 Proof of Proposition 2

By the independence assumption, I can solve the firms attention choices for aggregate demand shock and the productivity shock separately.

In the case of demand shocks, the signals take the form $s_{j,q,t} = q_t + e_{j,q,t}$. To derive firms' attention choices, it is instructive to first express the firms' ex ante expected utility as a function of their attention choices. Note that firm j's prior uncertainty about q_t is simply σ_q^2, and denote firm j's the posterior uncertainty as $\sigma_{q|s_j} \equiv var(q_t|s_{j,q,t})$. The firm j's attention problem is then

$$\max_{\{s_{j,q,t}\in\mathcal{S}_f^t\}} \mathbb{E}_t^f \left[-\frac{\theta-1}{2}\left(\mathbb{E}[p_{j,t}^*|s_{j,q,t}] - p_{j,t}^*\right)^2 - \mu^f \mathcal{I}\left(q_t; s_{j,q,t}\right)\right]$$
$$= \frac{1}{2} \max_{\sigma_{q|s_j}^2 \le \sigma_q^2} \left[-(\theta-1)\lambda_{f,q}^2 \sigma_{q|s_j}^2 - \mu^f \ln \frac{\sigma_q^2}{\sigma_{q|s_j}^2}\right] \tag{A3}$$

For every realization of the signal at time t, the firm will set price $p_{j,t} = \mathbb{E}[p_{j,t}^*|s_{j,q,t}]$. Hence, the expected profit depends on the expected square deviation of $\mathbb{E}[p_{j,t}^*|s_{j,q,t}]$ from $p_{j,t}^*$, which reduces to the conditional variance in (A3).

Upon reception of a signal $s_{j,q,t} = q_t + e_{j,q,t}$, the price $p_{j,t} = \mathbb{E}[p_{j,t}^*|s_{j,q,t}]$ maximizes the expected profit for any given posterior belief. Bayesian updating with Gaussian prior uncertainty and signals delivers

$$\mathbb{E}\left[p_{j,t}^*|s_{j,q,t}\right] = \xi_{f,q}\lambda_{f,q}[q_t + e_{j,q,t}]$$

where $\xi_{f,q} \equiv (1 - \sigma_{q|s_j}^2/\sigma_q^2) \in [0,1]$ is the attention weight on the signal. I can now rewrite the problem (A3) in terms of choice variable $\xi_{f,q}$

$$\max_{\xi_{f,q}\in[0,1]} \left[-\left(\theta-1\right)\lambda_{f,q}^2\left(1-\xi_{f,q}\right)\sigma_q^2 - \mu^f \ln \frac{1}{1-\xi_{f,q}}\right]$$

Solving gives the expression in Equation (3.18a)

$$\xi_{f,q} = \max\left(0, 1 - \frac{\mu^f}{(\theta-1)\lambda_{f,q}^2\sigma_q^2}\right)$$

By the same procedure, I can solve the attention problem for supply shocks a_t.

In the case of productivity shocks, the firm's attention problem is

$$\max_{\sigma_{a|s_j}^2 \le \sigma_a^2} \left[-\left(\theta-1\right)\lambda_{f,a}^2\sigma_{a|s_j}^2 - \mu^f \ln\left(\frac{\sigma_a^2}{\sigma_{a|s_j}^2}\right)\right]$$

where $\lambda_{f,a} = -\frac{1+\eta}{\gamma+\eta}$, σ_a^2 is the prior variance of firm j's belief about the productivity shock and $\sigma_{a|s_j}^2$ denotes the posterior variance.

Upon reception of a signal $s_{j,a,t} = a_t + e_{j,a,t}$, the price $p_{j,t} = \mathbb{E}[p_{j,t}^*|s_{j,a,t}]$ maximizes the expected profit for any given posterior belief. Bayesian updating with Gaussian prior uncertainty and signals delivers

$$\mathbb{E}\left[p_{j,t}^*|s_{j,a,t}\right] = \xi_{f,a}\lambda_{f,a}[a_t + e_{j,a,t}]$$

where $\xi_{f,a} \equiv (1-\sigma^2_{a|s_j}/\sigma^2_a) \in [0,1]$, and $\lambda_{f,a}\xi_{f,a}$ reflects the attention weight on the signal. I can now rewrite the firms' attention problem in terms of choice variable $\xi_{f,a}$

$$\max_{\xi_{f,a}\in[0,1]} \left[-(\theta-1)\,\lambda^2_{f,a}\,(1-\xi_{f,a})\,\sigma^2_a - \mu^f \ln \frac{1}{1-\xi_{f,a}}\right]$$

Solving gives the expression in Equation (3.18b)

$$\xi_{f,a} = \max\left(0, 1-\frac{\mu^f}{(\theta-1)\lambda^2_{f,a}\sigma^2_a}\right)$$

Combining the results together gives the Proposition 2.

A.5 Proof of Corollary 3

Under the optimal signal design, firms optimally choose to receive a single signal of the optimal price, i.e., $s_{j,t} = p^*_{j,t} + e_{j,t} = \lambda_{f,q}q_t + \lambda_{f,a}a_t + e_{j,t}$ where $e_{j,t}$ is the attention error. Upon receiving this signal, the price $p_{j,t} = \mathbb{E}[p^*_{j,t}|s_{j,t}]$ maximizes the expected profit for any given posterior belief. Therefore, the objective can be expressed as

$$\begin{aligned}
&\max_{\{s_{j,t}\in\mathcal{S}^t_f\}} \mathbb{E}^f_t\left[-\frac{\theta-1}{2}\left(\mathbb{E}[p^*_{j,t}|s_{j,t}]-p^*_{j,t}\right)^2 - \mu^f\mathcal{I}\left(q_t,a_t;s_{j,t}\right)\right] \\
&= \frac{1}{2}\max_{\sigma^2_{p|s}\le\sigma^2_p}\left[-(\theta-1)\sigma^2_{p|s} - \mu^f\ln\left(\frac{\sigma^2_p}{\sigma^2_{p|s}}\right)\right]
\end{aligned}$$

where $\sigma^2_p \equiv \lambda^2_{f,q}\sigma^2_q + \lambda^2_{f,a}\sigma^2_a$ denotes the prior uncertainty about $p^*_{j,t}$ and $\sigma^2_{p|s}$ denotes the posterior uncertainty. Solve the model, the firm sets a price according to

$$p_{j,t} = \xi_f\left(p^*_{j,t} + e_{j.t}\right) = \xi_f\left(\lambda_{f,q}q_t + \lambda_{f,a}a_t + e_{j,t}\right) \tag{A4}$$

with

$$\xi_f = \max\left(0, 1-\frac{\mu^f}{(\theta-1)\sigma^2_p}\right)$$

From Equation (A4), the weights on the demand shock (q_t) and the supply shock (a_t) are $\xi_f\lambda_{f,q}$ and $\xi_f\lambda_{f,a}$, respectively.

B Proofs for quantitative model

B.1 Approximation of Households' Utility

First, using the flow budget constraint (4.2) to substitute for labor in the utility function and expressing all variables in terms of log-deviations from the non-stochastic steady state yields the following expression for the period utility of household i in period t:

$$u = \left(\frac{\bar{C}^{1-\gamma}}{1-\gamma} e^{(1-\gamma)c_{i,t}} - \frac{\left[\frac{\bar{P}\bar{C}e^{p_t + c_{i,t}} + \bar{B}e^{b_{i,t}} - \bar{R}\bar{B}e^{i_{t-1}+b_{i,t-1}} - \bar{D}e^{d_t} - \bar{T}e^{\tau_t}}{\bar{W}e^{w_t}} \right]^{1+\eta}}{1+\eta} \right)$$

here, the lowercase letters denote the log-deviations of the corresponding variables. $c_{i,t}$ is the consumption by household i, $\tilde{b}_{i,t}$ denotes the real bond holdings by household i, $\tilde{d}_t$ is the real dividend, and $\tilde{\tau}_t$ is the real transfer (tax if negative). Moreover, define ω_B, ω_W, ω_D and ω_T as the steady state ratios

$$(\omega_B, \omega_W, \omega_D, \omega_T) = \left(\frac{\bar{B}}{\bar{C}\bar{P}}, \frac{\bar{W}\bar{L}}{\bar{C}\bar{P}}, \frac{\bar{D}}{\bar{C}\bar{P}}, \frac{\bar{T}}{\bar{C}\bar{P}} \right)$$

In period t, household i chooses $v_t \equiv (\tilde{b}_{i,t}, c_{i,t})'$, the choices made in previous period represented by $v_{t-1} = (\tilde{b}_{i,t-1}, 0)'$. Households take other variables as given $\zeta_t \equiv [\tilde{d}_t, i_{t-1}, \tilde{w}_t, \tilde{\tau}_t, \pi_t]'$.

A log-quadratic approximation to the expected discounted sum of period utility around the non-stochastic steady state yields

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_i^h \left[\frac{1}{2} (v_t - v_t^*)' \Theta_0 (v_t - v_t^*) + (v_t - v_t^*) \Theta_1 \left(v_{t+1} - v_{t+1}^* \right) \right] \tag{A1}$$

where

$$\Theta_0 = -\bar{C}_i^{1-\gamma} \begin{bmatrix} \frac{\eta}{\omega_W} \left[1 + \frac{1}{\beta} \right] \omega_B^2 & \frac{\eta}{\omega_W} \omega_B \\ \frac{\eta}{\omega_W} \omega_B & \left(\gamma + \frac{\eta}{\omega_W} \right) \end{bmatrix}, \qquad \Theta_1 = \bar{C}^{1-\gamma} \begin{bmatrix} \frac{\eta}{\omega_W} \omega_B^2 & \frac{\eta}{\omega_W} \omega_B \\ 0 & 0 \end{bmatrix}$$

The sequence of optimal bond holdings under full information is given by

$$\omega_B \left(\frac{1}{\beta} \tilde{b}_{i,t-1}^* - \tilde{b}_{i,t}^* \right) + c_{i,t}^* = \mathbb{E}_t \left[\omega_B \left(\frac{1}{\beta} \tilde{b}_{i,t}^* - \tilde{b}_{i,t+1}^* \right) + c_{i,t+1}^* \right] \tag{A2}$$

and the optimality choice for consumption

$$-\omega_B \left(\frac{1}{\beta} \tilde{b}_{i,t-1}^* - \tilde{b}_{i,t}^* \right) + \left(\gamma \frac{\omega_W}{\eta} + 1 \right) c_{i,t}^* = \omega_W \left(\frac{1}{\eta} + 1 \right) \tilde{w}_t + \left[\frac{1}{\beta} \omega_B (i_{t-1} - \pi_t) + \omega_D \tilde{d}_t + \omega_T \tilde{\tau}_t \right] \tag{A3}$$

Together with the log-linearised budget constraint

$$c_{i,t} = \omega_W (\tilde{w}_t + l_{i,.t}) + \frac{1}{\beta} \omega_B (i_{t-1} - \pi_t) + \omega_B \left(\frac{1}{\beta} \tilde{b}_{i,t-1} - \tilde{b}_{i,t} \right) + \omega_D \tilde{d}_t + \omega_D \tilde{\tau}_t \tag{A4}$$

Under full information, combine the optimality choice for consumption (A3) with the optimal bond holdings (A2), I get the usual inter-temporal Euler equation

$$c_{i,t}^* = \mathbb{E}_t \left[c_{i,t+1}^* - \frac{1}{\gamma} \left(i_t - \pi_{t+1} \right) \right]$$

Combine the budget constraint (A4) with the optimality condition for consumption choice (A3) gives the usual intro-temporal Euler equation

$$\tilde{w}_t = \gamma c_{i,t}^* + \eta l_{i,t}^* \tag{A5}$$

To solve for the optimal bond holdings under full information, I do the transformation following Maćkowiak and Wiederholt (2023). First, using Equation (A5) to substitute for $l_{i,t}$ in the budget constraint (A4) and rearranging yields the equation

$$\left(1 + \omega_W \frac{\gamma}{\eta}\right) c_{i,t}^* = \omega_W \left(1 + \frac{1}{\eta}\right) \tilde{w}_t + \omega_B \left(\frac{1}{\beta} \left(\tilde{b}_{i,t-1}^* + i_{t-1} - \pi_t \right) - \tilde{b}_{i,t}^* \right) + \omega_D \tilde{d}_t + \omega_D \tilde{\tau}_t$$

Sum from $t = 0$ to infinity and discount by β

$$\left(1 + \omega_W \frac{\gamma}{\eta}\right) \sum_{s=t}^{t+N} \beta^{s-t} c_{i,s}^* = \omega_B \frac{1}{\beta} \tilde{b}_{i,t-1}^* + \sum_{s=t}^{t+N} \beta^{s-t} \left[z_s \right] - \omega_B \beta^N \tilde{b}_{i,t+N}^* \tag{A6}$$

Here $z_t \equiv \omega_W \left(1 + \frac{1}{\eta}\right) \tilde{w}_t + \frac{1}{\beta} \omega_B \left(i_{t-1} - \pi_t \right) + \omega_D \tilde{d}_t + \omega_D \tilde{\tau}_t$.

Taking the expectation on both sides of Equation (A6), and as $N \to \infty$, I get

$$\left(1 + \omega_W \frac{\gamma}{\eta}\right) \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t \left[c_{i,s}^* \right] = \omega_B \frac{1}{\beta} \tilde{b}_{i,t-1}^* + \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t \left[z_s \right] \tag{A7}$$

Next, using the Euler Equation and the law of iterated expectations yields

$$\sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t \left[c_{i,s}^* \right] = \frac{1}{1-\beta} c_{i,t}^* + \frac{1}{\gamma} \frac{1}{1-\beta} \sum_{s=t+1}^{\infty} \beta^{s-t} \mathbb{E}_t \left(r_{s-1} - \pi_s \right) \tag{A8}$$

Combining the Equation (A8) with the budget constraint (A4) yields

$$\omega_B \tilde{b}_{i,t}^* = \omega_B \tilde{b}_{i,t-1}^* + z_t - (1-\beta) \sum_{s=t}^{t+N} \beta^{s-t} \mathbb{E}_t \left[z_s \right] + \left(1 + \omega_W \frac{\gamma}{\eta}\right) \frac{1}{\gamma} \sum_{s=t+1}^{\infty} \beta^{s-t} \mathbb{E}_t \left(r_{s-1} - \pi_s \right) \tag{A9}$$

Note that the off-diagonal element of Θ_0 in Equation (A1) is non-zero, implying that a suboptimal bond holdings $b_{i,t}^*$ will affect the optimal consumption choice $c_{i,t}^*$ and vice versa. Moreover, the second term in Equation (A1) indicates that a suboptimal bond holding today will affect tomorrow's bond holding decisions. This intra-and inter-relationships complicate the problem. Therefore, similar to Maćkowiak and Wiederholt (2023), I do the following transformation such that I could express Equation (A1) as[29]

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_i^h \left[\frac{1}{2} (v_t - v_t^*)' \Theta_0 (v_t - v_t^*) + (v_t - v_t^*) \Theta_1 \left(v_{t+1} - v_{t+1}^* \right) \right]$$
$$= \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{i,-1} \left[\frac{1}{2} \left(x_{i,t} - x_{i,t}^* \right)' \Theta \left(x_{i,t} - x_{i,t}^* \right) \right] \tag{A10}$$

where instead of choosing directly $v_t = (\tilde{b}_{i,t}, c_{i,t})'$, I assume the household i chooses the a transformation of v_t:

$$x_{i,t} = \begin{pmatrix} \omega_B \left(\tilde{b}_{i,t} - \tilde{b}_{i,t-1} \right) \\ -\omega_B \left(\frac{1}{\beta} \tilde{b}_{i,t-1} - \tilde{b}_{i,t} \right) + \left(\gamma \frac{\omega_W}{\eta} + 1 \right) c_{i,t} \end{pmatrix}$$

And the Θ is diagonal, i.e., the suboptimal choice of the first element in $x_{i,t}$ will not affect the optimal choice of the second element in $x_{i,t}$

$$\Theta = -\bar{C}^{1-\gamma} \begin{bmatrix} \frac{\eta}{\omega_W} \left[1 - \frac{1}{\left(1+\omega_W \frac{\gamma}{\eta}\right)} \right] \frac{1}{\beta} & 0 \\ 0 & \frac{\eta}{\omega_W} \frac{1}{\left(1+\omega_W \frac{\gamma}{\eta}\right)} \end{bmatrix}$$

And the optimal choice of $x_{i,t}^*$ under full information is

$$x_{i,t}^* = \begin{pmatrix} z_t - (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t [z_s] + \frac{\beta}{\gamma} \left(1 + \omega_W \frac{\gamma}{\eta} \right) \sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t (i_s - \pi_{s+1}) \\ \omega_W \left(\frac{1}{\eta} + 1 \right) \tilde{w}_t + \left[\frac{1}{\beta} \omega_B (i_{t-1} - \pi_t) + \omega_D \tilde{d}_t + \omega_T \tilde{\tau}_t \right] \end{pmatrix}$$

B.2 Solution algorithm under rational inattention

In this economy, firms want to track their optimal price $p_{j,t}^*$ given by Equation (4.11), while households want to track their optimal $x_{i,t}^*$ given by Equation (4.9). It is evident from Equation (4.11) and (4.9) that the optimal actions are determined in the equilibrium. However, as these are Gaussian processes and by Wold's theorem, these processes can be decomposed into its $MA(\infty)$ representation, in particular,

$$p_{j,t}^* = \Phi_a(L) \varepsilon_t^a + \Phi_u(L) \varepsilon_t^u$$

$$x_{i,t}^* = \Psi_a(L) \varepsilon_t^a + \Psi_u(L) \varepsilon_t^u$$

where $\Phi_a(.)$, $\Phi_u(.)$, $\Gamma_a(.)$ and $\Gamma_u(.)$ are lag polynomials. However, to bypass the issue of unit root, follow Afrouzi and Yang (2021), I define $\tilde{\varepsilon}_t^u \equiv (1-L)^{-1} \varepsilon_t^u = \sum_{k=0}^{\infty} \varepsilon_{t-k}^u$. I re-write

[29] The proof for this is quite detailed and extensive, so for brevity, it hasn't been included in this appendix. However, I can provide it upon request.

the state-space representation as

$$p_{j,t}^{*} = \Phi_a(L)\varepsilon_t^a + \phi_u(L)\tilde{\varepsilon}_t^u$$

$$x_{i,t}^{*} = \Psi_a(L)\varepsilon_t^a + \psi_u(L)\tilde{\varepsilon}_t^u$$

where $\phi_u(L) = (1-L)\Phi_u(L)$ and $\psi_u(L) = (1-L)\Psi_u(L)$ are in l_2, and thus the processes can now be approximated arbitrarily precisely with truncation.

The equilibrium should be determined uniquely by the history of monetary shocks and productivity shocks. Define $\nu_t = (\varepsilon_t^a, \varepsilon_t^u)$ and $\tilde{\nu}_t = (\varepsilon_t^a, \tilde{\varepsilon}_t^u)$, and let $\vec{g}_t \equiv (\nu_t, \nu_{t-1}, \ldots, \nu_{t-(L+1)})$ and $\vec{\mathrm{g}}_t \equiv (\tilde{\nu}_t, \tilde{\nu}_{t-1}, \ldots, \tilde{\nu}_{t-(L+1)})$, with $\vec{g}_t = (I - \Lambda M')\,\vec{\mathrm{g}}_t$, where I is an identity matrix, Λ is a diagonal matrix with $\Lambda_{(2k,2k)} = 1$ and $\Lambda_{(2k-1,2k-1)} = 0$ for all $k = 1, 2, \ldots, L$, and M is a shift matrix. Note that the exogenous processes can be represented by

$$\begin{aligned} a_t &= H_a'\vec{\mathrm{x}}_t, \quad H_a' = (1, 0, \rho_a, 0, \rho_a^2, 0, \ldots, \rho_a^{L-1}, 0) \\ \varepsilon_t^u &= H_u'\vec{\mathrm{x}}_t, \quad H_u' = (0, 1, 0, 0, 0, 0, \ldots, 0, 0) \end{aligned}$$

The optimal price can be represented by $p_{j,t}^{*} \approx H_{p,(n)}'\vec{\mathrm{g}}_t$, the optimal action for households can be represented by $x_{i,t}^{*} \approx H_{x,(n)}'\vec{\mathrm{g}}_t$, and the objective is to iterate and to find the $H_{p,(n)}'$ and $H_{x,(n)}'$. In particular, given the guess $H_{(p,(n-1))}$ and $H_{(x,(n-1))}$, the optimal actions are

$$p_{j,t}^{*} = H_{(p,(n-1))}'\vec{\mathrm{g}}_t; \qquad x_{1,i,t}^{*} = H_{(x1,(n-1))}'\vec{\mathrm{g}}_t; \qquad x_{2,i,t}^{*} = H_{(x2,(n-1))}'\vec{\mathrm{g}}_t;$$

Here $x_{1,i,t}^{*}$ and $x_{2,i,t}^{*}$ denote the first and second element in the optimal action $x_{i,t}^{*}$. If the government debt are held constant, then it is optimal to pay no attention towards $x_{1,i,t}^{*}$ as the interest rate is determined such that there will be no change in the total bond holdings, and $x_{1,i,t}^{*} = 0$ for any shocks. However, if the government debt can absorb some of the fiscal imbalances, then households will pay attention to $x_{1,i,t}^{*}$. For simplicity, the derivation here considers the case where the bond is held constant.

Aggregating over firms and households, I get the aggregate price level and aggregate change in bond holdings and aggregate consumption. For example,

$$\begin{aligned} p_t = \int_0^1 p_{j,t}dj = H_{p,(n-1)}' \int_0^1 \mathbb{E}_{j,t}\left[\vec{\mathrm{g}}_t\right] dj &\approx H_{p,(n-1)}' \left[\sum_{k=0}^{\infty} \left[\left(I - K_{(n)}Y_{(n)}'\right) A\right]^k K_{(n)}Y_{(n)}'M'^k\right] \vec{\mathrm{g}}_t \\ &= H_{p,(n-1)}'X_{(n)}\vec{\mathrm{g}}_t \equiv H_p'\vec{\mathrm{g}}_t \end{aligned}$$

By same procedure, I get

$$x_{2,t} = \int_0^1 x_{2,i,t}di \approx H_{(x2,(n-1))}'Z_{(n)}\vec{\mathrm{g}}_t = H_{(x2)}'\vec{\mathrm{g}}_t$$

Follow directly, I get an expression for inflation, and total consumption.

$$\pi_t = H'_\pi \vec{g}_t = \left[H'_p(I-\Lambda M')^{-1}(I-M')\right]\vec{g}_t$$
$$c_t = H'_c \vec{g}_t = \frac{1}{\left(\gamma\frac{\omega_W}{\eta}+1\right)} H'_{x2}(I-\Lambda M')^{-1}\vec{g}_t$$

By the production function $y_t = a_t + l_t$ and goods market clears, the aggregate labor demand is $l_t = H'_l \vec{g}_t = (H_c - H_a)'\vec{g}_t$. And the interest rate is determined by the Taylor rule 4.5

$$i_t = H'_i \vec{g}_t = \left((1-\rho)\left(\phi_\pi H'_\pi + \phi_x\left(H'_c - \frac{1+\eta}{\gamma+\eta}H'_a\right)\right) + H'_u\right)(I-\rho M')^{-1}\vec{g}_t$$

I then solve for the implied representation for other variables in the model.

$$\omega_t = H'_\omega \vec{g}_t = \frac{\eta}{\omega_W}\left(-\frac{1}{\gamma}\left(1+\omega_W\frac{\gamma}{\eta}\right)(H'_i - H'_\pi M)(I-M')^{-1} - (H'_c - \omega_W H'_l)\right)\vec{g}_t$$
$$d_t = H'_d \vec{g}_t = \frac{1}{\omega_D}\left(H'_c - (1-\frac{1}{\theta})\omega_W(H'_\omega + H'_l)\right)\vec{g}_t$$
$$\tau_t = H'_\tau \vec{g}_t = \frac{1}{\omega_T}\left(-\frac{1}{\beta}\omega_B(H'_i M' - H'_\pi) - \frac{1}{\theta}\omega_W(H'_\omega + H'_l)\right)\vec{g}_t$$

Given these variables, I use Equation (4.11) and Equation (4.9) to update my guess for the MA processes of $H_{p,(n)}$ and $H_{x2,(n)}$

$$H_{p,(n)} = \left((H'_\omega + H'_p(I-\Lambda M')^{-1} - H'_a)(I-\Lambda M')\right)'$$
$$H_{x2,(n)} = \left(\left(\frac{1}{\beta}\omega_B(H'_i M' - H'_\pi) + \omega_D H'_d + \omega_T H'_\tau\right) + \omega_W(1+\frac{1}{\eta})H'_\omega\right)(I-\Lambda M')^{-1})'$$

I repeat above procedures until convergence of both $H_{p,(n)}$ and $H_{x2,(n)}$.

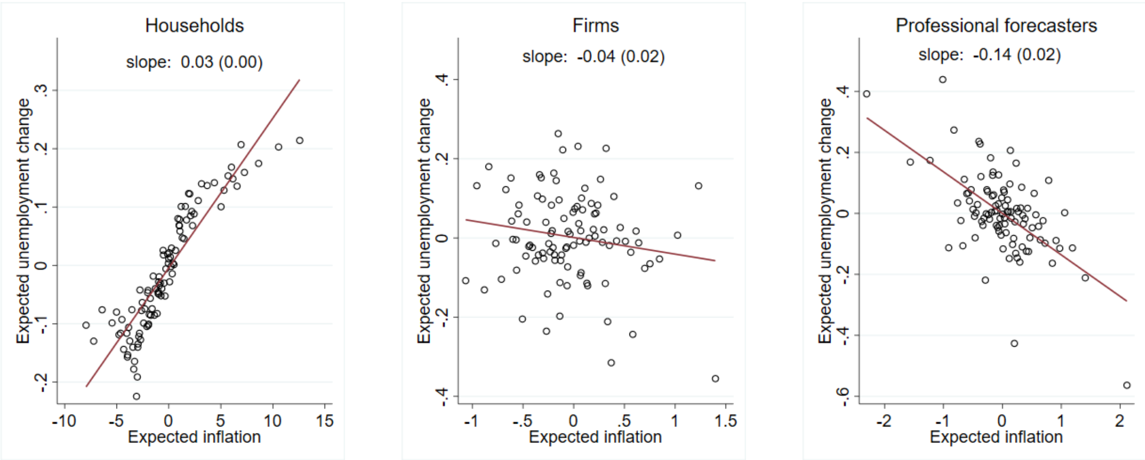
C Appendix Figure and Tables

Table A.1: Perceived Relationship between Inflation and Growth

	Growth Forecasts			
	Households		Firms	Professional forecasters
	Full Sample	Great Moderation		
Inflation Forecasts	-0.038^{***}	-0.034^{***}	0.039	0.109^{***}
	(0.001)	(0.001)	(0.020)	(0.023)
Observations	232,848	143,680	337	2,886
R^2	0.022	0.017	0.002	0.016

Note: The table provides statistics for the Figure 1.

Figure A.1: Correlation between expected inflation and expected unemployment change



Notes: Each panel plots a bin-scatter for the joint distribution of expectations for change in unemployment rate and inflation in the next year across different economic agents in the United States. For each variable, I take out the time fixed effect so that all variables are mean zero.
Data Sources: Michigan Survey of Consumers; The Livingston Survey; The Survey of Professional Forecasters.