Mid-term Evaluation
High level image understanding using logical and morphological approaches

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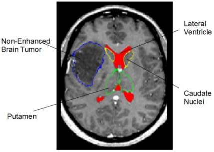
Outline

Motivation



- An abnormal structure is present in the brain.
- A peripheral non-enhanced tumor is present in the right hemisphere.

- A person pulling a pole towards the ground.
- A middle phase of pole vaulting.

[J. Atif *et al.*, 2014] Explanatory reasoning for image understanding using formal concept analysis and description logics, *IEEE Transactions on Systems, Man and Cybernetics, 2014*.
[S. Espinosa *et al.*, 2007] Multimedia Interpretation as Abduction, *20th International Workshop on Description Logics*.

Related work

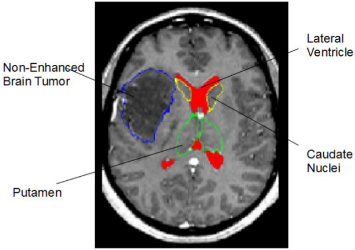
- Graph-based approaches
  - Bayesian network [S. Nikolopoulos *et al.*, 2009].
  - And-Or graph image grammar [F. Han *et al.*, 2009] and [K. Tu *et al.*, 2014].
- Logic-based approaches
  - Aggregation concepts (additional rules) [S. Espinosa *et al.*, 2007].
  - Knowledge representation based on formal concept analysis and reasoning based on morphological method [J. Atif *et al.*, 2014].

Our goal

- More expressive logic-based representation.
- Implicit information.
- Adaptive minimality criteria.

High level interpretation

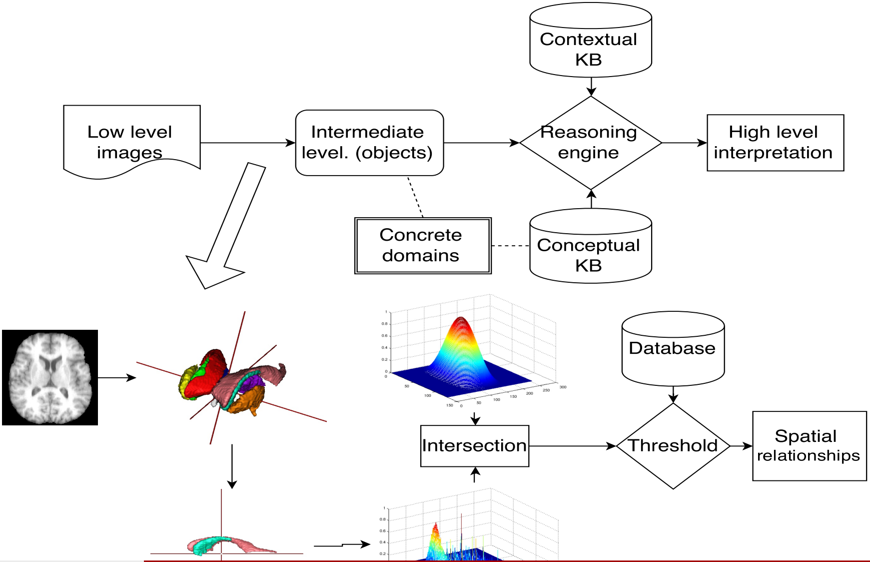
- Diagnostic problem of a pathological brain image.
- Extraction of semantic descriptions from a given image in application terminology.



Components

- Knowledge representation.
- Fuzzy representation.
- Qualitative spatial reasoning.
- Abductive reasoning.

General schema



Outline

Description Logics (Syntax)

Signature and constructors ($\mathcal{ALC}$)

$Sig = (N_C, N_R, N_I)$: sets of concepts, roles and individuals.

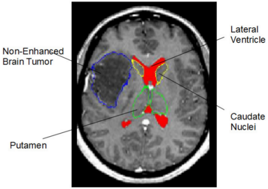
$Con = \{\neg\ (negation), \sqcap\ (conjunction), \sqcup\ (disjunction),$
$\exists\ (existential\ restriction), \forall\ (universal\ restriction).\}$

Example

$BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNI$

$\exists isPartOf.Brain \sqcap \neg BrainStructure$

$LeftHemisphere \sqcup RightHemisphere$



Description Logics (Semantics)

Semantics

An interpretation is a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$.

- $\Delta^{\mathcal{I}}$ is a domain (non-empty set).
- $\cdot^{\mathcal{I}}$ is a function that maps a concept to a subset of domain ($\Delta^{\mathcal{I}}$) and a role to a binary relation over the domain ($\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$).

Knowledge Base $\mathcal{K}$

Terminological Box ($\mathcal{T}$) and Assertional Box ($\mathcal{A}$). $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$

- A set of general inclusion axioms ($C \sqsubseteq D$).
- A set of assertional facts ($a : C$, $\langle a, b \rangle : r$).

Knowledge base example

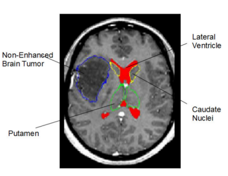
Example

$$
\begin{aligned}
TBox = \{ Hemisphere &\sqsubseteq \exists isPartOf.Brain \\
BrainStructure &\sqsubseteq \exists isPartOf.Brain \\
BrainDisease &\sqsubseteq \exists isPartOf.Brain \sqcap \neg BrainStructure \\
Tumor &\sqsubseteq BrainDisease \\
LVl &\sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl \\
LVr &\sqsubseteq BrainStructure \sqcap \exists(leftOf \sqcap closeTo).CNr \\
CNl &\sqsubseteq BrainStructure \\
CNr &\sqsubseteq BrainStructure
\end{aligned}
$$

Description of experts' background knowledge.

Representing the observation.

$$
\begin{aligned}
ABox = \{ a &: CNl \\
b &: Unknown\ Object \\
c &: Brain \\
\langle a, b \rangle &: leftOf, closeTo \\
\langle b, c \rangle &: isPartOf \}
\end{aligned}
$$



Need of qualitative spatial reasoning

- Qualitative information in human expression.
- Reliable information in brain images.

Constructor	Syntax	Semantics	Example
Atomic role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$	$leftOf$
Inverse role	r^{-}	$\{(x,y),\ x \in \Delta^{\mathcal{I}},\ y \in \Delta^{\mathcal{I}} \mid (y,x) \in r^{\mathcal{I}}\}$	$leftOf^{-}$
Role negation	$\neg r$	$\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus r^{\mathcal{I}}$	$\neg leftOf$
Role composition	$r_1 \circ r_2$	$\{(x,z), x \in \Delta^{\mathcal{I}},\ z \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, (x,y) \in r_1^{\mathcal{I}} \text{ and } (y,z) \in r_2^{\mathcal{I}}\}$	$isPartOf \circ isPartOf$
Role conjunction	$r_1 \sqcap r_2$	$r_1^{\mathcal{I}} \cap r_2^{\mathcal{I}}$	$leftOf \sqcap closeTo$
Role disjunction	$r_1 \sqcup r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$	$closeTo \sqcup farFrom$
Role inclusion	$r_1 \sqsubseteq r_2$	$r_1^{\mathcal{I}} \subseteq r_2^{\mathcal{I}}$	$adjacent \sqsubseteq closeTo$
Role equivalence	$r_1 \equiv r_2$	$r_1^{\mathcal{I}} = r_2^{\mathcal{I}}$	$leftOf \equiv rightOf^{-}$

Proposition ($\mathcal{ALCHI}_{\mathcal{R}^+}$)

- Inverse relation: $leftOf \equiv rightOf^{-}$, $hasPart \equiv isPartOf^{-}$
- Transitive relation: $isPartOf \circ isPartOf \sqsubseteq isPartOf$
- Symmetric relation: $closeTo \equiv closeTo^{-}$

Description Logics (Reasoning services)

- Subsumption checking: $\mathcal{T} \models C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model of $\mathcal{I}$ of $\mathcal{T}$.
- Concept satisfiability: C is satisfiable with respect to $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}} \neq \emptyset$.

$$\mathcal{T} \models C \sqsubseteq D \quad \textit{iff} \quad \mathcal{T} \not\models C \sqcap \neg D$$

Illustration of QSR

Definition (Most specific concept)

Given a TBox $\mathcal{T}$ and an associated interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ in a DL $\mathcal{L}$, let $X \subseteq \Delta^{\mathcal{I}}$ be a subset of the interpretation space and E a defined concept in $\mathfrak{C}(\mathcal{L})$. The concept E is defined as the most specific concept of X w.r.t. $\mathcal{I}$ if:

- $X \subseteq E^{\mathcal{I}}$.
- for every defined concept $F \in \mathfrak{C}(\mathcal{L})$ with $X \subseteq F^{\mathcal{I}}$, we have $E \sqsubseteq_{\mathcal{T}} F$.

$O \equiv \exists(leftOf^{-} \sqcap closeTo^{-}).CNl \sqcap \exists isPartOf.Brain$

$$H \equiv LVl \sqcap \exists isPartOf.Hemisphere$$

$$\mathcal{T} \models H \sqsubseteq O \quad iff \quad \mathcal{T} \not\models H \sqcap \neg O$$

$$LVl \sqcap \exists isPartOf.Hemisphere \sqcap \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl \sqcup \forall isPartOf.\neg Brain$$

Tableau method

Tableau method

- Starts from $\mathcal{L}(x) = \{C\}$ with expansion rules to construct a model of C where
  - x is an element of interpretation of the concept C to test,
  - $\mathcal{L}(x)$ is a set of concepts and $\mathcal{E}(\langle x, y \rangle)$ is a set of roles.
- Terminates when
  - a *clash* occurs if $\{D, \neg D\}$ exists in a $\mathcal{L}(\cdot)$,
  - no rules can be applied.

Expansion rules

- Conjunction
  - if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and if $\{C_1, C_2\} \nsubseteq \mathcal{L}(x)$, then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{C_1, C_2\}$.
- Existential Quantification
  - if $\exists r.C \in \mathcal{L}(x)$ and x has no successor y with $C \notin \mathcal{L}(y)$, then create a new node y with $\mathcal{E}(\langle x, y \rangle)$ and $\mathcal{L}(y) = \{C\}$
- ...

Tableau method (example)

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ BrainStructure,\ \exists(rightOf \sqcap closeTo).CNl\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ BrainStructure,\ \exists(rightOf \sqcap closeTo).CNl\}$$

$$\mathcal{L}(y) = \{CNl,\ \neg CNl\}$$

$$\mathcal{E}(\langle x, y\rangle) = \{rightOf, closeTo, leftOf^-, closeTo^-\}$$

$$\mathcal{E}(\langle y, x\rangle) = \{rightOf^-, closeTo^-, leftOf, closeTo\}$$

$$\boxtimes$$

Outline

Abductive reasoning

- Inference to the best explanation.

Abductive reasoning for image interpretation

Given a knowledge base $\mathcal{K}$ and an observation concept $\mathcal{O}$, the hypothesis $\mathcal{H}$ is an explanation of $\mathcal{O}$ if

$$\mathcal{K} \models \mathcal{H} \sqsubseteq \mathcal{O}$$

Satisfiability test

$$\mathcal{K} \not\models \mathcal{H} \sqcap \neg\mathcal{O}$$

Strategy

$$\mathcal{H} \sqcap \neg\mathcal{O} \quad (\textit{unsatisfiable}?)$$

An example application

$$
\begin{aligned}
\mathcal{T} = \{\ & \mathit{SmallDeformingTumor} \sqsubseteq \mathit{BrainTumor} \\
& \sqcap \exists \mathit{hasEnhancement}.\mathit{NonEnhanced} \\
& \sqcap \exists \mathit{hasBehavior}.\mathit{Infiltrating} \\
& \mathit{PeripheralDeformingTumor} \sqsubseteq \mathit{BrainTumor} \\
& \sqcap \exists \mathit{farFrom}.\mathit{LateralVentricle} \\
& \sqcap \exists \mathit{hasLocation}.\mathit{PeripheralCerebralHemisphere} \}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O} = \{\ & \exists \mathit{hasEnhancement}.\mathit{NonEnhanced} \\
& \sqcap \exists \mathit{farFrom}.\mathit{LateralVentricle} \\
& \sqcap \exists \mathit{hasLocation}.\mathit{PeripheralCerebralHemisphere} \}
\end{aligned}
$$

Knowledge Integration

Definition (Internalized concept)

Let $\mathcal{T}$ be a TBox and a set of axioms formulated as $C_i \sqsubseteq D_i$. The internalized concept of the TBox is defined as follows:

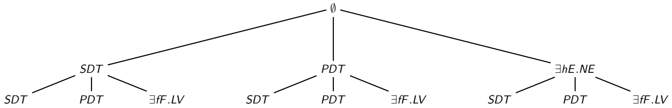
$$C_{\mathcal{T}} \equiv \sqcap_{(C_i \sqsubseteq D_i \in \mathcal{T})}(\neg C_i \sqcup D_i)$$

$$\begin{aligned} C_{\mathcal{T}} \sqcap \neg \mathcal{O} = \{ & (\neg SDT \sqcup (BT \sqcap \exists hE.NE \sqcap \exists hB.In)) \sqcap \\ & (\neg PDT \sqcup (BT \sqcap \exists fF.LV \sqcap \exists hL.PCH)) \sqcap \\ & (\forall hE.\neg NE \sqcup \forall fF.\neg LV \sqcup \forall hL.\neg PCH)\} \end{aligned}$$

Minimal hitting set

Six branches are open in the tableau at the end. In each one, the set of concepts that cannot be decomposed are:

$$
\begin{aligned}
H_1 &= \{SDT, PDT, \exists hE.NE\} \\
H_2 &= \{SDT, PDT, \exists fF.LV\} \\
H_3 &= \{SDT, PDT, \exists hL.PCH\} \\
H_4 &= \{SDT, \neg BT, \forall fF.\neg LV, \forall hL.\neg PCH, \exists hE.NE\} \\
H_5 &= \{PDT, \neg BT, \forall hE.\neg NE, \forall hB.\neg In, \exists fF.LV\} \\
H_6 &= \{PDT, \neg BT, \forall hE.\neg NE, \forall hB.\neg In, \exists hL.PCH\}
\end{aligned}
$$



Minimality criteria

Selecting the preferred explanation

- Consistency: $\mathcal{K} \cup H$ is consistent. e.g. $SDT \sqcap \neg BT$, $PDT \sqcap \forall fF \neg LV$.
- Relevance: $\mathcal{O}$ is not entailed by H ($H \nvDash \mathcal{O}$). e.g. irrelevant hypothesis: $\exists hE.NE \sqcap \exists fF.LV \sqcap \exists hL.PCH$
- Semantic minimality: $\nexists$ H_i such that $H_i \vDash H$.
  For an abduction problem $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$, and $\{\langle P_1, \ldots, P_n \rangle\}$ potential hypotheses,
  - P_i is a $\sqsubseteq -minimal$ explanation if there does not exist an explanation P_j for $\mathcal{P}$ such that $P_i \sqsubseteq P_j$.

Explanations

- $PeripheralDeformingTumor \sqcap \exists hasEnhancement.NonEnhanced$
- $SmallDeformingTumor \sqcap \exists farFrom.LateralVentricle$
  $\sqcap \exists hasLocation.PeripheralCerebralHemisphere$

Outline

Conclusions and perspectives

Conclusion

- A complete image interpretation system.
- A more expressive Description Logic language.
- Adaptive tableau method for image interpretation.

Perspectives

- Abductive reasoning.
  - Generation of hypotheses (Iterative way).
  - Selection of the "best" hypothesis with a criterion.
- Concrete domain.
- Fuzzy logic.

Q & A

Thank you
Questions?

Extra tableau

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\ \neg Hemisphere \sqcup \exists isPartOf.Brain\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain,\ \forall isPartOf.\neg Brain,\ \neg Hemisphere\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\ \exists isPartOf.Brain\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\ \exists isPartOf.Brain\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$
$$\mathcal{L}(z) = \{Brain, \neg Brain, \forall isPartOf.\neg Brain\}$$
$$\mathcal{E}(\langle y, z\rangle) = \{isPartOf\}$$

$$\boxtimes$$