Mid-term Evaluation High level image understanding using logical and morphological approaches

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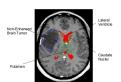






- Introduction
- 2 Knowledge representation and reasoning
- 3 Abductive reasoning
- Conclusion and perspectives







- An abnormal structure is present in the brain.
- A peripheral non-enhanced tumor is present in the right hemisphere.
- A person pulling a pole towards the ground.
- A middle phase of pole vaulting.

[J. Atif et al., 2014] Explanatory reasoning for image understanding using formal concept analysis and description logics, IEEE Transactions on Systems, Man and Cybernetics, 2014.

[S. Espinosa et al., 2007] Multimedia Interpretation as Abduction, 20th International Workshop on Description Logics.





- Graph-based approaches
 - Bayesian network [S. Nikolopoulos et al., 2009].
 - And-Or graph image grammar [F. Han et al., 2009] and [K. Tu et al., 2014].
- Logic-based approaches
 - Aggregation concepts (additional rules) [S. Espinosa et al., 2007].
 - Knowledge representation based on formal concept analysis and reasoning based on morphological method [J. Atif et al., 2014].

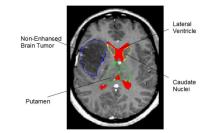
Our goal

- More expressive logic-based representation.
- Implicit information.
- Adaptive minimality criteria.



High level interpretation

- Diagnostic problem of a pathological brain image.
- Extraction of semantic descriptions from a given image in application terminology.



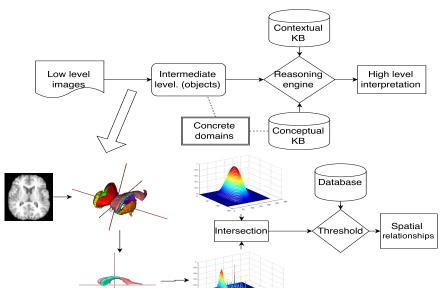
Components

- Knowledge representation.
- Fuzzy representation.
- Qualitative spatial reasoning.
- Abductive reasoning.

General schema



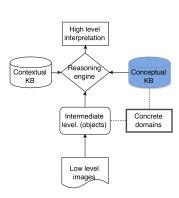








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Description Logics (Syntax)



Signature and constructors (ALC)

 $Sig = (N_C, N_R, N_I)$: sets of concepts, roles and individuals. $Con = \{\neg (negation), \neg (conjunction), \cup (disjunction), \exists (existential restriction), \forall (universal restriction).\}$

Example

 $BrainStructure \sqcap \exists (rightOf \sqcap closeTo).CNI \exists isPartOf.Brain \sqcap \neg BrainStructure$ $LeftHemisphere \sqcup RightHemisphere$





Description Logics (Semantics)

Semantics

An interpretation is a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$.

- $\Delta^{\mathcal{I}}$ is a domain (non-empty set).
- $\cdot^{\mathcal{I}}$ is a function that maps a concept to a subset of domain $(\Delta^{\mathcal{I}})$ and a role to a binary relation over the domain $(\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}})$.

Knowledge Base ${\mathcal K}$

Terminological Box (\mathcal{T}) and Assertional Box (\mathcal{A}) . $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$

- A set of general inclusion axioms ($C \sqsubseteq D$).
- A set of assertional facts $(a : C, \langle a, b \rangle : r)$.



Knowledge base example



Example

```
TBox = \{Hemisphere \sqsubseteq \exists isPartOf.Brain \ BrainStructure \sqsubseteq \exists isPartOf.Brain \ BrainDisease \sqsubseteq \exists isPartOf.Brain \sqcap \neg BrainStructure \ Tumor \sqsubseteq BrainDisease \ LVI \sqsubseteq BrainStructure \sqcap \exists (rightOf \sqcap closeTo).CNI \ LVr \sqsubseteq BrainStructure \sqcap \exists (leftOf \sqcap closeTo).CNr
```

 $CNI \sqsubseteq BrainStructure$ $CNr \sqsubseteq BrainStructure$ Description of experts' background knowledge.

Description of the observation.

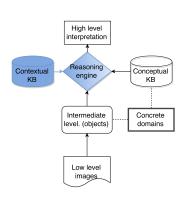
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ABox = \{a : CNI \\ b : Unknown \ Object \\ c : Brain \\ \langle a,b \rangle : leftOf, closeTo \\ \langle b,c \rangle : isPartOf\}
```







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Qualitative spatial representation (QSR)



Need of qualitative spatial reasoning

- Qualitative information in human expression.
- Reliable information in brain images.

Constructor	Syntax	Semantics	Example
Atomic role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} imes \Delta^{\mathcal{I}}$	leftOf
Inverse role	r ⁻	$\{(x,y), x \in \Delta^{\mathcal{I}}, y \in \Delta^{\mathcal{I}} (y,x) \in r^{\mathcal{I}}\}$	leftOf [—]
Role negation	$\neg r$	$\Delta^{\mathcal{I}} imes \Delta^{\mathcal{I}} \setminus r^{\mathcal{I}}$	¬leftOf
Role composition	$r_1 \circ r_2$	$\left \{(x,z), x \in \Delta^{\mathcal{I}}, \ z \in \Delta^{\mathcal{I}} \exists y \in \Delta^{\mathcal{I}}, (x,y) \in r_1^{\mathcal{I}} \ \text{and} \ (y,z) \in r_2^{\mathcal{I}} \} \right $	isPartOf ∘ isPartOf
Role conjunction	$r_1 \sqcap r_2$	$r_1^\mathcal{I}\cap r_2^\mathcal{I}$	leftOf □ closeTo
Role disjunction	$r_1 \sqcup r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$	closeTo ⊔ farFrom
Role inclusion	$r_1 \sqsubseteq r_2$	extstyle ext	adjacent ⊑ closeTo
Role equivalence	$r_1 \equiv r_2$	$r_1^{\mathcal{I}} = r_2^{\mathcal{I}}$	$leftOf \equiv rightOf^-$

Proposition $(\mathcal{ALCHI}_{\mathcal{R}^+})$

- Inverse relation: $leftOf \equiv rightOf^-$, $hasPart \equiv isPartOf^-$
- Transitive relation: *isPartOf isPartOf* □ *isPartOf*
- Symmetric relation: closeTo ≡ closeTo⁻



Description Logics (Reasoning services)



- Subsumption checking: $\mathcal{T} \models C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model of \mathcal{I} of \mathcal{T} .
- Concept satisfiability: C is satisfiable with respect to $\mathcal T$ if there exists a model $\mathcal I$ of $\mathcal T$ such that $C^{\mathcal I} \neq \emptyset$.

$$\mathcal{T} \models C \sqsubseteq D$$
 iff $\mathcal{T} \not\models C \sqcap \neg D$

Definition (Most specific concept)

Given a TBox \mathcal{T} and an associated interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ in a DL \mathcal{L} , let $X \subseteq \Delta^{\mathcal{I}}$ be a subset of the interpretation space and E a defined concept in $\mathfrak{C}(\mathcal{L})$. The concept E is defined as the most specific concept of X w.r.t. \mathcal{I} if:

- \bullet $X \subseteq E^{\mathcal{I}}$.
- for every defined concept $F \in \mathfrak{C}(\mathcal{L})$ with $X \subseteq F^{\mathcal{I}}$, we have $E \sqsubseteq_{\mathcal{T}} F$.

$$O \equiv \exists (\textit{leftOf}^- \sqcap \textit{closeTo}^-).\textit{CNI} \sqcap \exists \textit{isPartOf}.\textit{Brain}$$

$$H \equiv LVI \sqcap \exists isPartOf.Hemisphere$$

$$\mathcal{T} \vDash H \sqsubseteq O \text{ iff } \mathcal{T} \nvDash H \sqcap \neg O$$

 $LVI \sqcap \exists isPartOf. Hemisphere \sqcap \forall (leftOf ^ \sqcap closeTo ^). \neg CNI \sqcup \forall isPartOf. \neg Brain$



Tableau method

- Starts from $\mathcal{L}(x) = \{C\}$ with expansion rules to construct a model of C where
 - x is an element of interpretation of the concept C to test,
 - $\mathcal{L}(x)$ is a set of concepts and $\mathcal{E}(\langle x, y \rangle)$ is a set of roles.
- Terminates when
 - a clash occurs if $\{D, \neg D\}$ exists in a $\mathcal{L}(\cdot)$,
 - no rules can be applied.

Expansion rules

- Conjunction
 - if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and if $\{C_1, C_2\} \not\subset \mathcal{L}(x)$, then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{C_1, C_2\}$.
- Existential Quantification
 - if $\exists r. C \in \mathcal{L}(x)$ and x has no successor y with $C \notin \mathcal{L}(y)$, then create a new node y with $\mathcal{E}(\langle x, y \rangle)$ and $\mathcal{L}(y) = \{C\}$
-



Tableau method (example)

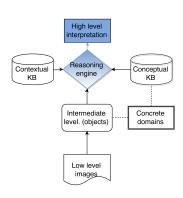


```
\mathcal{L}(x) = \{LVI \cap \exists isPartOf. Hemisphere \cap (\forall (leftOf \cap \neg closeTo^-). \neg CNI \cup \forall isPartOf. \neg Brain)\}
           \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere,
                                                                                             \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere,
                  \forall (leftOf^- \sqcap closeTo^-). \neg CNI,
                                                                                                          ∀isPartOf.¬Brain}
\neg LVI \sqcup (BrainStructure \sqcap \exists (rightOf \sqcap closeTo).CNI))
           \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere,
                  \forall (leftOf^- \sqcap closeTo^-), \neg CNI.
                            BrainStructure.
                   \exists (rightOf \sqcap closeTo).CNI \}
           \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere,
                  \forall (leftOf^- \sqcap closeTo^-). \neg CNI,
                            BrainStructure.
                   \exists (rightOf \sqcap closeTo).CNI \}
                      \mathcal{L}(y) = \{CNI, \neg CNI\}
  \mathcal{E}(\langle x, y \rangle) = \{ rightOf, closeTo, leftOf^-, closeTo^- \}
  \mathcal{E}(\langle v, x \rangle) = \{ rightOf^-, closeTo^-, leftOf, closeTo \}
```





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Abductive reasoning



• Inference to the best explanation.

Abductive reasoning for image interpretation

Given a knowledge base $\mathcal K$ and an observation concept $\mathcal O$, the hypothesis $\mathcal H$ is an explanation of $\mathcal O$ if

$$\mathcal{K} \models \mathcal{H} \sqsubset \mathcal{O}$$

Satisfiability test

$$\mathcal{K} \nvDash \mathcal{H} \sqcap \neg \mathcal{O}$$

Strategy

$$\mathcal{H} \sqcap \neg \mathcal{O}$$
 (unsatisfiable?)



An example application



```
\mathcal{T} = \{ SmallDeformingTumor \sqsubseteq BrainTumor \}
                                    □ ∃hasEnhancement.NonEnhanced
                                    \sqcap \exists hasBehavior.Infiltrating
   Peripheral Deforming Tumor \sqsubseteq Brain Tumor
                                    \sqcap \exists farFrom.LateralVentricle
                                    □ ∃hasLocation.PeripheralCerebralHemisphere}
            \mathcal{O} = \{ \exists hasEnhancement.NonEnhanced \}
                    \sqcap \exists farFrom. I ateral Ventricle
                    □ ∃hasLocation.PeripheralCerebralHemisphere}
```



Knowledge Integration

Definition (Internalized concept)

Let \mathcal{T} be a TBox and a set of axioms formulated as $C_i \sqsubseteq D_i$. The internalized concept of the TBox is defined as follows:

$$C_{\mathcal{T}} \equiv \sqcap_{(C_i \sqsubseteq D_i \in \mathcal{T})} (\neg C_i \sqcup D_i)$$

$$C_{\mathcal{T}} \sqcap \neg \mathcal{O} = \{ (\neg SDT \sqcup (BT \sqcap \exists hE.NE \sqcap \exists hB.In)) \sqcap \\ (\neg PDT \sqcup (BT \sqcap \exists fF.LV \sqcap \exists hL.PCH)) \sqcap \\ (\forall hE.\neg NE \sqcup \forall fF.\neg LV \sqcup \forall hL.\neg PCH) \}$$



Generating potential hypotheses



Six branches are open in the tableau at the end. In each one, the set of concepts that cannot be decomposed are:

$$\begin{split} H_1 &= \{SDT, PDT, \exists hE.NE\} \\ H_2 &= \{SDT, PDT, \exists fF.LV\} \\ H_3 &= \{SDT, PDT, \exists hL.PCH\} \\ H_4 &= \{SDT, \neg BT, \forall fF. \neg LV, \forall hL. \neg PCH, \exists hE.NE\} \\ H_5 &= \{PDT, \neg BT, \forall hE. \neg NE, \forall hB. \neg In, \exists fF.LV\} \\ H_6 &= \{PDT, \neg BT, \forall hE. \neg NE, \forall hB. \neg In, \exists hL.PCH\} \end{split}$$

Minimal hitting set (Complexity: NP-hard.)





Minimality criteria

Selecting the preferred explanation

- Consistency: $\mathcal{K} \cup H$ is consistent. e.g. $SDT \sqcap \neg BT$, $PDT \sqcap \forall fF \neg LV$.
- Relevance: \mathcal{O} is not entailed by H ($H \not\models \mathcal{O}$). e.g. irrelevant hypothesis: $\exists hE.NE \sqcap \exists fF.LV \sqcap \exists hL.PCH$
- Semantic minimality: \nexists H_i such that $H_i \models H$. For an abduction problem $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$, and $\{\langle P_1, \dots, P_n \rangle\}$ potential hypotheses,
 - P_i is a \sqsubseteq -minimal explanation if there does not exist an explanation P_j for \mathcal{P} such that $P_i \sqsubseteq P_j$.

Explanations

- PeripheralDeformingTumor □ ∃hasEnhancement.NonEnhanced
- SmallDeformingTumor □ ∃farFrom.LateralVentricle □∃hasLocation.PeripheralCerebralHemisphere



Conclusions and perspectives

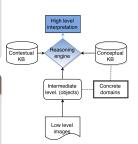


Conclusion

- A complete image interpretation system.
- A more expressive Description Logic language.
- Adaptive tableau method for image interpretation.

Perspectives

- Abductive reasoning.
 - Generation of hypotheses (Iterative way).
 - Selection of the "best" hypothesis with a criterion.
- Concrete domain.
- Fuzzy logic.







Thank you Questions?



Extra tableau



```
\mathcal{L}(x) = \{LVI \cap \exists isPartOf. Hemisphere \cap (\forall (leftOf \cap \neg closeTo^-). \neg CNI \cup \forall isPartOf. \neg Brain)\}
            \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere,
                                                                            \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere.\}
                  \forall (leftOf^- \sqcap closeTo^-), \neg CNI.
                                                                                          ∀isPartOf.¬Brain}
\neg LVI \sqcup (BrainStructure \sqcap \exists (rightOf \sqcap closeTo), CNI)
                                                                           \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere.
                                                                                          ∀isPartOf.¬Brain}
                                                                    \mathcal{L}(v) = \{ Hemisphere, \neg Brain, \forall isPartOf, \neg Brain, \}
                                                                               \neg Hemisphere \sqcup \exists isPartOf.Brain
                                                                                      \mathcal{E}(\langle x, y \rangle) = \{ isPartOf \}
                                            \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere,
                                                                                                            \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere,
                                                          ∀isPartOf.¬Brain}
                                                                                                                          ∀isPartOf.¬Brain}
                                                  \mathcal{L}(y) = \{Hemisphere, \neg Brain,
                                                                                                    \mathcal{L}(v) = \{Hemisphere, \neg Brain, \forall isPartOf. \neg Brain, \}
                                                          ∀isPartOf.¬Brain.
                                                                                                                           ∃isPartOf .Brain}
                                                             ¬Hemisphere}
                                                                                                                      \mathcal{E}(\langle x, y \rangle) = \{ isPartOf \}
                                                      \mathcal{E}(\langle x, v \rangle) = \{ isPartOf \}
                                                                                                            \mathcal{L}(x) = \{LVI, \exists isPartOf.Hemisphere.
                                                                                                                          ∀isPartOf.¬Brain}
                                                                                                    \mathcal{L}(v) = \{ Hemisphere, \neg Brain, \forall isPartOf, \neg Brain. \}
                                                                                                                           ∃isPartOf .Brain}
                                                                                                                      \mathcal{E}(\langle x, y \rangle) = \{ isPartOf \}
                                                                                                        \mathcal{L}(z) = \{Brain, \neg Brain, \forall isPartOf. \neg Brain\}
                                                                                                                      \mathcal{E}(\langle v, z \rangle) = \{ isPartOf \}
```