High-level image interpretation using logical and morphological approaches

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1 Introduction

High-level semantics extraction from an image is an interesting research area for automatic image understanding in artificial intelligence. Many related fields like image annotation, activity recognition and decision-support systems take advantage of semantic content. As advanced as AI has become, it still remains a big challenge for computers to accomplish complex understanding tasks as humans do. Digital image itself is a numerical representation which does not represent explicitly semantic information. Moreover, beyond a single object understanding based on low level features such as colors and forms, we focus on a complex description which relies on context information like spatial relations between diverse objects as well as prior knowledge on the application domain. For instance, in the context of medical applications, the understanding task can be formulated as giving an abstract description of a pathological brain volume, such as in Figure 1. According to different levels of anatomical prior knowledge on brain pathology, two possible descriptions could be given:

- an abnormal structure is present in the brain,
- a peripheral non-enhanced tumor is present in the right hemisphere.

In this thesis, a high-level interpretation is regarded as an explanation of what we have seen in the image. This process is an inference based on prior knowledge to link the abstract description and the observed context of the scene.

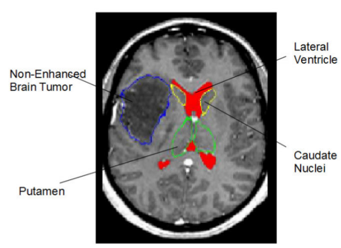

Figure 1: A slice of a pathological brain volume (MRI acquisition), where some structures are annotated.

1.1 Problem formulation

According to the objective pointed out in the previous part, the aim is to extract high-level semantic information from a given image and translate it at a linguistic level. Concretely, we are interested in the interpretation of cerebral images with tumors. The high-level information corresponds to the presence of diverse types of pathologies as well as descriptions of brain structures and spatial relations among them in a brain image. In the context of this thesis, the decision process is modeled as an abductive reasoning [2] using a logical formalism, which is an inference mechanism from facts to explanations. The objective of this thesis is to build a generic logic based formalism as well as to develop an appropriate reasoning process for image interpretation, allowing us to extract a set of suitable candidates as potential hypotheses for a given image and to select the “best” one by a defined criterion. In image interpretation, spatial relationships are important when objects of similar appearance are present in the image, especially in magnetic resonance imaging (MRI). Such relationships have then to be included in the representation and in the reasoning process.

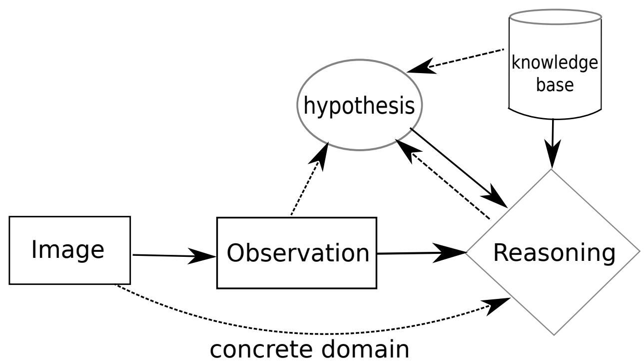

Figure 2: A general schema of image interpretation task in the thesis.

I will think about it and improve this part and the schema for the next version. Figure 2 shows the major components of our framework in this thesis. The given image is translated into symbolic representations in terms of logical form at the beginning. The image can also serve as the concrete domain in both the knowledge base and the reasoning process. Concrete domain is used as a real model to represent abstract terminologies in image space. A hypothesis of an description might be generated from the observation or the axioms in the knowledge base. The relations between the hypothesis and reasoning are two directions, which allows validating the hypothesis with the help of standard reasoning and building a possible hypothesis within non-standard reasoning processes.

To summarize the ongoing and future work, we need to answer the following questions:

- *How to model knowledge and formalize an appropriate representation in a given application domain? (Section 2)*
- *How to connect image level representation and symbolic level representation? (Section 3)*
- *How to overcome the semantic gap between numerical representation and qualitative representation of spatial relationships? (Section 3)*
- *How to generate hypotheses to explain the observed scene? (Section 4 and Section 5)*
- *How to define a criterion to choose a “best” explanation in our case? (Section 4 and Section 5)*

1.2 Related work

Recognition of perceptual objects and scene understanding, which translate low level signal information into meaningful semantic information, belong to one of the fundamental abilities of human beings. Semantics is important in image analysis, for various tasks such as image annotation [43], event detection [31] and diagnostic problems [3, 4]. In some specific domains, like medical imaging [3, 9, 18, 34] and remote sensing [17, 47], image interpretation combines image processing with artificial intelligence techniques to derive reasonable semantics. Prior knowledge is intensively used by experts who interpret visually an image. Evidently it should then also be used by machines to associate semantics with the image. However, image interpretation still faces some difficulties, one of which is how to accurately associate perceptual data with appropriate concepts. Without an expert knowledge, such a link cannot be established. This relation between visual perception and high-level linguistic expression is called *semantic gap* [25].

As a high level process of exploiting semantic in the scene, image interpretation involves two levels:

- relating low level features to semantics (from pixels to semantic information) [9, 18, 26, 34].
- inferring the description from the semantic image content (from semantics to explanation) [3, 16].

Roughly speaking, the first level describes what is happening while the second one describes how it is happening [45]. The first level has been mainly studied in the field of multiple objects recognition. Image interpretation maps regions or groups of regions onto labels corresponding to semantic concepts (e.g. labels of anatomical structures for medical images). Various approaches employ Bayesian networks with a combination of semantics and probabilistic inference mechanisms [32, 37, 41]. These techniques provide inference mechanisms by attempting to construct co-occurrence objects and contextual information with a probabilistic model for reasoning.

Further, a hierarchical representation of knowledge base is proposed, called image grammar [46, 51]. The grammar is a structured knowledge represented by an And-Or graph. In this graph, a global description of a scene is decomposed into parts, objects until primitive pixel patches from top to bottom. An And-node consists of a set of successive components and an Or-node is composed by alternative nodes. A parsing method is proposed as inference within a probabilistic model in each node [23, 50].

The second level consists in reasoning at the language (knowledge) level. For the purpose of giving an adequate explanation, the second level is a logic-based reasoning to depict the image with a deep and abstract description from the point of view of an expert. There is not much work on image interpretation using logical knowledge representation and reasoning. However, formal language based on logic formalism has strong associated semantics for knowledge representation as well as reasoning processes. An aggregation concept is proposed in [16] to represent a complex event or scene concerning occurrence objects, as well as spatial and temporal constraints configuration. According to these defined aggregation and specific rules, a high-level interpretation is able to be inferred [35]. In addition, the results using Bayesian networks and image grammars are limited to defined descriptions. A complex description can also be generated when non-explicitly presented in the knowledge base [3]. At this level, a high-level semantics, required both background knowledge and contextual information, can be reasoned out from explicit observation in an image. The high-level semantics in terms of a complex description within an expressive logic formalism is still an open problem for image interpretation. In this thesis, we are interested in proposing an adapted logical formalism for the knowledge representation as well as a specific reasoning method for image interpretation inference.

2 Preliminaries

2.1 Ontology

Experts' knowledge is expressed in terms of diverse vocabulary of special domain in natural language which is difficult to be interpreted by machines. In order to facilitate an automated reasoning process with a background knowledge base, a structural semantic based model is an effective means to represent prior knowledge. The term ontology is derived from philosophy and then used for the purpose of expressing common sense knowledge in computer science [1]. Since then, ontologies were adopted for image interpretation tasks [7, 26, 44]. Ontologies are defined as "*a formal specification of a shared conceptualization*" [42], which deal with modeling a universal and reusable knowledge among different applications for a specific domain. Ontologies were also studied for reasoning service within its expressive formal description. An ontology mainly contains *individuals*, *concepts*, *properties* and *axiom rules*. These components enable the background knowledge to be understood and processable by machines.

2.2 Description Logics

As mentioned above, ontologies require a formal representation language and well-defined semantics for reasoning services. Description Logics (DLs) is a family of knowledge representation logical formalisms, which is seen as good candidates for ontologies [24]. The basic elements of Description logics are concepts (unary predicates), roles (binary predicates) and individuals. Besides the formal knowledge representation, another important feature of DLs is their ability of reasoning. Implicit information can be inferred from explicit knowledge description, such as satisfiability checking [5]. In this part, we introduce syntax and semantics of a description logic language $\mathcal{ALC}$ as well as its reasoning services.

2.2.1 Syntax and semantics

We first recall the syntax and semantics of the basic language of Description Logics ($\mathcal{ALC}$) [5].

Definition 1 (Signature)**.** *The syntax of a Description Logic is defined over a signature, which is defined as three disjoint sets $Sig = (N_C,\ N_R,\ N_I)$. N_C is a set of concept names that refers to a set of entities with the same characteristics. N_I is a set of individuals that contains instances of the concepts in N_C. N_R is a set of role names that refers to the binary relationships between two individuals or two concepts.*

Definition 2 (Concept expression)**.** *The set of concept expression is recursively built from the signature as follows:*

- *all the concept names, as well as $\top$ (top concept) and $\bot$ (bottom concept) are concepts,*
- *if C and D are two concepts in N_C and r is a role in N_R then $\neg C$ (negation), $C \sqcap D$ (conjunction), $C \sqcup D$ (union), $\exists r.C$ (existential quantification), $\forall r.C$ (universal quantification) are also concepts.*

Let $\mathfrak{C}$ be the infinite set of all the concepts that can be defined using constructors and signature elements.

Definition 3 (Terminological box (TBox) and assertional box (ABox))**.** *A general concept inclusion axiom (GCI) is an expression of the form $C \sqsubseteq D$ for two concepts. An equality is an expression of the form $C \equiv D$. An equality can be written in terms of GCI: $C \sqsubseteq D$ and $D \sqsubseteq C$. A TBox is a finite set of GCIs (an equality is expressed by two GCIs), denoted by $\mathcal{T}$.*

An ABox is a set of individual assertions: $a : C$, $b : D$ and $(a, b) : r$, where $a \in N_I$ and $b \in N_I$ are two instances of concepts C and D, called concept assertions, and the binary relation between a and b is an assertion of role r, called role assertion. An ABox is denoted by $\mathcal{A}$.

A knowledge base is a pair of TBox and ABox: $\mathcal{K} = (\mathcal{T}, \mathcal{A})$.

Definition 4 (Interpretation of $\mathcal{ALC}$)**.** *An interpretation $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ provides the semantics of concepts and roles. $\Delta^\mathcal{I}$ is a non-empty set which indicates the entire "world" of the application domain. $\cdot^\mathcal{I}$ is an interpretation function which maps concept and individual symbols to $\Delta^\mathcal{I}$ and roles to $\Delta^\mathcal{I} \times \Delta^\mathcal{I}$.*

- *Every concept $C \in N_C$ is interpreted as a subset of $\Delta^\mathcal{I}$, represented by $C^\mathcal{I} \subseteq \Delta^\mathcal{I}$.*
- *Every role r is interpreted as a subset of $\Delta^\mathcal{I} \times \Delta^\mathcal{I}$, denoted as $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$.*
- *Every individual $a \in N_I$ is interpreted as an element in the set $\Delta^\mathcal{I}$, denoted as $a^\mathcal{I} \in \Delta^\mathcal{I}$.*

The interpretation for concept expressions and axioms in the knowledge base are shown in Table 5. Let Φ be a set of axioms, an interpretation $\mathcal{I}$ is a model of Φ if Φ holds (true) in the context of $\mathcal{I}$.

Constructor	Syntax	Semantics	Example
Atomic Concept	C	$C^\mathcal{I} \subseteq \Delta^\mathcal{I}$	$Human$
Negation	$\neg C$	$\Delta^\mathcal{I} \backslash C^\mathcal{I}$	$\neg Human$
Top	$\top$	$\top^\mathcal{I} = \Delta^\mathcal{I}$	*All*
Bottom	$\bot$	$\bot^\mathcal{I} = \emptyset^\mathcal{I}$	$Nothing$
Conjunction	$(C \sqcap D)$	$C^\mathcal{I} \cap D^\mathcal{I}$	$Human \sqcap Male$
Disjunction	$(C \sqcup D)$	$C^\mathcal{I} \cup D^\mathcal{I}$	$Female \sqcup Male$
Universal Qualification	$\forall r.C$	$\{x \in \Delta^\mathcal{I} \mid \forall y \in \Delta^\mathcal{I} : \langle x, y\rangle \in r^\mathcal{I} \textit{ implies } y \in C^\mathcal{I}\}$	$\forall hasChild.Human$
Existential Restriction	$\exists r.C$	$\{x \in \Delta^\mathcal{I} \mid \exists y \in \Delta^\mathcal{I} : \langle x, y\rangle \in r^\mathcal{I} \textit{ and } y \in C^\mathcal{I}\}$	$\exists hasChild.Female$
Subsumption	$C \sqsubseteq D$	$C^\mathcal{I} \subseteq D^\mathcal{I}$	$Man \sqsubseteq Human$
Concept definition	$C \equiv D$	$C^\mathcal{I} = D^\mathcal{I}$	$Father \equiv Man \sqcap \exists hasChild.Huma$
Concept Assertion	$a : C$	$a^\mathcal{I} \in C^\mathcal{I}$	$John : Man$
Role Assertion	$(a, b) : r$	$\langle a^\mathcal{I}, b^\mathcal{I}\rangle \in r^\mathcal{I}$	$(John, Lea) : hasChild$

Table 1: Syntax and interpretations of $\mathcal{ALC}$ [5].

An example of a knowledge base referring to brain anatomy is as follows, where LVl and LVr denote left and right lateral ventricles and left and right caudate nuclei are denoted by CNl and CNr. The general knowledge is represented in the TBox, where describes basic axioms of the background knowledge. The ABox represent the assertions, which are the facts in the observation (ex. information extracted from an image).

$$
\begin{aligned}
TBox = \{Hemisphere &\sqsubseteq \exists isPartOf.Brain \\
BrainStructure &\sqsubseteq \exists isPartOf.Brain \\
BrainDisease &\sqsubseteq \exists isPartOf.Brain \sqcap \neg BrainStructure \\
Tumor &\sqsubseteq BrainDisease \\
LVl &\sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl \\
LVr &\sqsubseteq BrainStructure \sqcap \exists(leftOf \sqcap closeTo).CNr \\
CNl &\sqsubseteq BrainStructure \\
CNr &\sqsubseteq BrainStructure
\end{aligned}
$$

$$
\begin{aligned}
ABox = \{a &: CNl \\
b &: Unknown\ Object \\
c &: Brain \\
\langle a, b \rangle &: leftOf, closeTo \\
\langle b, c \rangle &: isPartOf\}
\end{aligned}
$$

This knowledge base example demonstrates a practical way to represent brain anatomy. For instance, $LVl \sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl$ expresses that the left lateral ventricle belongs to the brain structure which is on the right of and close to the left caudate nucleus. In the ABox, a, b, c are individuals corresponding to observed objects in the image. $a : CNl$ is a concept assertion and $\langle b, c \rangle : isPartOf$ is a role assertion, expressing that b is a part of c.

2.2.2 Reasoning services

Implicit information which is not explicitly defined in the knowledge base needs to be inferred with reasoning services. Reasoning services in Description Logic are decision procedures based on a knowledge base model. The basic reasoning on concept in Description Logics is subsumption checking (written as $\mathcal{T} \models C \sqsubseteq D$) and concept satisfiability checking (written as $\mathcal{T} \not\models C \equiv \bot$). Subsumption checking is a decision procedure to check whether a concept D is more general than another concept C. Checking satisfiability of a concept C is a decision procedure to determine whether C has a model with respect to the TBox. Complex reasoning services are built based on the basic ones. For example, classification is a decision procedure to find subconcept and superconcept relationships between concepts in a given terminology. This allows us to find a position in terminological hierarchy. Therefore, classification can be reduced to subsumption checking of each pair of concepts in the given terminology. The definitions of subsumption and satisfiability of a concept are introduced as follows [5]:

- subsumption checking: $\mathcal{T} \models C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model of $\mathcal{I}$ of $\mathcal{T}$.
- concept satisfiability: C is satisfiable with respect to $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}} \neq \emptyset$.

All the reasoning problems like subsumption, classification, consistency checking, can be reduced to a concept satisfiability problem [5].

2.3 Tableau method reasoning

The tableau algorithm is an efficient decision procedure for the concept satisfiability problem [6, 21, 36]. This method tries to construct a model of a concept C with respect to the given terminological knowledge. All the concepts are required to be expressed in negation normal form (NNF).

Definition 5. *(Negation normal form) Negation normal form is a form of concept expression such that the negation constructor appears only before atomic concepts. The rules of transformation are described as follows:*

- $\neg(\neg C) \equiv C$,
- $\neg(C \sqcup D) \equiv \neg C \sqcap \neg D$,
- $\neg(C \sqcap D) \equiv \neg C \sqcup \neg D$,
- $\neg(\exists r.C) \equiv \forall r.\neg C$,
- $\neg(\forall r.C) \equiv \exists r.\neg C$

For example, the negation normal form of the concept $\neg(BrainStructure \sqcap \exists leftOf.CNl)$ is $\neg BrainStructure \sqcup \forall leftOf.\neg CNl$.

Definition 6 (A tableau for $\mathcal{ALC}$)**.** *Let D be an $\mathcal{ALC}$ concept in NNF and let R_D be the set of roles in $\mathcal{ALC}$, a tableau T for D is defined as a triplet $(\mathbf{S}, \mathcal{L}, \mathcal{E})$, where $\mathbf{S}$ is a set of interpretation elements; $\mathcal{L}$ relates each interpretation element to a set of concepts occurring in D (from $\mathbf{S}$ to $\mathcal{P}(\mathfrak{C})$); $\mathcal{E}$ relates each pair of interpretation elements to a set of roles in R_D (from $\mathbf{S} \times \mathbf{S}$ to $\mathcal{P}(R_D)$).*

The decision procedure to check the satisfiability of a given concept D is based on constructing a model using the tableau method. Let x and y be two interpretation elements in $\mathbf{S}$ ($x, y \in \mathbf{S}$), C, E be two concepts occurring in D and $r \in R_D$. The model is constructed as a tree structure where each node corresponds to an element of interpretation $x \in \Delta^{\mathcal{I}}$. The node is labeled with a set of concepts $\mathcal{L}(x)$. The edge between the nodes x and y is labeled with corresponding roles $r \in \mathcal{E}(\langle x, y\rangle)$. The following properties hold:

- *if $C \in \mathcal{L}(x)$, then $\neg C \notin \mathcal{L}(x)$.*
- *if $C \sqcap E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ and $E \in \mathcal{L}(x)$.*
- *if $C \sqcup E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ or $E \in \mathcal{L}(x)$.*
- *if $\exists r.C \in \mathcal{L}(x)$, then there exists some $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y\rangle)$ and $C \in \mathcal{L}(y)$.*
- *if $\forall r.C \in \mathcal{L}(x)$, then for all $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y\rangle)$, $C \in \mathcal{L}(y)$.*

To check the satisfiability of a concept D, the tableau method is initialized by a root node associated with an interpretation element x and $D \in \mathcal{L}(x)$. The tableau is expanded with new nodes for $\exists r.C$. The edge linking two nodes is labeled with a role r. Each node is updated by adding or removing elements in $\mathcal{L}(x)$ and $\mathcal{E}(\langle x, y\rangle)$ according to following rules:

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \nsubseteq \mathcal{L}(x)$, then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C_1, C_2\}$.

$$\begin{array}{c} \mathcal{L}(x) = \{C_1 \sqcap C_2\} \\ | \\ \mathcal{L}(x) = \{C_1 \sqcap C_2,\ C_1,\ C_2\} \end{array}$$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \cap \mathcal{L}(x) \neq \emptyset$, then $\mathcal{L}(x) \rightarrow \mathcal{L}(x) \cup \{C\}$ *for some* $C \in \{C_1, C_2\}$.

$$\begin{array}{ccc} & \mathcal{L}(x) = \{C_1 \sqcup C_2\} & \\ \swarrow & & \searrow \\ \mathcal{L}(x) = \{C_1 \sqcup C_2,\ C_1\} & & \mathcal{L}(x) = \{C_1 \sqcup C_2,\ C_2\} \end{array}$$

$\exists$-rule: if $\exists r.C \in \mathcal{L}(x)$ and there does not exist a y such that $\mathcal{E}(\langle x, y\rangle)$ and $C \in \mathcal{L}(y)$, then create a new node y with $\mathcal{E}(\langle x, y\rangle)$ and $\mathcal{L}(y) = \{C\}$.

$$\begin{array}{c} \mathcal{L}(x) = \{\exists r.C\} \\ | \\ \mathcal{L}(x) = \{\exists r.C\} \\ \mathcal{L}(y) = \{C\} \\ \mathcal{E}(\langle x, y\rangle) = \{r\} \end{array}$$

$\forall$-rule: if $\forall r.C \in \mathcal{L}(x)$ and there exists a y such that $\mathcal{E}(\langle x, y\rangle)$ and $C \notin \mathcal{L}(y)$, then $\mathcal{L}(y) \to \mathcal{L}(y) \cup \{C\}$.

$$\begin{array}{c} \mathcal{L}(x) = \{\forall r.C\} \\ \mathcal{L}(y) = \{D\} \\ \mathcal{E}(\langle x, y\rangle) = \{r\} \\ | \\ \mathcal{L}(x) = \{\forall r.C\} \\ \mathcal{L}(y) = \{C,\ D\} \\ \mathcal{E}(\langle x, y\rangle) = \{r\} \end{array}$$

The tableau is said to be complete when there exists a clash in some node x or none of the rules mentioned above can be applied in the tableau. For a given concept D, D *is satisfiable* if the tableau is complete without a clash, otherwise D *is unsatisfiable*.

Definition 7. *(Clash) A tableau contains a clash if, for a node x and a concept C, $\{C, \neg C\} \subseteq \mathcal{L}(x)$.*

$$\begin{array}{c} \mathcal{L}(x) = \{C,\ \neg C\} \\ | \\ \boxtimes \end{array}$$

3 Qualitative spatial reasoning

Qualitative spatial relations have been studied in different views: topological relations (e.g. "contains"), directional relative relations (e.g. "left of"), distances (e.g. "close to") as well as more complex relations (e.g. "between") [9, 19, 26, 30]. These relations are frequently used at a linguistic level by humans when describing a scene [20]. Even though quantitative information is more precise, human cannot use it as accurately as a machine. Hence to qualitative representations to describe relations between spatial objects are crucial. The aim of our study is to apply human knowledge to the description of qualitative relationships and the reasoning tasks with spatial objects in a complex scene. In this thesis, we consider a part of topology (inclusion relations and adjacency), direction and distance for spatial representation in a 3D space. Qualitative spatial reasoning deals with the following questions, among others:

- Can we recognize an object from known objects and their spatial relations?
- Which relationships are satisfied between two objects when their relationships are not explicitly described in a given knowledge base?
- Is a recognized spatial arrangement of a scene consistent with the given knowledge of the scene?

The reasoning task can be summarized as follows:

1. Determining whether an object satisfies a spatial configuration, where an object is described by an observation of spatial arrangement and a spatial configuration is defined using expert knowledge. Then the task is considered as a consistency checking of the observed object with respect to the spatial configuration in the knowledge base.
2. Determining the relationship between two objects from other spatial arrangements. The implicit relations between two objects can be inferred by other known spatial relations in a spatial arrangement.

3. Determining the consistency of a spatial arrangement in a given configuration of the scene with respect to a specific domain knowledge. This task verifies the consistency between the observation and the spatial configuration defined using expert knowledge.

In this section, we discuss different representations of spatial relations and illustrate our formalism to perform spatial reasoning.

3.1 State of the art

Spatial relationship is an important factor for image interpretation in brain images due to the similar appearance among brain structures [9]. Chen *et al.* discussed a broad range of spatial relation representations [10]. Different spatial calculi are summarized for various aspects of space (topology, direction, distance, object shape, etc.). Basic qualitative spatial relations are summarized in Table 2.

Topological relations [38]	Directional relations	Distance relations
dc (“disconnected from”)	left of	far from
eq (“equal with”, “identical”)	right of	close to
po (“intersect with”, “partially overlaps”)	above	
ec (“external connected with”,“touches”, “adjacent”)	below	
tpp (“tangential proper part of”)	in front of	
tppi (“inversion of tpp”)	behind	
ntpp (“not tangential proper part of”)		
ntppi (“inversion of ntpp”)		

Table 2: Basic spatial relations.

3.1.1 Topology

Topology has been mostly investigated in qualitative spatial representation and the most popular representation is based on *Region Connection Calculus* (RCC) [11]. The collection $\{dc, eq, po, ec, tpp, tppi, ntpp, ntppi\}$ is a set of disjoint exhaustive topological relations defined as RCC8 [38]. Between any two objects in topological space, only one of eight relations can hold. Therefore, a useful reasoning mechanism of RCC8 based on a composition table is proposed in [14] (Table 3). Let A, B, C be three objects in a topological space, both A, B and B, C are adjacent (ec). Then the possible relations between A, C can be found within the table. This reasoning mechanism allows answering the second and third questions of qualitative spatial reasoning problems. However, the composition table only gives possible relations and many of composition rules give no information like $dc(A, B)$ and $dc(B, C)$ (all the relations possibly hold between A and C from the composition table). Further, the RCC8 representation and composition table were constructed for determining a satisfaction problem in a specific arrangement [48, 49]. Unfortunately, the inference within this kind of representation is undecidable [48]. Afterwards, Lutz *et al.* exploited qualitative reasoning in concrete domains with constraint satisfaction problems [33]. In the context of brain anatomy, only the inclusion relations are included in the work in [39]. The property of transitivity is emphasized in this context.

	dc (B,C)	**ec (B,C)**	*eq* (B,C)	*ntpp* (B,C)	*tpp* (B,C)	*ntppi* (B,C)	*tppi* (B,C)	*op* (B,C)
dc (A,B)	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc*	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*
ec (A,B)	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*	**dc or ec or ntpp or tpp or op**	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc*	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*
eq (A,B)	*dc*	*ec*	*eq*	*ntpp*	*tpp*	*ntppi*	*tppi*	*op*
ntpp (A,B)	*dc*	*dc*	*ntpp*	*ntpp*	*ntpp*	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*
tpp (A,B)	*dc*	*dc* or *ec*	*tpp*	*ntpp*	*tpp* or *ntpp*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *eq* or *tpp* or *tppi* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*
ntppi (A,B)	*dc* or *ec* or *ntppi* or *tppi* or *op*	*ntppi* or *tppi* or *op*	*ntppi*	*eq* or *ntpp* or *tpp* or *tppi* or *ntppi* or *op*	*tppi* or *ntppi* or *op*	*ntppi*	*ntppi*	*tppi* or *ntppi* or *op*
tppi (A,B)	*dc* or *ec* or *ntppi* or *tppi* or *op*	*ec* or *ntppi* or *tppi* or *op*	*tppi*	*ntpp* or *tpp* or *op*	*eq* or *tpp* or *tppi* or *op*	*ntppi*	*ntppi* or *tppi*	*ntppi* or *tppi* or *op*
op (A,B)	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*op*	*ntpp* or *tpp* or *op*	*ntpp* or *tpp* or *op*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*

Table 3: The 64 compositions of binary topological relations between objects A and C via the third object B. Extracted from [14].

3.1.2 Direction

Directional relations are seen as primarily important spatial relationships in brain anatomical images [9, 18, 26, 34]. As mentioned in [9], a direction is defined by a *target object*, a *reference object* and a *reference system*. In an 3D space, directional relations are represented by six basic terms (e.g.: right, above, behind, etc.). In order to compute directional relationships between two objects, many methods such as histograms of angles and morphological approaches are summarized in [9]. Fuzzy representation is employed in these approaches. For instance, a fuzzy set (fuzzy landscape) representing the region of space where a given directional relation with a reference object is satisfied can be computed using morphological dilatation of the reference object. The satisfaction degree of the direction between the target object and the reference object is evaluated by comparison between the target object and the fuzzy landscape. In crisp logic, many researches are based on two categories: point based and extend object based (bounding box) [10]. However, both of two representation ignore the influence of objects forms. The representation is not as accurate to human intuition as fuzzy representation.

3.1.3 Distance

Qualitative representation of the distance between two objects depends on the metric measures about the geometrical information and a scale system of the specific domain. The measure is often calculated in terms of Euclidean distance, considering two objects as two points. In a complex scene, which the form cannot be ignored, distance consists of minimum distance, mean distance and Hausdorff distance etc [9]. One useful scale system for convert numerical measures to qualitative representation is that using fuzzy set for satisfaction degree of different qualitative representation [9].

3.2 Qualitative spatial reasoning in $\mathcal{ALCHI}_{\mathcal{R}^+}$

In this section, we give the logical formalism to represent qualitative spatial relationships as roles and corresponding properties in terms of role axioms within a role box. Then, we illustrate how the qualitative spatial reasoning is developed using the logical formalism.

3.2.1 Syntax and semantics of roles

Definition 8. *(Role syntax) Let N_R be a set of role names, the inverse roles and the negation of roles are represented by r^- and $\neg r$. Complex roles are characterized with $r_1 \sqcap r_2$ and $r_1 \sqcup r_2$. Role axioms are used for modeling properties of roles such as inclusion ($r_1 \sqsubseteq r_2$) and role composition ($r_1 \circ r_2$).*

Table 4 describes a main syntax and semantics of roles for Description Logics.

Constructor	Syntax	Semantics
Atomic role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
Inverse role	r^-	$\{\langle x, y\rangle,\ x \in \Delta^{\mathcal{I}},\ y \in \Delta^{\mathcal{I}} \mid \langle y, x\rangle \in r^{\mathcal{I}}\}$
Role negation	$\neg r$	$\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus r^{\mathcal{I}}$
Role composition	$r_1 \circ r_2$	$\{\langle x, z\rangle, x \in \Delta^{\mathcal{I}},\ z \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, \langle x, y\rangle \in r_1^{\mathcal{I}} \text{ and } \langle y, z\rangle \in r_2^{\mathcal{I}}\}$
Role conjunction	$r_1 \sqcap r_2$	$r_1^{\mathcal{I}} \cap r_2^{\mathcal{I}}$
Role disjunction	$r_1 \sqcup r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$
Role inclusion	$r_1 \sqsubseteq r_2$	$r_1^{\mathcal{I}} \subseteq r_2^{\mathcal{I}}$
Role equivalence	$r_1 \equiv r_2$	$r_1^{\mathcal{I}} = r_2^{\mathcal{I}}$

Table 4: Basic relations

Definition 9. *(Role inclusion axiom) A role inclusion axiom is defined in the form:*

$$r \sqsubseteq s,$$

where r and s are basic roles in N_R or complex roles built with role constructors. A role equivalence $r \equiv s$ can be rewritten in the form $r \sqsubseteq s$ and $s \sqsubseteq r$.

Definition 10. *(RBox) A role box, denoted by RBox, is a finite set of axioms for N_R based on a set of roles $\mathbf{R} = N_R \cup \{r^- | r \in N_R\}$, where r^- represents the inversion of role r. Based on role inclusion and role equivalence, role axioms characterize role properties as follows:*

- *role composition:* $u \circ v \sqsubseteq r_1 \sqcup \cdots \sqcup r_n$ *with* $n \geqslant 1$, *which is interpreted as* $(u \circ v)^{\mathcal{I}} \subseteq r_1^{\mathcal{I}} \cup \cdots \cup r_n^{\mathcal{I}}$. *If there exist three interpretation elements* $x, y, z \in \Delta^{\mathcal{I}}$, $\langle x, y\rangle \in u^{\mathcal{I}}$ *and* $\langle y, z\rangle \in v^{\mathcal{I}}$ *implies* $\langle x, z\rangle \in r_1^{\mathcal{I}} \cup \cdots \cup r_n^{\mathcal{I}}$.

- *transitive role:* $r \circ r \sqsubseteq r$, *which is interpreted as* $(r \circ r)^{\mathcal{I}} \subseteq r^{\mathcal{I}}$. *If there exist three interpretation elements* $x, y, z \in \Delta^{\mathcal{I}}$, $\langle x, y\rangle \in r^{\mathcal{I}}$ *and* $\langle y, z\rangle \in r^{\mathcal{I}}$ *implies* $\langle x, z\rangle \in r^{\mathcal{I}}$.

- *inverse roles:* $u \equiv v^{-}$, *which is interpreted as* $u^{\mathcal{I}} = v^{-\mathcal{I}}$. *If there exist two interpretation elements* $x, y \in \Delta^{\mathcal{I}}$, *then* $\langle x, y\rangle \in u^{\mathcal{I}}$ *iff* $\langle y, x\rangle \in v^{\mathcal{I}}$.

- *symmetric role:* $r \equiv r^{-}$, *which is interpreted as* $r^{\mathcal{I}} = r^{-\mathcal{I}}$. *If there exist two interpretation elements* $x, y \in \Delta^{\mathcal{I}}$, *then* $\langle x, y\rangle \in r^{\mathcal{I}}$ *iff* $\langle y, x\rangle \in r^{\mathcal{I}}$.

- *disjoint roles:* $u \sqsubseteq \neg v$. *which is interpreted as* $u^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \setminus v^{\mathcal{I}}$ *or* $u^{\mathcal{I}} \cap v^{\mathcal{I}} = \emptyset$. *If there exist two interpretation elements* $x, y \in \Delta^{\mathcal{I}}$, $\langle x, y\rangle \in u^{\mathcal{I}}$ *implies* $\langle x, y\rangle \notin v^{\mathcal{I}}$.

The knowledge base used for spatial reasoning in our framework is built with three blocks: terminologies (TBox), role axioms (RBox) and assertions (ABox) ($\mathcal{K} = \{\mathcal{T}, \mathcal{R}, \mathcal{A}\}$).

A decidable DL language $\mathcal{ALCHI}_{\mathcal{R}_+}$ [24] is described in the following for the qualitative spatial reasoning, where the spatial relations are represented by roles and the properties of these spatial relations can be represented by the axioms in the RBox.

The table of syntax and semantics of $\mathcal{ALCHI}_{\mathcal{R}_+}$ is shown as follows:

Name	Syntax	Semantics
Top	$\top$	$\Delta^{\mathcal{I}}$
Bottom	$\bot$	$\emptyset$
Negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
Conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
Disjunction	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
Existential restriction	$\exists r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, \langle x, y\rangle \in r^{\mathcal{I}} \textit{ and } y \in C^{\mathcal{I}}\}$
Universal restriction	$\forall r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \forall\ y \in \Delta^{\mathcal{I}}, \langle x, y\rangle \in r^{\mathcal{I}} \textit{ implies } y \in C^{\mathcal{I}}\}$
Atomic role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
Inverse role	r^{-}	$\{\langle x, y\rangle,\ x \in \Delta^{\mathcal{I}},\ y \in \Delta^{\mathcal{I}} \mid \langle y, x\rangle \in r^{\mathcal{I}}\}$
Role composition	$r_1 \circ r_2$	$\{\langle x, z\rangle, x \in \Delta^{\mathcal{I}},\ z \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, \langle x, y\rangle \in r_1^{\mathcal{I}} \textit{ and } \langle y, z\rangle \in r_2^{\mathcal{I}}\}$
Role conjunction	$r_1 \sqcap r_2$	$r_1^{\mathcal{I}} \cap r_2^{\mathcal{I}}$
Role disjunction	$r_1 \sqcup r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$
Role inclusion	$r_1 \sqsubseteq r_2$	$r_1^{\mathcal{I}} \subseteq r_2^{\mathcal{I}}$
Role equivalence	$r_1 \equiv r_2$	$r_1^{\mathcal{I}} = r_2^{\mathcal{I}}$
Subsumption	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all $\mathcal{I}$
Concept Definition	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$ for all $\mathcal{I}$
Concept assertion	$a : C$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
Role assertion	$(a, b) : r$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$

Table 5: Syntax and interpretations of $\mathcal{ALCHI}_{\mathcal{R}_+}$.

A tableaux method tries to check satisfiability of a concept D by finding a model for D. The tableaux is constructed by applying a set of expansion rules. The model contains a set of interpretation elements and associated concepts for each interpretation element. These concepts are restricted to subsets of subconcepts of D ($sub(D)$). The subconcept of a concept D is defined as follows:

Definition 11 (Subconcept [24])**.** *A subconcept of a concept* D *is the concept occurring in* D. $sub(\cdot)$ *is the* *set of all subconcepts:*

$$\begin{aligned} sub(A) &= \{A\} \; for\ concept\ names\ A \in N_C \\ sub(C \sqcap E) &= \{C \sqcap E\} \cup sub(C) \cup sub(E) \\ sub(C \sqcup E) &= \{C \sqcup E\} \cup sub(C) \cup sub(E) \\ sub(\exists r.C) &= \{\exists r.C\} \cup sub(C) \\ sub(\forall r.C) &= \{\forall r.C\} \cup sub(C) \end{aligned}$$

For example,

$$\begin{aligned} sub(\exists leftOf.CNl \sqcap \exists closeTo.CNl) = \{&\exists leftOf.CNl \sqcap \exists closeTo.CNl, \\ &\exists leftOf.CNl, \\ &\exists closeTo.CNl, \\ &CNl\} \end{aligned}$$

Definition 12 ($\mathcal{ALCHI}_{\mathcal{R}+}$ tableaux [24])**.** *Let D be an $\mathcal{ALCHI}_{\mathcal{R}+}$ concept in negation normal form (NNF) and let R_D be the set of roles in $\mathcal{ALCHI}_{\mathcal{R}+}$, a tableau T for D is defined as a triple $(\mathbf{S}, \mathcal{L}, \mathcal{E})$, where $\mathbf{S}$ is a set of interpretation elements; $\mathcal{L}$ relates each interpretation element to a set of concepts occurring in D ($\mathcal{L} : \mathbf{S} \to \mathcal{P}(sub(D))$ [1]); $\mathcal{E}$ relates each pair of interpretation elements to a set of roles in R_D ($\mathcal{E} : \mathbf{S} \times \mathbf{S} \to \mathcal{P}(R_D)$).*

The decision procedure to check the satisfiability of a given concept D is based on constructing a model using the tableau method. Let x and y be two interpretation elements in $\mathbf{S}$ ($x,\ y \in \mathbf{S}$), C, E be two concepts occurring in D and $r \in R_D$. The model is constructed as a tree structure where each node corresponds to an element of interpretation $x \in \Delta^{\mathcal{I}}$. The node is labeled with a set of concepts $\mathcal{L}(x)$. The edge between the nodes x and y is labeled with corresponding roles $r \in \mathcal{E}(\langle x, y\rangle)$. The following conditions hold:

1. *if $C \in \mathcal{L}(x)$, then $\neg C \notin \mathcal{L}(x)$.*
2. *if $C \sqcap E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ and $E \in \mathcal{L}(x)$.*
3. *if $C \sqcup E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ or $E \in \mathcal{L}(x)$.*
4. *if $\exists r.C \in \mathcal{L}(x)$, then there exists some $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y\rangle)$ and $C \in \mathcal{L}(y)$.*
5. *if $\forall r.C \in \mathcal{L}(x)$, then for all $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y\rangle)$, $C \in \mathcal{L}(y)$.*
6. *if $\forall r.C \in \mathcal{L}(x)$, for all $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y\rangle)$ and r is a transitive role*
7. *$r \in \mathcal{E}(\langle x, y\rangle)$ iff $r^- \in \mathcal{E}(\langle y, x\rangle)$.*
8. *if $r \in \mathcal{E}(\langle x, y\rangle)$ and $r \sqsubseteq v$ (or $r^- \sqsubseteq v^-$) then $v \in \mathcal{E}(\langle x, y\rangle)$.*

3.3 From image to symbolic representation

The transformation of representation from low-level numerical data to symbolic level is first phase of the framework. Concretely, objects and contextual information are extracted and represented within terminologies using an ABox. Further, the symbolic representation can be used for reasoning service in the logical formalism.

[1] $\mathcal{P}(sub(D))$ is the power set of $sub(D)$.

3.4 Example

The complete knowledge base is given as follows:

$$
\begin{aligned}
TBox = \{Hemisphere &\sqsubseteq \exists isPartOf.Brain \\
BrainStructure &\sqsubseteq \exists isPartOf.Brain \\
BrainDisease &\sqsubseteq \exists isPartOf.Brain \sqcap \neg BrainStructure \\
Tumor &\sqsubseteq BrainDisease \\
LVl &\sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl \\
LVr &\sqsubseteq BrainStructure \sqcap \exists(leftOf \sqcap closeTo).CNr \\
CNl &\sqsubseteq BrainStructure \\
CNr &\sqsubseteq BrainStructure\}
\end{aligned}
$$

The role axioms are described as:

$$
\begin{aligned}
RBox = \{rightOf &\equiv leftOf^- \\
above &\equiv below^- \\
closeTo &\equiv closeTo^- \\
farFrom &\equiv farFrom^- \\
isPartOf \circ isPartOf &\sqsubseteq isPartOf \\
hasPart \circ hasPart &\sqsubseteq hasPart \\
isPartOf &\equiv hasPart^-\}
\end{aligned}
$$

The ABox represents the observation of structures in an image and the relationships between them. In this example, both recognized and unrecognized structures are represented by individuals. Spatial relations between the unknown structure and recognized structures are represented by roles. For instance, a region is recognized as the left caudate nucleus (CNl), denoted by a. The region of brain is denoted by c. An unknown region is segmented and their relationships are computed. Such an observation can be represented as

$$
\begin{aligned}
ABox = \{a &: CNl \\
b &: Unknown\ Object \\
c &: Brain \\
\langle a, b\rangle &: leftOf, closeTo \\
\langle b, c\rangle &: isPartOf\}
\end{aligned}
$$

In this example, the ABox describes an observation of a given scene and the objective is to find a reasonable description of the unknown object b. A possible hypothesized description is $LVl \sqcap \exists isPartOf.Hemisphere$. The hypothesis can be verified by a concept subsumption checking: $\mathcal{K} \models H \sqsubseteq O$, where H is the explained concept for the observation O. To check subsumption of two concepts H and O, $\mathcal{K} \models H \sqcap \neg O \sqsubseteq \bot$ is required to prove that $H \sqcap \neg O$ is unsatisfiable.

In this example, we could derive $\langle b, a\rangle : leftOf^-, closeTo^-$ from $\langle a, b\rangle : leftOf, closeTo$. Then most specific concept (see Definition 13) of b can be formulated as $O \equiv \exists(leftOf^- \sqcap closeTo^-).CNl \sqcap \exists isPartOf.Brain$ and $H \equiv LVl \sqcap \exists isPartOf.Hemisphere$. Let x be the interpretation element of the concept $H \sqcap \neg O$.

The tableau is initialized with $\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap \forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain\}$. According to Definition 12, $\sqcap$ and $\sqcup$ rule (rule 2 and rule 3) are applied and we obtain:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl\} \qquad \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

To integrate terminological knowledge, axioms like $C \sqsubseteq D$ in the TBox can be internalized into single concepts $(\neg C \sqcup D)$ and added to $\mathcal{L}(x)$. Here, for the sake of simplicity of demonstration, we only add the internalization of the axiom $LVl \sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl$ for the first branch.

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\} \end{array} \qquad \begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall isPartOf.\neg Brain\} \end{array}$$

We then apply $\sqcap$ and $\sqcup$ rule (rule 2 and rule 3) again on the first branch:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\} \end{array} \qquad \begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall isPartOf.\neg Brain\} \end{array}$$

$$\begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ \neg LVl\} \\ | \\ \boxtimes \end{array} \qquad \begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ BrainStructure, \\ \exists(rightOf \sqcap closeTo).CNl \end{array}$$

A clash $(LVl,\ \neg LVl)$ is detected in the first part of the first branch (closed). We then apply $\exists$ rule (rule 4) on the second part:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\} \end{array} \qquad \begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall isPartOf.\neg Brain\} \end{array}$$

$$\begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ \neg LVl\} \\ | \\ \boxtimes \end{array} \qquad \begin{array}{c} \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ BrainStructure, \\ \exists(rightOf \sqcap closeTo).CNl \\ | \\ \mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere, \\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl, \\ BrainStructure, \\ \exists(rightOf \sqcap closeTo).CNl \\ \mathcal{L}(y) = \{CNl\} \\ \mathcal{E}(\langle x, y\rangle) = \{rightOf, closeTo\} \end{array}$$

Because of inverse role axiom in the RBox, we can add inverse roles in $\mathcal{E}(\langle x, y\rangle)$ (rule 7) and apply $\forall$ rule (rule 5) on the second part:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl,\\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}\end{array}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl,\\ \neg LVl\}\end{array}$$

$$\boxtimes$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl,\\ BrainStructure,\\ \exists(rightOf \sqcap closeTo).CNl\end{array}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl,\\ BrainStructure,\\ \exists(rightOf \sqcap closeTo).CNl\\ \mathcal{L}(y) = \{CNl,\ \neg CNl\}\\ \mathcal{E}(\langle x, y\rangle) = \{rightOf, closeTo, leftOf^{-}, closeTo^{-}\}\\ \mathcal{L}(\langle y, x\rangle) = \{rightOf^{-}, closeTo^{-}, leftOf, closeTo\}\end{array}$$

$$\boxtimes$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\end{array}$$

The first branch of the tableau is closed because of the clash of CNl and $\neg CNl$ in the second part. We then explore the second branch. At first we apply the $\exists$ rule (rule 4) and then $\forall$ rule (rule 5):

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall(leftOf^{-} \sqcap closeTo^{-}).\neg CNl,\\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}\end{array}$$

$$\boxtimes$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\end{array}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\\ \mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain\}\\ \mathcal{E}(\langle x, y\rangle) = \{isPartOf\}\end{array}$$

The axiom $Hemisphere \sqsubseteq \exists isPartOf.Brain$ is internalized and added into $\mathcal{L}(y)$. Then we continue to extend the second branch with expansion rules (rule 2,3,4,6,7):

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}\end{array}$$

$$\boxtimes$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\end{array}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\\ \mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain\\ \neg Hemisphere \sqcup \exists isPartOf.Brain\}\\ \mathcal{E}(\langle x, y\rangle) = \{isPartOf\}\end{array}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\\ \mathcal{L}(y) = \{Hemisphere, \neg Brain,\\ \forall isPartOf.\neg Brain,\\ \neg Hemisphere\}\\ \mathcal{E}(\langle x, y\rangle) = \{isPartOf\}\end{array}$$

$$\boxtimes$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\\ \mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\\ \exists isPartOf.Brain\}\\ \mathcal{E}(\langle x, y\rangle) = \{isPartOf\}\end{array}$$

$$\begin{array}{c}\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\\ \forall isPartOf.\neg Brain\}\\ \mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\\ \exists isPartOf.Brain\}\\ \mathcal{E}(\langle x, y\rangle) = \{isPartOf\}\\ \mathcal{L}(z) = \{Brain, \neg Brain, \forall isPartOf.\neg Brain\}\\ \mathcal{E}(\langle y, z\rangle) = \{isPartOf\}\end{array}$$

$$\boxtimes$$

In both two parts of the second branch, we get clashes ($Hemisphere$ and $\neg Hemisphere$ in $\mathcal{L}(y)$ for the first part, $Brain$ and $\neg Brain$ in $\mathcal{L}(z)$ for the second part). This implies that we cannot find a model for the concept $H \sqcap \neg O$. Therefore, it is unsatisfiable and we can conclude that $K \models H \sqsubseteq O$ and $LVl \sqcap \exists isPartOf.Hemisphere$ is a potential explanation of the observation.

4 Abductive reasoning

Abductive reasoning is a backward-chaining inference, which consists in generating hypotheses and finding the "best" explanation on the basis of observation. Unlike the inference operation of standard reasoning presented in Section 2, abductive reasoning is a non-monotonic reasoning because the conclusion is not necessarily correct. New knowledge should be added in order to positively entail the observation. Medical image interpretation can be expressed as an abductive reasoning mechanism. When facing a pathological brain image, an expert has to resort to his knowledge of pathological anatomy, in order to give an explanation of the observed image. In this section, we will introduce how abduction is applied in image interpretation from two aspects (generation of hypotheses and selection).

4.1 State of the art of abductive reasoning

The term "Abduction" was first proposed by Charles S. Peirce in philosophy. Afterwards, abduction was developed in artificial intelligence and cognitive science. Aliseda [2] gave a general overview of abduction in propositional logic and proposed tableaux methods for abduction. Further, in the context of Description Logics, four types of abduction problems are described by Elisenbroich [15]. Let $\mathcal{L}$ be a DL, $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ be a knowledge base in $\mathcal{L}$, C, D two concepts in $\mathcal{L}$ and suppose that they are satisfiable with respect to $\mathcal{K}$. The logical formalisms of abduction in DLs are represented as follows:

- Concept abduction: given an observation concept O, a hypothesis is a concept H such that $\mathcal{K} \vDash H \sqsubseteq O$.
- TBox abduction: let $C \sqsubseteq D$ be satisfiable w.r.t $\mathcal{K}$, the hypothesis is a set of axioms $S_T = \{E_i \sqsubseteq F_i \mid i \leq n\}$ such that $\mathcal{K} \cup S_T \vDash C \sqsubseteq D$.
- ABox abduction: let S_a be a set of assertions representing the observation, a hypothesis is a set S_b of ABox assertions such that $\mathcal{K} \cup S_b \vDash S_a$.
- Knowledge base abduction: let ϕ be a consistent set of ABox or TBox assertions w.r.t. $\mathcal{K}$. A solution of knowledge base abduction, considered as a combination of TBox abduction and ABox abduction, is any finite set $S = \{\psi_i \mid i \leq n\}$ such that $\mathcal{K} \cup S \vDash \phi$.

An image interpretation tasks was regarded as an abduction problem in [3, 22, 35, 40]. In [35], DL-safe rules were proposed to map high level concepts and occurrence objects in the scene and their relationships. The rules ensure the expressivity and preserve the decidability of the reasoning. However, only the concept defined in the rules can be inferred using the formalism. In [3], the image interpretation was formulated as a concept abduction problem. The DL is expressed in $\mathcal{EL}$. The knowledge base is processed using formal concept analysis and the abductive reasoning is tackled by a recursive erosion on the lattice based representation. In this way, not only defined concepts but also undefined complex concepts can be inferred.

The tableau method was first adapted in Description Logics formalisms for a market matchmaking problem [12]. Colucci *et al.* modeled this problem as a concept abduction in the DL $\mathcal{ALN}$ [12], where the observations are the demand and the supply is treated as the explanation for the meet of the request. The tableau method has also been studied by Halland *et al.* in [28] for a TBox abduction problem. For a TBox abduction problem, a TBox axiom in the form $\phi = C \sqsubseteq D$ is an explanation to enforce the entailment of the observation, which is also in the form of a TBox subsumption form. Similar to the tableau method for the concept abduction, if the disjunction of two concepts A_1 and $\neg A_2$ can create a clash of the tableau, then $A_2 \sqsubseteq A_1$ is considered as a potential explanation.

Klarman *et al.* [29] present the tableau method for the ABox abduction in $\mathcal{ALC}$. This method integrates logic reasoning techniques of the first-order logic. First, knowledge and observation are transformed into first-order logic. Then, a tableau in the context of the first-order logic is built and solutions are selected in the open branches. The results are transformed into Description Logic from the first-order logic in the end. In [13, 27], the authors also focus on ABox abduction problems. Unlike the traditional work in [29], the authors in [27] propose the explanation with fresh individuals which are not suitable to use a traditional set inclusion minimality to choose a preferred explanation. In [27], Du *et al.* introduced a tractable approach to ABox abduction, called the query abduction problem. This problem focuses on giving the explanations, which are new facts neither in the observation nor in the assertional knowledge, for an observation. The observation is represented by a boolean conjunctive query in the form $\exists x \Phi(x, c)$, where $\Phi(x, c)$ is a conjunction of concepts assertions and role assertions and x represents a variable and c represents an individual. However, the potential hypotheses are restricted to atomic concepts and roles in the DL.

We then move on the other aspect of abduction problem: the selection problem. As a set of syntactical candidates generated using the tableau method, the selection relies on explicit restrictions for choosing the "best" explanation. Restrictions concerns filtering out inappropriate hypotheses, for instance, inconsistent hypothesis (H_1 such that $\mathcal{K} \cup H_1 \vDash \emptyset$) and independent hypothesis (H_1 entails the observation independently without background knowledge, such that $H_1 \vDash O$). These types of hypotheses need to be removed. In addition, minimality criteria are required to select the "best" among the filtered candidates. Though the desired candidates are selected, the solutions can be infinite. Therefore, defining minimality criteria is an important manner to find a preference among all the potential hypotheses. Bienvenu discussed a set of basic minimality criteria for abductive reasoning in [8].

4.2 Abductive reasoning for image interpretation

As introduced above, the tableau method is an effective way to find an explanation given the observation. In this work, we apply this general strategy to image interpretation expressed as a concept abduction problem.

An observed image is represented by an ABox, which is supposed to be consistent with the knowledge base. In the concept abduction problem, the observation concept is constructed on the basis of the individual that selected to be explained and contextual information in the ABox. The knowledge base is in the DL $\mathcal{ALC}$, but the explanations are generated in a restricted language.

The tableau method has proven effective for tackling abductive reasoning problems. We then consider extending and automating this approach for image interpretation. The work is divided into two parts: generation of hypotheses and selection of the "best" explanation using a minimality criterion.

Following the processing steps presented in Section 3, the given image observation is translated into an ABox. An unknown object is represented by a most specific concept (see Definition 13). This concept converts contextual information in the ABox to an appropriate concept to represent the object.

Definition 13 (**Most specific concept** [3]). *Given a TBox $\mathcal{T}$ and an associated interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, let $X \subseteq \Delta^{\mathcal{I}}$ be a subset of the interpretation space and E a defined concept in $\mathcal{T}$. The concept E is defined as the most specific concept of X w.r.t. $\mathcal{I}$ if:*

- $X \subseteq E^{\mathcal{I}}$.
- *for every defined concept F with $X \subseteq F^{\mathcal{I}}$, we have $E \sqsubseteq_{\mathcal{T}} F$.*

An example of ABox is given as follows:

$$
\begin{aligned}
\mathcal{A}_{obs} = \{t_1 &: BrainTumor\\
e_1 &: NonEnhanced\\
l_1 &: LateralVentricle\\
p_1 &: PeripheralCerebralHemisphere\\
(t_1, e_1) &: hasEnhancement\\
(t_1, l_1) &: farFrom\\
(t_1, p_1) &: hasLocation\}.
\end{aligned}
$$

The most specific concept of the individual t_1 is :

$$
\begin{aligned}
BrainTumor &\sqcap \exists hasEnhancement.NonEnhanced\\
&\sqcap \exists farFrom.LateralVentricle\\
&\sqcap \exists hasLocation.PeripheralCerebralHemisphere
\end{aligned}
$$

As all observed objects in the ABox can be formulated by the most specific concept, our problem is modeled as a concept abduction. $\mathcal{K} \vDash H \sqsubseteq O$. H is an explanation of the given observation O if H is subsumed by O w.r.t. $\mathcal{K}$. The subsumption problem can be converted into a test of satisfiability which requires to prove that $H \sqcap \neg O$ is unsatisfiable. According to the strategy proposed by Aliseda [2], a potential hypothesis H is the concept which makes the tableau of $H \sqcap \neg O$ closed as a consequence.

In the context of acyclic TBox, the classic tableau method integrates axioms of the TBox using the normalization process. This optimization technique is suitable for forward-chaining inference. For instance, a concept D can be inferred by getting a concept C with the axiom $C \sqsubseteq D$ in a deduction way since a model of the concept C is also a model of D. However, this is not suitable for a backward-chaining inference, which intends to find a concept C as a hypothesis for D. A possible solution is to add internalized concept (see Definition 14) in the tableau.

Definition 14 (Internalized concept [5]). *Let $\mathcal{T}$ be a TBox and a set of axioms formulated as $C_i \sqsubseteq D_i$. The internalized concept of the TBox is defined as follows:*

$$C_{\mathcal{T}} \equiv \sqcap_{(C_i \sqsubseteq D_i \in \mathcal{T})} (\neg C_i \sqcup D_i)$$

If $C_i \sqsubseteq D_i$, then $\top \sqsubseteq \neg C_i \sqcup D_i$ and $C_{\mathcal{T}} \equiv \top$. As a consequence, all interpretations of the TBox $\mathcal{T}$ is equivalent to interpretations of the internalized concept $C_{\mathcal{T}}$. Therefore, every interpretation elements belongs to $C_{\mathcal{T}}^{\mathcal{I}}$. Its use has a result in $C \equiv C \sqcap C_{\mathcal{T}}$.

We reformulate the subsumption in terms of satisfiability: the concept $H \sqcap \neg D$ is not satisfiable w.r.t. $\mathcal{T}$, where H is an explanation, D is an observation, $\mathcal{T}$ is a TBox. This problem can be reduced by testing the satisfiability of a concept $H \sqcap \neg D \sqcap C_{\mathcal{T}}$, where $C_{\mathcal{T}}$ is the internalized concept of $\mathcal{T}$. The concept H that causes unsatisfiability of $H \sqcap \neg D \sqcap C_{\mathcal{T}}$ is a potential hypothesis, i.e. the tableau built from this concept is closed. We follow this strategy and propose an extension of the work by Colucci *et al.* in [12].

Each interpretation element in the tableau has now four label function (instead of $\mathcal{L}(x)$ and $\mathcal{E}(x, y)$ in Definition 6): $\mathbf{T}(x)$, $\mathbf{F}(x)$, $\mathbf{T}(x, y)$, $\mathbf{F}(x, y)$, where x, y are interpretation elements in $\Delta^{\mathcal{I}}$. The are defined as follows:

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a knowledge base, $x^{\mathcal{I}}$, $y^{\mathcal{I}}$ interpretation elements, C, D two concepts et r, s two role in the given DL, we have:

- $\mathbf{T}(x)$ represents a set of concepts such that $x^{\mathcal{I}}$ is one of their interpretations: $C \in \mathbf{T}(x)$ iff $x^{\mathcal{I}} \in C^{\mathcal{I}}$.
- $\mathbf{F}(x)$ represents a set of concepts such that $x^{\mathcal{I}}$ is not one of their interpretations: $D \in \mathbf{F}(x)$ iff $x^{\mathcal{I}} \notin D^{\mathcal{I}}$.
- $\mathbf{T}(x, y)$ represents a set of roles between x and y: $r \in \mathbf{T}(x, y)$ iff $\langle x^{\mathcal{I}}, y^{\mathcal{I}} \rangle \in r^{\mathcal{I}}$.
- $\mathbf{F}(x, y)$ represents a set of unsatisfiable roles between x and y: $s \in \mathbf{F}(x, y)$ iff $\langle x^{\mathcal{I}}, y^{\mathcal{I}} \rangle \notin s^{\mathcal{I}}$.

In the initialization step, the root node of the tableau is initialized with the concept $C_{\mathcal{T}} \sqcap \neg O$. As $C_{\mathcal{T}} \sqcap \neg O$ belongs to $\mathbf{T}(1)$, we add its negation to $\mathbf{F}(1)$. This technique avoids adding the negation before concepts selected to generate contradictions in the table. We can prove the equivalence between $C \in \mathbf{T}(x)$ and $\neg C \in \mathbf{F}(x)$. Suppose that for $x^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, $x^{\mathcal{I}}$ is an interpretation of a concept C, and $x^{\mathcal{I}}$ is also an interpretation of the concept of $\neg C$. So $x^{\mathcal{I}}$ is an interpretation of the concept $C \sqcap \neg C \equiv \bot$. There is no such interpretation. Thus, if $x^{\mathcal{I}} \in C^{\mathcal{I}}$, then $x^{\mathcal{I}} \notin (\neg C)^{\mathcal{I}}$.

We assume that the concepts are simplified in the normal form of negation. For a concept $C \in \mathcal{ALC}$, the normal form of a negated concept of $\neg C$ is denoted by $\overline{C}$. The expansion rules used in our work are presented here:

1. Conjunction
   - **T**) if $C \sqcap D \in \mathbf{T}(x)$, we add C and D in $\mathbf{T}(x)$.
   - **F**) if $C \sqcup D \in \mathbf{F}(x)$, we add C and D in $\mathbf{F}(x)$.
2. Disjunction
   - **T**) if $C \sqcup D \in \mathbf{T}(x)$, the branch is divided into two $(\mathbf{T}(x_1), \mathbf{T}(x_2))$. $\mathbf{T}(x_1) = \mathbf{T}(x) \cup \{C\}$ and $\mathbf{T}(x_2) = \mathbf{T}(x) \cup \{D\}$
   - **F**) if $C \sqcap D \in \mathbf{F}(x)$, the branch is divided into two $(\mathbf{F}(x_1), \mathbf{F}(x_2))$. $\mathbf{F}(x_1) = \mathbf{F}(x) \cup \{C\}$ and $\mathbf{F}(x_2) = \mathbf{F}(x) \cup \{D\}$
3. Existential restriction
   - **T**) if $\exists r.C \in \mathbf{T}(x)$ and there does not exist a y such that $r \in \mathbf{T}(x, y)$ and $C \in \mathbf{T}(y)$, we create a new interpretation element y then add r in $\mathbf{T}(x, y)$, and C in $\mathbf{T}(y)$.
   - **F**) if $\forall r.C \in \mathbf{F}(x)$ and there does not exist a y such that $r \in \mathbf{T}(x, y)$ and $C \in \mathbf{T}(y)$, we create a new interpretation element y then add r in $\mathbf{T}(x, y)$, and C in $\mathbf{F}(y)$.
4. Universal restriction
   - **T**) if $\forall r.C \in \mathbf{T}(x)$ and for all y such that $r \in \mathbf{T}(x, y)$ and $C \notin \mathbf{T}(y)$, we add C in $\mathbf{T}(y)$.
   - **F**) if $\exists r.C \in \mathbf{F}(x)$ and for all y such that $r \in \mathbf{T}(x, y)$ and $C \notin \mathbf{T}(y)$, we add C in $\mathbf{F}(y)$.
5. Replacement of axioms in $\mathcal{T}$

T) if $A \in \mathbf{T}(x)$ and $A \equiv C \in \mathcal{T}$, we add C in $\mathbf{T}(x)$.

T) if $\neg A \in \mathbf{T}(x)$ and $A \equiv C \in \mathcal{T}$, we add $\overline{C}$ in $\mathbf{T}(x)$.

F) if $\neg A \in \mathbf{F}(x)$ and $A \equiv C \in \mathcal{T}$, we add $\overline{C}$ in $\mathbf{F}(x)$.

F) if $A \in \mathbf{F}(x)$ and $A \equiv C \in \mathcal{T}$, we add C in $\mathbf{F}(x)$.

The contradiction in the adapted form is classified in two types: homogeneous clash and heterogeneous clash.

Definition 15 (**Clash** [12])**.** *1. A branch is defined as a homogeneous clash if:*

- $\bot \in \mathbf{T}(x)$ *or* $\top \in \mathbf{F}(x)$.
- $\{A, \neg A\} \in \mathbf{T}(x)$ *or* $\{A, \neg A\} \in \mathbf{F}(x)$.

2. A branch is defined as a heterogeneous clash if:

- $\{A \text{ or } \neg A\} \in \mathbf{T}(x) \cap \mathbf{F}(x)$.

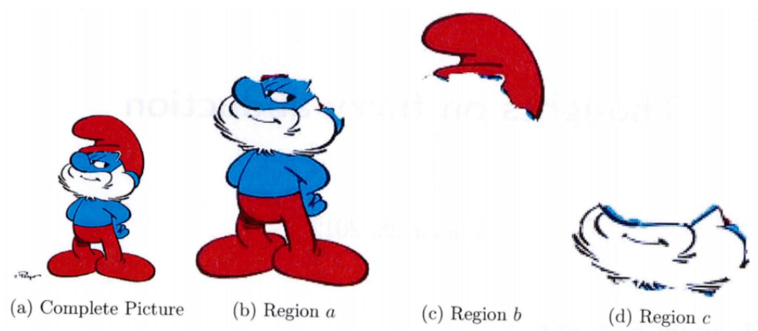

Figure 3: The smurf and its segmentation of different elements.

We illustrate this procedure by the example of the interpretation of the image of the Smurf (Figure 3). In the TBox, we describe the background knowledge that a leader smurf has a beard and wears a red hat as follows:

$$\begin{aligned}\mathcal{T} = \{&SmurfLeader \sqsubseteq \exists hasPart.Beard \sqcap \exists hasOnTop.RedHat,\\ &RedHat \equiv Hat \sqcap \exists hasColor.Red\}\end{aligned}$$

Suppose that we can recognize three parts a, b, c in an image and the observation is encoded by the following ABox:

$$\begin{aligned}\mathcal{A}_{obs} = \{&(a,b) : hasOnTop,\\ &(a,c) : hasPart,\\ &b : Hat,\\ &b : \exists hasColor.Red,\\ &c : Beard\}.\end{aligned}$$

Then, the most specific concept of a is constructed as:

$$D = \exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$$

In our approach of the construction of the tableau, the initial node consists of two complex concepts. One concept is satisfiable in $\mathbf{T}(1)$. Here, the concept is empty because we does not specify a constraint. The other is not satisfiable concept in $\mathbf{F}(1)$. Here is the negation of the conjunction of $C_{\mathcal{T}}$ and $\neg O$.

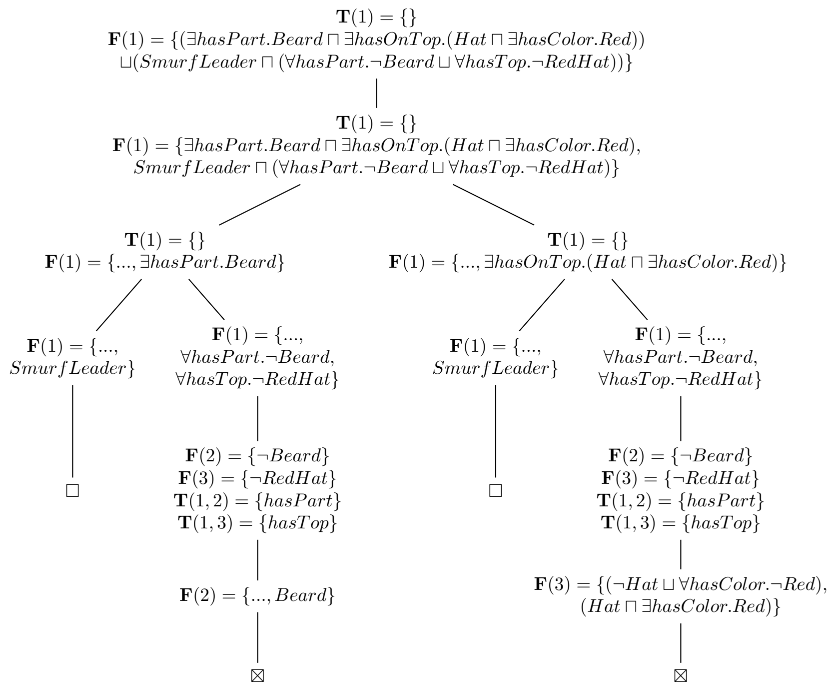

Figure 4: The process of constructing the tableau by applying expansion rules.

By applying expansion rules, the construction process of the tableau is shown in Figure 4. The hypotheses are generated from open branches. In this example, two sets of concepts are:

$$H_1 = \{SmurfLeader,\ \exists hasPart.Beard\}$$
$$H_2 = \{SmurfLeader,\ \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)\}$$

The concepts in these two sets are basic elements to build a hypothesis H. The hypothesis H is considered as a concept in $\mathbf{T}(1)$. To close the tableau, we can take these concepts in $\mathbf{F}(1)$ to generate a heterogeneous clash. The first branch is closed if one takes the concept $SmurfLeader$, $\exists hasPart.Beard$ or the combination of these two concepts $SmurfLeader \sqcap \exists hasPart.Beard$. The concept $SmurfLeader$ is also a concept for closing the second branch. We can then consider that $H \equiv SmurfLeader$ is a potential hypothesis. Apart from this case, $\exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $SmurfLeader \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $SmurfLeader \sqcap \exists hasPart.Beard$ are also potential hypotheses.

4.2.1 Tableau closure (minimal hitting set)

At this stage, we conducted a construction procedure of the tree model using the tableau method. Then we have a set of concepts for each open branch in the tableau. One potential hypothesis is the combination of these concepts. The considered concepts can close the table if at least one concept is selected in each branch. To avoid redundancy, we want to take the minimum hitting set. The definition of hitting set is introduced below.

Definition 16. *(Hitting set) Let $\{S_1, \ldots, S_n\}$ be a collection of sets. A hitting set T is a subset $T \subseteq \cup_{i=1}^{n} S_i$ such that T contains at least one element of each set in the collection $T \cap S_i \neq \emptyset$ $(1 \leq i \leq n)$.*

Such a minimal hitting set of concepts guarantees a syntactical hypothesis in the given DL. To ensure the consistency and relevance of an explanation, the following properties are required to be satisfied for each potential hypothesis:

Relevant $\mathcal{K} \vDash \mathcal{H} \sqsubseteq \mathcal{O}$

Consistent $\mathcal{K}, \mathcal{H}$ are consistent.

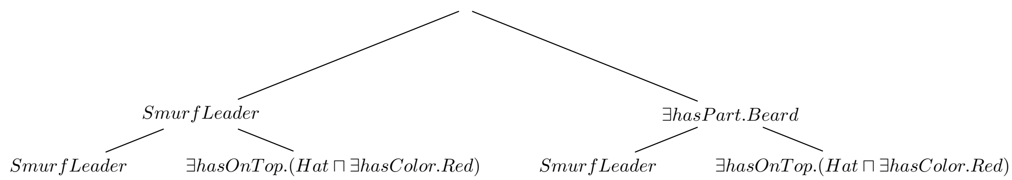

Figure 5: Hitting set construction tree.

Explainable $\mathcal{K}, \nvDash \mathcal{O}, \mathcal{H} \nvDash \mathcal{O}$

An exhaustive algorithm is proposed for selecting the minimal hitting set:
Input: A collection of sets $\{S_1, \ldots, S_n\}$;
Output: A collection of hitting sets $\mathcal{H}$;
Initialization $\mathcal{H} = \{\}$;
Root initialization.;
For (i From 1 to n) **do:**
    Create a new leaf for every S_i in each branch;
    An intermediate hypothesis H_j is the conjunction of all the concepts in the same branch;
    Delete the branch j if H_j is inconsistent w.r.t. the TBox;
End
The conjunction of all concepts in each branch j represents a potential hypothesis H_j;
return: $\mathcal{H}$;

Algorithm 1: Exhaustive search algorithm of selecting hitting sets.

We illustrate the algorithm by the example of Smurf. Note that we got two sets of concepts in the previous step. Applying this algorithm, a tree is initialized with a empty root. Then we construct recursively the tree by adding all concepts in the set h_i as new leaves (Figure 5). In this case, all assumptions are consistent with TBox: $H_1 = SmurfLeader$, $H_2 = \exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $H_3 = SmurfLeader \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $H_4 = SmurfLeader \sqcap \exists hasPart.Beard$. The second hypothesis is removed because $H = \exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$ is not an independent explainable hypothesis ($H \vDash \mathcal{O}$).

Assumptions obtained in this algorithm is consistent and minimal syntactically. A preference explanation depends on a minimality criterion that will be presented in the next subsection.

4.3 Minimality criteria

5 Perspectives

Several directions will be presented in this section:

- concrete domain with fuzzy logic. Concrete domains are necessary when semantic truth concept has no corresponding space in concrete domain.
- how to generate potential hypotheses.
- how to select the "best" explanation with an appropriate minimality criterion.
- perspective of publications.

6 Activities

- project LOGIMA
- seminars
- doctoral formation

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