Mid-term Thesis Report
High-level image interpretation using logical and morphological approaches

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March 20, 2015

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Thesis started in October 2013

1 Introduction

High-level semantics extraction from an image is an important research area in artificial intelligence. Many related fields like image annotation, activity recognition and decision-support systems take advantage of semantic content. Recognition of perceptual objects and scene understanding, which translate low level signal information into meaningful semantic information, belong to one of the fundamental abilities of human beings. As advanced as AI has become, it still remains a big challenge for computers to accomplish complex understanding tasks as humans do. Digital image itself is a numerical representation which does not represent explicitly semantic information. Prior knowledge is intensively used by experts who interpret visually an image. It should then also be used by machines to associate semantics with the image. However, image interpretation still faces some difficulties, one of which is how to accurately associate perceptual data with appropriate concepts. Without an expert knowledge, such a link is difficult to be established. This relation between visual percepts and high-level linguistic expression is called *semantic gap* [29]. In this work, beyond a single object understanding based on low level features such as color and shape, we focus on a complex description which relies on context information like spatial relations between diverse objects as well as prior knowledge on the application domain. For instance, in the context of medical applications, the understanding task can be formulated as giving an abstract description of a pathological brain volume, such as in Figure 1. According to different levels of anatomical prior knowledge on brain pathology, two possible descriptions could be given:

- an abnormal structure is present in the brain,
- a peripheral non-enhanced tumor is present in the right hemisphere.

In this thesis, a high-level interpretation is regarded as an explanation of what we have seen in the image. This process is a backward-chaining inference based on prior knowledge to link the abstract description and the observed context of the scene.

1.1 Problem formulation

According to the objective pointed out in the introduction, our aim is to extract high-level semantic information from a given image and translate it at a linguistic level. Concretely, we are interested in the interpretation of cerebral images with tumors. The high-level information corresponds to the presence of diverse types of pathologies as well as descriptions of brain structures and spatial relations among them in a brain image. In the context of this thesis, the decision process is modeled as an abductive reasoning [2] using a logical formalism, which is an inference mechanism from facts to explanations. The objective of this thesis is to build a generic logic-based formalism as well as to develop an appropriate reasoning process for image interpretation,

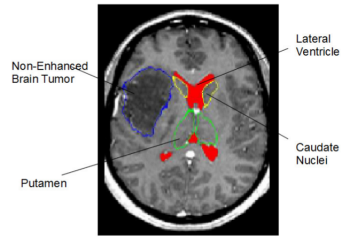

Figure 1: A slice of a pathological brain volume (MRI acquisition), where some structures are annotated.

allowing us to extract a set of suitable candidates as potential hypotheses for a given image and to select the "best" one according to a defined criterion. In image interpretation, spatial relationships are important when objects of similar appearance are present in the image, especially in magnetic resonance imaging (MRI). Such relationships have then to be included in the representation and in the reasoning process.

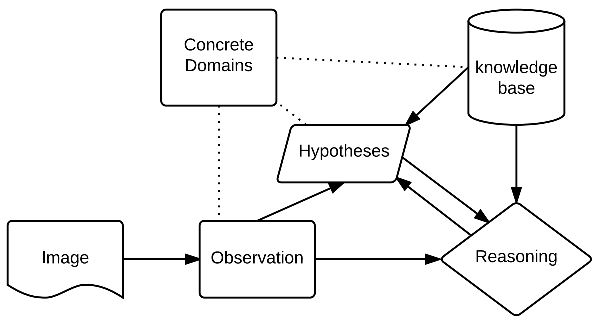

Figure 2: A general schema of image interpretation in the thesis.

Figure 2 shows the major components of our framework in this thesis. The main components encompass an observation of a given image, a prior knowledge base of the application domain and the reasoning service for the purpose of image interpretation. The given image is translated into symbolic representations in terms of logical formulas by segmentation and recognition of objects using image processing tools. Meanwhile, it can also serve as instances on the concrete domain. Concrete domains [10], considered as a real world model (e.g. image space) linked with abstract terminologies, is as well a useful part which benefits from complementary information of abstract level of knowledge in the image representation. During the past period, concrete domains have not been exploited and will be studied in the future. Hypotheses are formulated with the help of the reasoning process taking both the observation and the background knowledge into account. The relations between the hypotheses and the reasoning are in two directions, which allows validating the hypothesis with the help of standard reasoning and building a possible hypothesis within backward-chaining reasoning processes.

To summarize the ongoing and future work, we need to answer the following questions:

- *How to model the prior knowledge and formalize an appropriate representation in a given application domain? (Section 2)*
- *How to connect the image level representation and the symbolic level representation? (Section 3)*
- *How to overcome the semantic gap between numerical representation and qualitative representation of spatial relationships? (Section 3)*
- *How to generate hypotheses to explain the observed scene? (Section 4 and Section 5)*
- *How to define a criterion to choose a "best" explanation in our case? (Section 4 and Section 5)*

1.2 Related work

Semantics extraction is important in image analysis, for various tasks such as image annotation [46], event detection [33] and diagnostic problems [3, 4]. In some specific domains, like medical imaging [3, 9, 21, 36] and remote sensing [20, 50], image interpretation combines image processing with artificial intelligence techniques to derive reasonable semantics.

As a high level process of exploiting semantic in the scene, image interpretation involves two levels:

- relating low level features to semantics (from pixels to semantic information) [9, 21, 30, 36].
- inferring the abstract high-level description from the semantic image content (from semantics to explanation) [3, 19].

Roughly speaking, the first level describes what is happening while the second one describes how it is happening [48]. The first level has been mainly studied in the field of multiple objects recognition. Image interpretation maps regions or groups of regions onto labels corresponding to semantic concepts (e.g. labels of anatomical structures for medical images). Various approaches employ Bayesian networks with a combination of semantics and probabilistic inference mechanisms [34, 39, 44]. These techniques provide inference mechanisms by attempting to construct co-occurrence objects and contextual information with a probabilistic model for reasoning.

Further, a hierarchical representation of knowledge base is proposed, called image grammar [49, 54]. The grammar is a structured knowledge represented by an And-Or graph. In the And-Or graph, a global description of a scene is decomposed into parts, objects until primitive pixel patches from top to bottom. An And-node consists of a set of successive components and an Or-node is composed by alternative nodes. A parsing method is proposed as inference within a probabilistic model in each node [27, 53]. The best-fit description is selected according to the most probable model in each Or-node.

The second level consists in reasoning at the language (knowledge) level. For the purpose of giving an adequate explanation, the second level is a logic-based reasoning to depict the image with a deep and abstract description from the point of view of an expert. There is not much work on image interpretation using logical knowledge representation and reasoning. However, formal languages based on logical formalisms have strong associated semantics for knowledge representation as well as reasoning processes. An aggregation concept is proposed in [19] to represent a complex event or scene concerning occurring objects, as well as spatial and temporal constraints. According to these defined aggregation and specific rules, a high-level interpretation is inferred [37]. The special backward-chaining rules trigger the detection of the high-level description when its aggregated concepts are detected in the observation. Similar to Bayesian networks and image grammars approaches, the results are limited to defined descriptions. In addition, the approach requires to construct complementary rules apart from knowledge base. A complex description can also be generated when non-explicitly presented in the knowledge base [3]. The authors considered a less expressive logical formalism which is mostly used for medical knowledge representation. The high-level semantics in terms of a complex description within an expressive logic formalism is still an open problem for image interpretation. At this level, high-level semantics require both background knowledge and contextual information and can be inferred from explicit observation in an image.

Let us now discuss these works with respect to the objectives of this thesis. Using graphical models (e.g. Bayesian network or And-Or graph) is a key idea to represent semantic objects consisting of low level image features and contextual information. Graphical models have proven their efficiency for integrating knowledge base and reasoning on dependence relationships incorporated with probabilistic models. However, this representation is restricted to explicit knowledge, since the network structure is represented using conditional distributions on dependence relationships where a semantic object is defined by dependence nodes (alternative choices or composition elements) in the graphical model. The reasoning power is limited too since it relies on this representation and on probabilistic models. In our framework, we choose a logic-based formalism, which is more powerful in both knowledge expressivity and reasoning services. As presented in the previous paragraph, logic-based image interpretation is more efficient for high-level semantic interpretation. Instead of limiting the expressivity of knowledge hierarchy by the dependence relationships, implicit knowledge can be inferred using more complex logical relations. In this thesis, we are interested in proposing an adapted logical formalism for the knowledge representation as well as a specific reasoning method for image interpretation.

2 Preliminaries

2.1 Ontologies

Experts' knowledge is usually expressed in terms of diverse vocabulary of a specific domain in natural language, which is difficult to be interpreted by machines. In order to facilitate automated reasoning process with a background knowledge base, a structural semantic based model is an efficient means to represent prior knowledge. The term ontology is derived from philosophy and then used for the purpose of expressing commonsense knowledge in computer science [1]. Since then, ontologies were adopted for image interpretation tasks [7, 30, 47]. Ontologies are defined as "*a formal specification of a shared conceptualization*" [45], which deal with modeling a universal and reusable knowledge among different applications for a specific domain. An ontology mainly contains *individuals*, *concepts*, *properties* and *axiom rules*. These components enable the background knowledge to be understandable and processable by machines.

2.2 Description Logics

As mentioned above, ontologies require a formal representation language and well-defined semantics for reasoning services. Description Logics (DLs) is a family of knowledge representation logical formalisms, which is seen as good candidates for ontologies [28]. The basic elements of Description logics are concepts (unary predicates), roles (binary predicates) and individuals. Besides the formal knowledge representation, another important feature of DLs is their ability of reasoning. Implicit information can be inferred from explicit knowledge description, such as satisfiability checking [5]. In this part, we introduce syntax and semantics of a description logic language $\mathcal{ALC}$ as well as its reasoning services.

2.2.1 Syntax and semantics

We first recall the syntax and semantics of the basic language of Description Logics ($\mathcal{ALC}$) [5].

Definition 1 (Signature)**.** *The syntax of a Description Logic is defined over a signature, which is defined as three disjoint sets* $Sig = (N_C,\ N_R,\ N_I)$. N_C *is a set of concept names that refers to a set of entities with the same characteristics.* N_I *is a set of individuals that contains instances of the concepts.* N_R *is a set of role names that refers to the binary relationships between two individuals or two concepts.*

Definition 2 (Concept expression)**.** *The set of concept expressions is recursively built from the signature as follows:*

- *all the concept names, as well as* $\top$ *(top concept) and* $\bot$ *(bottom concept) are concepts,*
- *if* C *and* D *are two concepts and* r *is a role in* N_R *then* $\neg C$ *(negation),* $C \sqcap D$ *(conjunction),* $C \sqcup D$ *(disjunction),* $\exists r.C$ *(existential quantification),* $\forall r.C$ *(universal quantification) are also concepts.*

Let $\mathfrak{C}$ *denotes the infinite set of all the concepts that can be defined using constructors and signature elements.*

Definition 3 (Terminological box (TBox) and assertional box (ABox))**.** *A general concept inclusion axiom (GCI) is an expression of the form* $C \sqsubseteq D$ *for two concepts. An equality is an expression of the form* $C \equiv D$. *An equality can be written in terms of GCI:* $C \sqsubseteq D$ *and* $D \sqsubseteq C$. *A TBox is a finite set of GCIs (an equality is expressed by two GCIs), denoted by* $\mathcal{T}$.

An ABox is a set of individual assertions: $a : C$, $b : D$ *and* $(a, b) : r$, *where* $a \in N_I$ *and* $b \in N_I$ *are two instances of concepts* C *and* D, *called concept assertions, and the binary relation between* a *and* b *is an assertion of role* r, *called role assertion. An ABox is denoted by* $\mathcal{A}$.

A knowledge base is a pair of TBox and ABox: $\mathcal{K} = (\mathcal{T}, \mathcal{A})$.

Definition 4 (Interpretation of $\mathcal{ALC}$)**.** *An interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ *provides the semantics of concepts and roles.* $\Delta^{\mathcal{I}}$ *is a non-empty set which indicates the entire "world" of the application domain.* $\cdot^{\mathcal{I}}$ *is an interpretation function which maps concept and individual symbols to* $\Delta^{\mathcal{I}}$ *and roles to* $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

- *Every concept* C *is interpreted as a subset of* $\Delta^{\mathcal{I}}$, *represented by* $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$.
- *Every role* r *is interpreted as a subset of* $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, *denoted as* $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$.

- *Every individual $a \in N_I$ is interpreted as an element in the set $\Delta^{\mathcal{I}}$, denoted as $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$.*

The interpretation for concept expressions and axioms in the knowledge base are shown in Table 1. Let Φ be a set of axioms, an interpretation $\mathcal{I}$ is a model of Φ if Φ holds in the context of $\mathcal{I}$.

Constructor	Syntax	Semantics	Example
Atomic concept	C	$C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$	$Human$
Negation	$\neg C$	$\Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}$	$\neg Human$
Top	$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$	All
Bottom	$\bot$	$\bot^{\mathcal{I}} = \emptyset^{\mathcal{I}}$	$Nothing$
Conjunction	$(C \sqcap D)$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$	$Human \sqcap Male$
Disjunction	$(C \sqcup D)$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$	$Female \sqcup Male$
Universal restriction	$\forall r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \forall y \in \Delta^{\mathcal{I}} : \langle x, y\rangle \in r^{\mathcal{I}} \text{ implies } y \in C^{\mathcal{I}}\}$	$\forall hasChild.Human$
Existential restriction	$\exists r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}} : \langle x, y\rangle \in r^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\}$	$\exists hasChild.Female$
Subsumption	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$	$Man \sqsubseteq Human$
Concept definition	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$	$Father \equiv Man \sqcap \exists hasChild.Human$
Concept assertion	$a : C$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$	$John : Man$
Role assertion	$(a, b) : r$	$\langle a^{\mathcal{I}}, b^{\mathcal{I}}\rangle \in r^{\mathcal{I}}$	$(John, Lea) : hasChild$

Table 1: Syntax and semantics of $\mathcal{ALC}$ [5].

An example of a knowledge base referring to brain anatomy is described as follows, where LVl and LVr denote left and right lateral ventricles and left and right caudate nuclei are denoted by CNl and CNr. The general knowledge is represented in the TBox, which describes basic axioms of the background knowledge. The ABox represents the assertions, involving the facts in the observation (e.g. information extracted from an image).

$$\begin{aligned} TBox = \{ Hemisphere &\sqsubseteq \exists isPartOf.Brain \\ BrainStructure &\sqsubseteq \exists isPartOf.Brain \\ BrainDisease &\sqsubseteq \exists isPartOf.Brain \sqcap \neg BrainStructure \\ Tumor &\sqsubseteq BrainDisease \\ LVl &\sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl \\ LVr &\sqsubseteq BrainStructure \sqcap \exists(leftOf \sqcap closeTo).CNr \\ CNl &\sqsubseteq BrainStructure \\ CNr &\sqsubseteq BrainStructure \end{aligned}$$

$$\begin{aligned} ABox = \{ a &: CNl \\ b &: Unknown\ Object \\ c &: Brain \\ \langle a, b\rangle &: leftOf, closeTo \\ \langle b, c\rangle &: isPartOf \} \end{aligned}$$

This knowledge base example demonstrates a practical way to represent brain anatomy. For instance, $LVl \sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl$ expresses that the left lateral ventricle belongs to the brain structure which is on the right of and close to the left caudate nucleus. In the ABox, a, b, c are individuals corresponding to observed objects in the image. $a : CNl$ is a concept assertion and $\langle b, c\rangle : isPartOf$ is a role assertion, expressing that b is a part of c.

2.2.2 Reasoning services

Implicit information which is not explicitly defined in the knowledge base needs to be inferred with reasoning services. Reasoning services in Description Logics are decision procedures based on a knowledge base model.

The basic reasoning on concepts in Description Logics is subsumption checking (written as $\mathcal{T} \models C \sqsubseteq D$) and concept satisfiability checking (written as $\mathcal{T} \not\models C \equiv \bot$). Subsumption checking is a decision procedure to check whether a concept D is more general than another concept C. Checking satisfiability of a concept C is a decision procedure to determine whether C has a model with respect to the TBox. Complex reasoning services are built based on these basic ones. For example, classification is a decision procedure amounting to finding subsumption relationships between concepts in a given terminology. This allows us to find a position of the given concept in the terminological hierarchy. Therefore, classification can be reduced to subsumption checking of each pair of concepts in the given terminology. The definitions of subsumption and satisfiability of a concept are introduced as follows [5]:

- subsumption checking: $\mathcal{T} \models C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model of $\mathcal{I}$ of $\mathcal{T}$.
- concept satisfiability: C is satisfiable with respect to $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}} \neq \emptyset$.

All the reasoning problems like subsumption and classification can be reduced to a concept satisfiability problem [5].

2.3 Tableau method reasoning

We present several auxiliary definitions that will be used later.

Definition 5 (Negation normal form)**.** *Negation normal form is a form of concept expression such that the negation constructor appears only before atomic concepts. The rules of transformation are described as follows:*

- $\neg(\neg C) \;\equiv\; C,$
- $\neg(C \sqcup D) \;\equiv\; \neg C \sqcap \neg D,$
- $\neg(C \sqcap D) \;\equiv\; \neg C \sqcup \neg D,$
- $\neg(\exists r.C) \;\equiv\; \forall r.\neg C,$
- $\neg(\forall r.C) \;\equiv\; \exists r.\neg C$

For example, the negation normal form of the concept $\neg(BrainStructure \sqcap \exists leftOf.CNl)$ is $\neg BrainStructure \sqcup \forall leftOf.\neg CNl$.

Definition 6 (Conjunctive normal form [14])**.** *Conjunctive normal form (CNF) is a form of concept expression such that complex concepts are required to be replaced by the conjunction of its superconcept taking TBox axioms into account. For a concept C and a TBox $\mathcal{T} = \{C \sqsubseteq D\}$, $CNF(C, \mathcal{T}) = C \sqcap D$.*

For example, the conjunctive normal form of the concept LVl w.r.t. the TBox described in Section 2.2.1 is $LVl \sqcap BrainStructure \sqcap \exists isPartOf.Brain \sqcap \exists(rightOf \sqcap closeTo).(CNl \sqcap BrainStructure \sqcap \exists isPartOf.Brain)$

Definition 7 (Subconcept [28])**.** *A subconcept of a concept D is the concept occurring in D. $sub(\cdot)$ is the set of all subconcepts:*

$$\begin{aligned}
sub(A) &= \{A\} \; for\ concept\ names\ A \in N_C \\
sub(C \sqcap E) &= \{C \sqcap E\} \cup sub(C) \cup sub(E) \\
sub(C \sqcup E) &= \{C \sqcup E\} \cup sub(C) \cup sub(E) \\
sub(\exists r.C) &= \{\exists r.C\} \cup sub(C) \\
sub(\forall r.C) &= \{\forall r.C\} \cup sub(C)
\end{aligned}$$

For example,

$$\begin{aligned}
sub(\exists leftOf.CNl \sqcap \exists closeTo.CNl) = \{&\exists leftOf.CNl \sqcap \exists closeTo.CNl, \\
&\exists leftOf.CNl, \\
&\exists closeTo.CNl, \\
&CNl\}
\end{aligned}$$

Definition 8 (Internalized concept [5])**.** *Let $\mathcal{T}$ be a TBox and a set of axioms formulated as $C_i \sqsubseteq D_i$. The internalized concept of the TBox is defined as follows:*

$$C_{\mathcal{T}} \equiv \sqcap_{(C_i \sqsubseteq D_i \in \mathcal{T})}(\neg C_i \sqcup D_i)$$

For example, the internalized concept of the axiom $LVl \sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl$ is $\neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)$.

The tableau algorithm is an efficient decision procedure for the concept satisfiability problem [6, 24, 38]. This method tries to construct a model of a concept C with respect to the given terminological knowledge. All the concepts are required to be expressed in negation normal form (NNF).

Definition 9 (A tableau for $\mathcal{ALC}$)**.** *Let D be an $\mathcal{ALC}$ concept in NNF and let R_D be the set of roles in $\mathcal{ALC}$, a tableau T for D is defined as a triplet $(\mathbf{S}, \mathcal{L}, \mathcal{E})$, where $\mathbf{S}$ is a set of interpretation elements; $\mathcal{L}$ relates each interpretation element to a set of concepts occurring in D ($\mathcal{L} : \mathbf{S} \to \mathcal{P}(sub(D))$ [1]); $\mathcal{E}$ relates each pair of interpretation elements to a set of roles in R_D ($\mathcal{E} : \mathbf{S} \times \mathbf{S} \to \mathcal{P}(R_D)$).*

The decision procedure to check the satisfiability of a given concept D is based on constructing a model using the tableau method. Let x and y be two interpretation elements in $\mathbf{S}$ ($x, y \in \mathbf{S}$), C, E be two concepts occurring in D and $r \in R_D$. The model is constructed as a tree structure where each node corresponds to an element of interpretation $x \in \Delta^{\mathcal{I}}$. The node is labeled with a set of concepts $\mathcal{L}(x)$. The edge between the nodes x and y is labeled with corresponding roles $r \in \mathcal{E}(\langle x, y \rangle)$. The following properties hold:

1. *if $C \in \mathcal{L}(x)$, then $\neg C \notin \mathcal{L}(x)$.*
2. *if $C \sqcap E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ and $E \in \mathcal{L}(x)$.*
3. *if $C \sqcup E \in \mathcal{L}(x)$, then $C \in \mathcal{L}(x)$ or $E \in \mathcal{L}(x)$.*
4. *if $\exists r.C \in \mathcal{L}(x)$, then there exists some $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y \rangle)$ and $C \in \mathcal{L}(y)$.*
5. *if $\forall r.C \in \mathcal{L}(x)$, then for all $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y \rangle)$, $C \in \mathcal{L}(y)$.*

To check the satisfiability of a concept D, the tableau method is initialized by a root node associated with an interpretation element x and $D \in \mathcal{L}(x)$. The tableau is expanded by creating new nodes, new edges and new branches as well as adding or removing elements in $\mathcal{L}(x)$ and $\mathcal{E}(\langle x, y \rangle)$ according to following rules:

$\sqcap$-rule: if $C_1 \sqcap C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \nsubseteq \mathcal{L}(x)$, then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{C_1, C_2\}$.

$$\begin{array}{c} \mathcal{L}(x) = \{C_1 \sqcap C_2\} \\ | \\ \mathcal{L}(x) = \{C_1 \sqcap C_2,\ C_1,\ C_2\} \end{array}$$

$\sqcup$-rule: if $C_1 \sqcup C_2 \in \mathcal{L}(x)$ and $\{C_1, C_2\} \cap \mathcal{L}(x) \neq \emptyset$, then $\mathcal{L}(x) \to \mathcal{L}(x) \cup \{C\}$ *for some* $C \in \{C_1, C_2\}$.

$$\begin{array}{ccc} & \mathcal{L}(x) = \{C_1 \sqcup C_2\} & \\ \swarrow & & \searrow \\ \mathcal{L}(x) = \{C_1 \sqcup C_2,\ C_1\} & & \mathcal{L}(x) = \{C_1 \sqcup C_2,\ C_2\} \end{array}$$

$\exists$-rule: if $\exists r.C \in \mathcal{L}(x)$ and there does not exist a y such that $\mathcal{E}(\langle x, y \rangle)$ and $C \in \mathcal{L}(y)$, then create a new node y with $\mathcal{E}(\langle x, y \rangle)$ and $\mathcal{L}(y) = \{C\}$.

[1] $\mathcal{P}(sub(D))$ is the power set of $sub(D)$.

$$
\begin{array}{c}
\mathcal{L}(x) = \{\exists r.C\} \\
| \\
\mathcal{L}(x) = \{\exists r.C\} \\
\mathcal{L}(y) = \{C\} \\
\mathcal{E}(\langle x, y\rangle) = \{r\}
\end{array}
$$

$\forall$-rule: if $\forall r.C \in \mathcal{L}(x)$ and there exists a y such that $\mathcal{E}(\langle x, y\rangle)$ and $C \notin \mathcal{L}(y)$, then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{C\}$.

$$
\begin{array}{c}
\mathcal{L}(x) = \{\forall r.C\} \\
\mathcal{L}(y) = \{D\} \\
\mathcal{E}(\langle x, y\rangle) = \{r\} \\
| \\
\mathcal{L}(x) = \{\forall r.C\} \\
\mathcal{L}(y) = \{C,\ D\} \\
\mathcal{E}(\langle x, y\rangle) = \{r\}
\end{array}
$$

Definition 10. *(Clash) A branch contains a clash (i.e. the branch is closed), when* $\{C, \neg C\} \subseteq \mathcal{L}(x)$ *for a node* x *and a concept* C*.*

$$
\begin{array}{c}
\mathcal{L}(x) = \{C,\ \neg C\} \\
| \\
\boxtimes
\end{array}
$$

A branch is said to be complete when there exists a clash in some node x or none of the rules mentioned above can be applied in the tableau. For a given concept D, D *is satisfiable* if all the branches in the tableau are complete and at least one branch is open, otherwise D *is unsatisfiable*.

3 Qualitative spatial reasoning

Qualitative spatial relations have been studied from different views: topological relations (e.g. “contains”), directional relative relations (e.g. “left of”), distances (e.g. “close to”) as well as more complex relations (e.g. “between”) [9, 22, 30, 32]. These relations are frequently used at a linguistic level by humans when describing a scene [23]. Even though quantitative information could be more precise, humans cannot use it as accurately as a machine. Therefore, how to represent qualitative information of human knowledge within the machine is a crucial research area in knowledge representation and reasoning. The aim of our study is to apply human knowledge to the description of qualitative relationships and the reasoning tasks with spatial objects in a complex scene. In this thesis, we consider some topological relations (inclusion and adjacency), directions and distances for spatial representation in a 3D space. Qualitative spatial reasoning deals with the following questions, among others:

- Can we recognize an object from known objects and their spatial relations?
- Which relationships are satisfied between two objects when their relationships are not explicitly described in a given knowledge base?
- Is a recognized spatial arrangement of a scene consistent with the given knowledge of the scene?

The reasoning tasks can be summarized as follows:

1. Determining whether an object satisfies a spatial configuration, where an object is described by an observation of spatial arrangement and a spatial configuration is defined using expert knowledge. Then the task is considered as a consistency checking of the observed object with respect to the spatial configuration in the knowledge base.

2. Determining the relationships between two objects from other spatial arrangements. The implicit relations between two objects can be inferred by other known spatial relations in a spatial arrangement.

3. Determining the consistency of a spatial arrangement in a given configuration of the scene with respect to a specific domain knowledge. This task verifies the consistency between the observation and the spatial configuration defined using expert knowledge.

In this section, we discuss different representations of spatial relations and illustrate our formalism to perform spatial reasoning.

3.1 State of the art

Chen *et al.* discussed a broad range of spatial relation representations [11]. Different spatial calculi are summarized for various aspects of space (topology, direction, distance, object shape, etc.). Basic qualitative spatial relations are summarized in Table 2.

Topological relations [40]	Directional relations	Distance relations
dc ("disconnected from")	left of	far from
eq ("equal with", "identical")	right of	close to
po ("intersect with", "partially overlaps")	above	
ec ("external connected with", "touches", "adjacent")	below	
tpp ("tangential proper part of")	in front of	
tppi ("tangential proper part inverse of")	behind	
ntpp ("non-tangential proper part of")		
ntppi ("non-tangential proper part inverse of")		

Table 2: Basic spatial relations.

3.1.1 Topology

Topology has been intensively investigated in qualitative spatial representation and the most popular representation is based on *Region Connection Calculus* (RCC) [12]. The collection $\{dc, eq, po, ec, tpp, tppi, ntpp, ntppi\}$ is a set of disjoint exhaustive topological relations defined as RCC8 [40]. Between any two objects in topological space, only one of eight relations can hold. Therefore, a useful reasoning mechanism of RCC8 based on a composition table is proposed in [17] (Table 3). Let A, B, C be three objects in a topological space, both A, B and B, C are adjacent (ec). Then the possible relations between A, C can be found within the table. This reasoning mechanism allows answering the second and third questions of qualitative spatial reasoning problems. However, the composition table only gives possible relations and many of composition rules give no information like $dc(A, B)$ and $dc(B, C)$ (all the relations possibly hold between A and C from the composition table). Furthermore, the RCC8 representation and composition table were constructed for determining a satisfaction problem in a specific arrangement [51, 52]. Unfortunately, the inference within this kind of representation is undecidable [51]. Lutz *et al.* exploited qualitative reasoning in concrete domains with constraint satisfaction problems [35]. The authors described a constraints network in the concrete domain based on RCC8 relations. The spatial reasoning is tackled by checking existence of an appropriate network in the concrete domain. In the context of brain anatomy, only the inclusion relations are included in the work in [42]. Therefore, inclusion relations and adjacency, considered in this thesis, are frequently used in the spatial knowledge of the brain anatomy. The property of transitivity is emphasized in this context.

	dc (B,C)	**ec (B,C)**	*eq* (B,C)	*ntpp* (B,C)	*tpp* (B,C)	*ntppi* (B,C)	*tppi* (B,C)	*op* (B,C)
dc (A,B)	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc*	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*
ec (A,B)	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*	**dc or ec or ntpp or tpp or op**	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc*	*dc*	*dc* or *ec* or *ntpp* or *tpp* or *op*
eq (A,B)	*dc*	*ec*	*eq*	*ntpp*	*tpp*	*ntppi*	*tppi*	*op*
ntpp (A,B)	*dc*	*dc*	*ntpp*	*ntpp*	*ntpp*	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*
tpp (A,B)	*dc*	*dc* or *ec*	*tpp*	*ntpp*	*tpp* or *ntpp*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *eq* or *tpp* or *tppi* or *op*	*dc* or *ec* or *ntpp* or *tpp* or *op*
ntppi (A,B)	*dc* or *ec* or *ntppi* or *tppi* or *op*	*ntppi* or *tppi* or *op*	*ntppi*	*eq* or *ntpp* or *tpp* or *tppi* or *ntppi* or *op*	*tppi* or *ntppi* or *op*	*ntppi*	*ntppi*	*tppi* or *ntppi* or *op*
tppi (A,B)	*dc* or *ec* or *ntppi* or *tppi* or *op*	*ec* or *ntppi* or *tppi* or *op*	*tppi*	*ntpp* or *tpp* or *op*	*eq* or *tpp* or *tppi* or *op*	*ntppi*	*ntppi* or *tppi*	*ntppi* or *tppi* or *op*
op (A,B)	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*op*	*ntpp* or *tpp* or *op*	*ntpp* or *tpp* or *op*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *ntppi* or *tppi* or *op*	*dc* or *ec* or *eq* or *ntpp* or *tpp* or *ntppi* or *tppi* or *op*

Table 3: The 64 compositions of binary topological relations between objects A and C via the third object B. Extracted from [17].

3.1.2 Direction

Directional relations are seen as primarily important spatial relationships in brain anatomical images [9, 21, 30, 36]. As mentioned in [9], a direction is defined by a *target object*, a *reference object* and a *reference system*. In a 3D space, directional relations are represented by six basic terms (e.g. right, above, behind, etc.). In order to compute directional relationships between two objects, many methods such as histograms of angles and morphological approaches are summarized in [9]. A fuzzy representation is employed in these approaches. For instance, a fuzzy set (fuzzy landscape) representing the region of space where a given directional relation with respect to a reference object is satisfied can be computed using a morphological dilation with an appropriate structuring element of the reference object. The satisfaction degree of the direction between the target object and the reference object is evaluated by comparison between the target object and the fuzzy landscape. In a crisp setting, two strategies are usually employed: point based and extend object based (bounding box) [11]. However, both representations ignore the influence of the object shape. Fuzzy representations better correspond to the human intuition than crisp ones.

3.1.3 Distance

Qualitative representation of the distance between two objects depends on the metric measures about the geometrical information of the specific domain. The measure is often evaluated in terms of Euclidean distance, considering two objects as two points. In a complex scene, which the shape cannot be ignored, distance can benefit from minimum distance, mean distance and Hausdorff distance, etc. [9]. One useful scale system to convert numerical measures into qualitative representations uses fuzzy sets for defining satisfaction degrees of different qualitative representations [9].

3.2 Qualitative spatial reasoning in $\mathcal{ALCHI}_{\mathcal{R}^+}$

In this section, we give the logical formalism to represent qualitative spatial relationships as roles and corresponding properties in terms of role axioms within a role box. Then, we illustrate how the qualitative spatial reasoning is developed using this logical formalism.

3.2.1 Syntax and semantics of roles

Definition 11. *(Role syntax) Let N_R be a set of role names, the inverse roles and the negation of roles are represented by r^- and $\neg r$. Complex roles are characterized with $r_1 \sqcap r_2$ and $r_1 \sqcup r_2$. Role axioms are used for modeling properties of roles such as inclusion ($r_1 \sqsubseteq r_2$) and role composition ($r_1 \circ r_2$).*

Table 4 describes a main syntax and semantics of roles for Description Logics.

Constructor	Syntax	Semantics
Atomic role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
Inverse role	r^-	$\{\langle x,y\rangle,\ x \in \Delta^{\mathcal{I}},\ y \in \Delta^{\mathcal{I}} \mid \langle y,x\rangle \in r^{\mathcal{I}}\}$
Role negation	$\neg r$	$\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus r^{\mathcal{I}}$
Role composition	$r_1 \circ r_2$	$\{\langle x,z\rangle, x \in \Delta^{\mathcal{I}},\ z \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, \langle x,y\rangle \in r_1^{\mathcal{I}}$ *and* $\langle y,z\rangle \in r_2^{\mathcal{I}}\}$
Role conjunction	$r_1 \sqcap r_2$	$r_1^{\mathcal{I}} \cap r_2^{\mathcal{I}}$
Role disjunction	$r_1 \sqcup r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$
Role inclusion	$r_1 \sqsubseteq r_2$	$r_1^{\mathcal{I}} \subseteq r_2^{\mathcal{I}}$
Role equivalence	$r_1 \equiv r_2$	$r_1^{\mathcal{I}} = r_2^{\mathcal{I}}$

Table 4: Basic role relations.

Definition 12. *(Role inclusion axiom) A role inclusion axiom is defined in the form:*

$$r \sqsubseteq s,$$

where r and s are basic roles in N_R or complex roles built with role constructors. A role equivalence $r \equiv s$ can be rewritten in the form $r \sqsubseteq s$ and $s \sqsubseteq r$.

Definition 13. *(RBox) A role box, denoted by RBox, is a finite set of axioms for N_R based on a set of roles $\mathbf{R} = N_R \cup \{r^- \mid r \in N_R\}$, where r^- represents the inverse of role r. Based on role inclusion and role equivalence, role axioms characterize role properties as follows:*

- *role composition:* $u \circ v \sqsubseteq r_1 \sqcup \cdots \sqcup r_n$ *with* $n \geqslant 1$*, which is interpreted as* $(u \circ v)^{\mathcal{I}} \subseteq r_1^{\mathcal{I}} \cup \cdots \cup r_n^{\mathcal{I}}$*. For all three interpretation elements* $x, y, z \in \Delta^{\mathcal{I}}$*,* $(\langle x, y\rangle \in u^{\mathcal{I}}$ *and* $\langle y, z\rangle \in v^{\mathcal{I}})$ *implies* $\langle x, z\rangle \in r_1^{\mathcal{I}} \cup \cdots \cup r_n^{\mathcal{I}}$*.*

- *transitive role:* $r \circ r \sqsubseteq r$*, which is interpreted as* $(r \circ r)^{\mathcal{I}} \subseteq r^{\mathcal{I}}$*. If there exist three interpretation elements* $x, y, z \in \Delta^{\mathcal{I}}$*,* $\langle x, y\rangle \in r^{\mathcal{I}}$ *and* $\langle y, z\rangle \in r^{\mathcal{I}}$ *implies* $\langle x, z\rangle \in r^{\mathcal{I}}$*.*

- *inverse roles:* $u \equiv v^-$*, which is interpreted as* $u^{\mathcal{I}} = v^{-\mathcal{I}}$*. For all two interpretation elements* $x, y \in \Delta^{\mathcal{I}}$*, then* $\langle x, y\rangle \in u^{\mathcal{I}}$ *iff* $\langle y, x\rangle \in v^{\mathcal{I}}$*.*

- *symmetric role:* $r \equiv r^-$*, which is interpreted as* $r^{\mathcal{I}} = r^{-\mathcal{I}}$*. For all two interpretation elements* $x, y \in \Delta^{\mathcal{I}}$*, then* $\langle x, y\rangle \in r^{\mathcal{I}}$ *iff* $\langle y, x\rangle \in r^{\mathcal{I}}$*.*

- *disjoint roles:* $u \sqsubseteq \neg v$*. which is interpreted as* $u^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \setminus v^{\mathcal{I}}$ *or* $u^{\mathcal{I}} \cap v^{\mathcal{I}} = \emptyset$*. For all two interpretation elements* $x, y \in \Delta^{\mathcal{I}}$*,* $\langle x, y\rangle \in u^{\mathcal{I}}$ *implies* $\langle x, y\rangle \notin v^{\mathcal{I}}$*.*

The knowledge base used for spatial reasoning in our framework is built with three blocks: terminologies (TBox), role axioms (RBox) and assertions (ABox) ($\mathcal{K} = \{\mathcal{T}, \mathcal{R}, \mathcal{A}\}$).

A decidable DL language $\mathcal{ALCHI}_{\mathcal{R}^+}$ [28] is described in the following for qualitative spatial reasoning, where the spatial relations are represented by roles and the properties of these spatial relations can be represented by the axioms in the RBox.

The table of syntax and semantics of $\mathcal{ALCHI}_{\mathcal{R}^+}$ is shown as follows:

Name	Syntax	Semantics
Top	$\top$	$\Delta^{\mathcal{I}}$
Bottom	$\bot$	$\emptyset$
Negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
Conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
Disjunction	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
Existential restriction	$\exists r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, \langle x, y\rangle \in r^{\mathcal{I}}$ *and* $y \in C^{\mathcal{I}}\}$
Universal restriction	$\forall r.C$	$\{x \in \Delta^{\mathcal{I}} \mid \forall\ y \in \Delta^{\mathcal{I}}, \langle x, y\rangle \in r^{\mathcal{I}}$ *implies* $y \in C^{\mathcal{I}}\}$
Atomic role	r	$r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
Inverse role	r^-	$\{\langle x, y\rangle,\ x \in \Delta^{\mathcal{I}},\ y \in \Delta^{\mathcal{I}} \mid \langle y, x\rangle \in r^{\mathcal{I}}\}$
Role composition	$r_1 \circ r_2$	$\{\langle x, z\rangle, x \in \Delta^{\mathcal{I}},\ z \in \Delta^{\mathcal{I}} \mid \exists y \in \Delta^{\mathcal{I}}, \langle x, y\rangle \in r_1^{\mathcal{I}}$ *and* $\langle y, z\rangle \in r_2^{\mathcal{I}}\}$
Role conjunction	$r_1 \sqcap r_2$	$r_1^{\mathcal{I}} \cap r_2^{\mathcal{I}}$
Role disjunction	$r_1 \sqcup r_2$	$r_1^{\mathcal{I}} \cup r_2^{\mathcal{I}}$
Role inclusion	$r_1 \sqsubseteq r_2$	$r_1^{\mathcal{I}} \subseteq r_2^{\mathcal{I}}$
Role equivalence	$r_1 \equiv r_2$	$r_1^{\mathcal{I}} = r_2^{\mathcal{I}}$
Subsumption	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for all $\mathcal{I}$
Concept definition	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$ for all $\mathcal{I}$
Concept assertion	$a : C$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
Role assertion	$(a, b) : r$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$

Table 5: Syntax and semantics of $\mathcal{ALCHI}_{\mathcal{R}^+}$.

We elaborate on Definition 9 for an $\mathcal{ALCHI}_{\mathcal{R}^+}$ tableau by adding new properties and associated rules as follows.

6. if $\forall r.C \in \mathcal{L}(x)$ and r is a transitive role, then for all $y \in \mathbf{S}$ such that $r \in \mathcal{E}(\langle x, y\rangle)$, $\forall r.C \in \mathcal{L}(y)$.

   $\forall r_{trans}$-rule: if $\forall r.C \in \mathcal{L}(x)$ which r is transitive and there exists a y such that $\mathcal{E}(\langle x, y\rangle)$, then $\mathcal{L}(y) \rightarrow \mathcal{L}(y) \cup \{\forall r.C\}$.

$$\begin{array}{c} \mathcal{L}(x) = \{\forall r.C\} \\ \mathcal{L}(y) = \{D\} \\ \mathcal{E}(\langle x, y\rangle) = \{r\} \\ | \\ \mathcal{L}(x) = \{\forall r.C\} \\ \mathcal{L}(y) = \{C, \forall r.C,\ D\} \\ \mathcal{E}(\langle x, y\rangle) = \{r\} \end{array}$$

7. $r \in \mathcal{E}(\langle x, y\rangle)$ iff $r^- \in \mathcal{E}(\langle y, x\rangle)$.
   r^--rule: if $r \in \mathcal{E}(\langle x, y\rangle)$, then $r^- \in \mathcal{E}(\langle y, x\rangle)$.

$$\begin{array}{c} \mathcal{E}(\langle x, y\rangle) = \{r\} \\ | \\ \mathcal{E}(\langle y, x\rangle) = \{r^-\} \end{array}$$

8. if $r \in \mathcal{E}(\langle x, y\rangle)$ and $r \sqsubseteq v$ (or $r^- \sqsubseteq v^-$) then $v \in \mathcal{E}(\langle x, y\rangle)$.
   $r_{\sqsubseteq}$-rule

$$\begin{array}{c} \mathcal{E}(\langle x, y\rangle) = \{r\} \\ | \\ \mathcal{E}(\langle x, y\rangle) = \{r, v\} \end{array}$$

3.3 From image to symbolic representation

This section deals with the computation of qualitative relationships given a segmented image and the construction of a symbolic representation for the observation. The transformation of representation space from low level numerical data to symbolic level is the first phase of the framework. Concretely, objects and spatial information are extracted and represented within terminologies using an ABox. Furthermore, the symbolic representation can be used for reasoning service in the logical formalism.

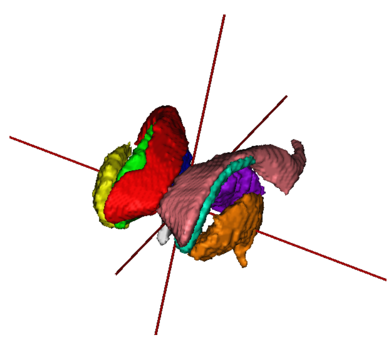

Figure 3: A segmented brain volume (structures are marked by different colors, e.g. the left lateral ventricle is in red.).

As shown in Figure 3, spatial relationships between structures in the brain is an important feature for the image interpretation. Here we consider directional relations, computed using angle histograms. The satisfaction degree for a given direction is computed as the intersection of the histogram (e.g. Figure 5) and the fuzzy set defining the semantics of direction (e.g. Figure 6).

We take the right caudate nucleus (target object) and the right lateral ventricle (reference object)[2] as an example to illustrate the computation. The structures are presented in Figure 4. The histogram of angles includes the count of angles of each pair of voxels in the right caudate nucleus and the right lateral ventricle. The angle is composed by a pair α_1 and α_2. α_1 takes value in $[-\pi, \pi]$ by measuring the angle between the projection of the segment joining two voxels on the x-y coordinate plane and the x-coordinate axis. α_2 measures the angle between the segment and the x-y projection, and takes value in $[-\pi/2, \pi/2]$. The histogram is normalized and contains 256 bins for α_1 and 128 bins for α_2 as shown in Figure 5. For each two pair of structures, six satisfaction degrees can be computed for each direction (i.e. "right", "left", "above", "below", "in front of", "behind".).

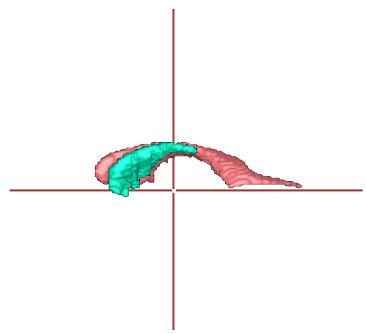

Figure 4: The right lateral ventricle and the right caudate nucleus seen from the right side.

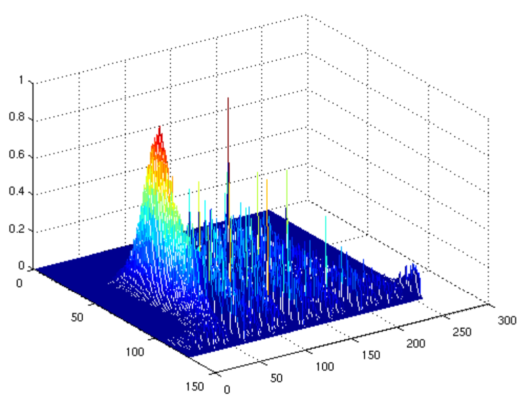

Figure 5: The histogram of angles between right lateral ventricle and right caudate nucleus.

Then we demonstrate how to choose a threshold in order to convert fuzzy relations to crisp ones. The learning process is based on a database of 50 brain MRIs. All the structures in a brain image in the database are manually segmented. This database is composed by 30 healthy images and 20 images presenting a brain tumor (with different localizations, types and sizes). The set of healthy images consists of the IBSR database ("Internet Brain Segmentation Repository")[3] and some images from the OASIS database ("Open Access

[2]Here we use "right" to represent structures in the right part of image for the simplicity of visualization (i.e. the structure in the right part of image is the left part in the brain.).

[3]The MR brain data sets and their manual segmentations were provided by the Center for Morphometric Analysis at Massachusetts General Hospital and are available at `http://www.cma.mgh.harvard.edu/ibsr/`.

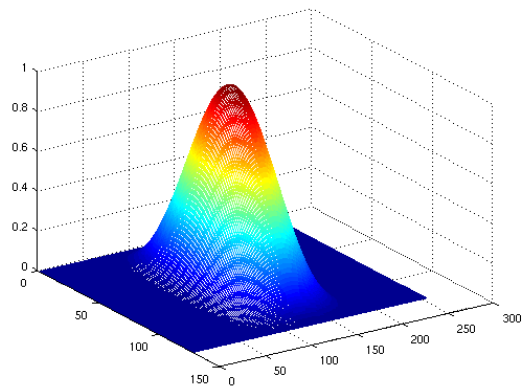

Figure 6: Fuzzy relation "right".

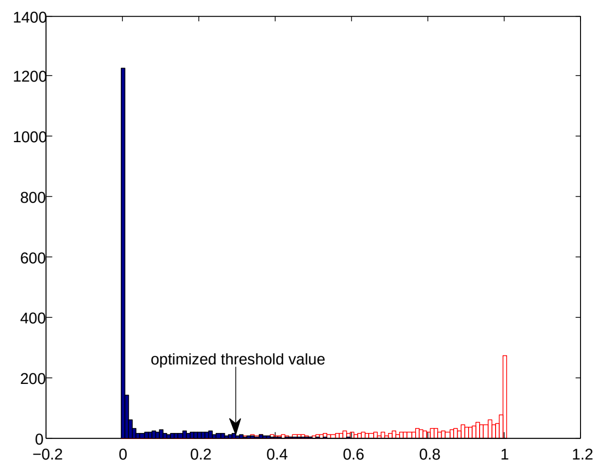

Figure 7: The histogram of intersection degree of the satisfied pairs (red bins) and the ones of the other part. The vertical arrow indicates the optimized threshold value.

Series of Imaging Studies")[4]. Manual segmentations are available for the IBSR database. All other images have been manually segmented and tumor segmentations have been validated by experts.

These segmentations are used for choosing a threshold t for every direction. We manually select all the pairs of segmented structures which satisfy the relation and all the unsatisfied pairs. We take the histogram of intersection degrees of the satisfied pairs (red bins) and the ones of the other part (blue bins) (Figure 7). Then the threshold is taken by minimizing the error of mis-classification: $E = \frac{\sharp(x_1 > threshold)}{\sharp(x_1)} + \frac{\sharp(x_2 < threshold)}{\sharp(x_2)}$ where x_1 (resp. x_2) represents unsatisfied pairs (resp. satisfied pairs) and $\sharp(x_1 > threshold)$ (resp. $\sharp(x_2 > threshold)$) represents the number of mis-classification of unsatisfied pairs (resp. the number of mis-classification of satisfied pairs).

The symbolic representation is generated in an ABox. All the detected structures and spatial relations are saved in a structural XML file using corresponding alias. A simple example is given in Figure 8.

[4] `http://www.oasis-brains.org`, built thanks to Pubmed Central submissions: P50 AG05681, P01 AG03991, R01 AG021910, P50 MH071616, U24 RR021382, R01MH56584.

```
−<rdf:RDF xml:base="files:///data/yiyang/dataset/Ontologies/observation">
  −<owl:NamedIndividual rdf:about="files:///data/yiyang/dataset/Ontologies/observation#ind5">
     <rdf:type rdf:resource="files:///data/yiyang/dataset/Ontologies/observation#LVr"/>
   </owl:NamedIndividual>
  −<owl:NamedIndividual rdf:about="files:///data/yiyang/dataset/Ontologies/observation#ind0">
     <rdf:type rdf:resource="files:///data/yiyang/dataset/Ontologies/observation#CDl"/>
   </owl:NamedIndividual>
  −<owl:NamedIndividual rdf:about="files:///data/yiyang/dataset/Ontologies/observation#ind1">
     <rdf:type rdf:resource="files:///data/yiyang/dataset/Ontologies/observation#CDr"/>
   </owl:NamedIndividual>
  −<owl:NamedIndividual rdf:about="files:///data/yiyang/dataset/Ontologies/observation#ind4">
     <rdf:type rdf:resource="files:///data/yiyang/dataset/Ontologies/observation#LVl"/>
   </owl:NamedIndividual>
  −<owl:NamedIndividual rdf:about="files:///data/yiyang/dataset/Ontologies/observation#ind1">
     <right rdf:resource="files:///data/yiyang/dataset/Ontologies/observation#ind5"/>
   </owl:NamedIndividual>
  −<owl:NamedIndividual rdf:about="files:///data/yiyang/dataset/Ontologies/observation#ind5">
     <right rdf:resource="files:///data/yiyang/dataset/Ontologies/observation#ind0"/>
   </owl:NamedIndividual>
  −<owl:NamedIndividual rdf:about="files:///data/yiyang/dataset/Ontologies/observation#ind5">
     <right rdf:resource="files:///data/yiyang/dataset/Ontologies/observation#ind4"/>
   </owl:NamedIndividual>
 </rdf:RDF>
<!-- Created with zsh script -->
```

Figure 8: A sample of the ABox.

3.4 Example

The complete knowledge base is given as follows:

$$\begin{aligned}
TBox = \{Hemisphere &\sqsubseteq \exists isPartOf.Brain\\
BrainStructure &\sqsubseteq \exists isPartOf.Brain\\
BrainDisease &\sqsubseteq \exists isPartOf.Brain \sqcap \neg BrainStructure\\
Tumor &\sqsubseteq BrainDisease\\
LVl &\sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl\\
LVr &\sqsubseteq BrainStructure \sqcap \exists(leftOf \sqcap closeTo).CNr\\
CNl &\sqsubseteq BrainStructure\\
CNr &\sqsubseteq BrainStructure\}
\end{aligned}$$

The role axioms are described as:

$$\begin{aligned}
RBox = \{rightOf &\equiv leftOf^-\\
above &\equiv below^-\\
closeTo &\equiv closeTo^-\\
farFrom &\equiv farFrom^-\\
isPartOf \circ isPartOf &\sqsubseteq isPartOf\\
hasPart \circ hasPart &\sqsubseteq hasPart\\
isPartOf &\equiv hasPart^-\}
\end{aligned}$$

The ABox represents the observation of structures in an image and the relationships between them. In this example, both recognized and unrecognized structures are represented by individuals. Spatial relations between the unknown structure (b) and recognized structures (a, c) are represented by roles. For instance, a region is recognized as the left caudate nucleus (CNl), denoted by a. The region of brain is denoted by c. An unknown region is segmented and their relationships are computed. Such an observation can be represented as:

$$\begin{aligned} ABox = \{a &: CNl \\ b &: Unknown\ Object \\ c &: Brain \\ \langle a, b\rangle &: leftOf, closeTo \\ \langle b, c\rangle &: isPartOf\} \end{aligned}$$

In this example, the ABox describes an observation of a given scene and the objective is to find a reasonable description of the unknown object b. Therefore, the relationships between b and other individuals are important for the reasoning. We demonstrate the spatial reasoning by a concept subsumption test using the tableau method in Appendix A. The subsumption test of the example in the appendix (which can be seen as an abductive reasoning) provides a tableau approach to answer the first question of the spatial reasoning problem (recognition of an unknown object using the background knowledge and the spatial arrangements in the observation). In the first step, we can derive $\langle b, a\rangle : leftOf^{-}, closeTo^{-}$ from $\langle a, b\rangle : leftOf, closeTo$ using the inverse property of roles. According to the definitions $rightOf \equiv leftOf^{-}$ and $closeTo \equiv closeTo^{-}$ in the RBox, the spatial relationships $rightOf$ and $closeTo$ between b and a can be derived. Another type of spatial reasoning consists in verifying inclusion relations by using the transitive property of transitive roles like $isPartOf$ in the tableau method. This type of reasoning answers the second question in a restricted way. For a transitive role (e.g. $isPartOf$), we can check the satisfiability of the relation between two objects via the third one. Rule 6 in Section 3.2 provides a corresponding rule to verify the implicit transitive relation in the tableau method. In the example, $\exists isPartOf.Hemisphere \sqsubseteq \exists isPartOf.Brain$ is verified. This example is modeled as a test of satisfiability problem which can be seen as the answer for the third question. These simple spatial reasonings are not as powerful as those in the references, however, we consider the objects as a set of pixels instead of approximating an object by a single pixel (the transitivity of direction does not hold anymore). In addition, spatial reasoning on compositions of relations in particular topological ones are still an open problem.

4 Abductive reasoning

Abductive reasoning is a backward-chaining inference, consisting in generating hypotheses and finding the "best" explanation of a given observation. New knowledge should be added in order to positively entail the observation. Image interpretation can be expressed as an abductive reasoning mechanism. When facing a pathological brain image, an expert has to resort to his knowledge of pathological anatomy, in order to give an explanation for the observed image. In this section, we will introduce how abduction is applied in image interpretation from two aspects (generation of hypotheses and selection of a preferred explanation).

4.1 State of the art of abductive reasoning

The term "Abduction" was first proposed by Charles S. Peirce in philosophy. Afterwards, abduction has been developed in artificial intelligence and cognitive science. Aliseda [2] gave a general overview of abduction in propositional logic and proposed tableau methods for abduction. In the context of Description Logics, four types of abduction problems are described by Elsenbroich [18]. Let $\mathcal{L}$ be a DL, $\mathcal{K} = \{\mathcal{T}, \mathcal{A}\}$ be a knowledge base in $\mathcal{L}$, C, D two concepts in $\mathcal{L}$ and suppose that they are satisfiable with respect to $\mathcal{K}$. The logical formalisms of abduction in DLs are represented as follows:

- Concept abduction: given an observation concept O, a hypothesis is a concept H such that $\mathcal{K} \models H \sqsubseteq O$.
- TBox abduction: let $C \sqsubseteq D$ be satisfiable w.r.t $\mathcal{K}$, the hypothesis is a set of axioms $S_T = \{E_i \sqsubseteq F_i \mid i \leq n\}$ such that $\mathcal{K} \cup S_T \models C \sqsubseteq D$.
- ABox abduction: let S_a be a set of assertions representing the observation, a hypothesis is a set S_b of ABox assertions such that $\mathcal{K} \cup S_b \models S_a$.

- Knowledge base abduction: let ϕ be a consistent set of ABox or TBox assertions w.r.t. $\mathcal{K}$. A solution of knowledge base abduction, considered as a combination of TBox abduction and ABox abduction, is any finite set $S = \{\psi_i \mid i \leq n\}$ such that $\mathcal{K} \cup S \vDash \phi$.

An explanation H for an abduction problem has the following properties [18]:

- Consistency: $\mathcal{K} \cup H$ is consistent.
- Relevance: ϕ is not entailed by H ($H \nvDash \phi$).
- Semantic minimality: there does not exist a hypothesis H_i such that $H_i \vDash H$.

Image interpretation task was regarded as an abduction problem in [3, 25, 37, 43]. In [37], DL-safe rules were proposed to map high-level concepts and occurrence objects in the scene and their relationships. The rules ensure the expressivity and preserve the decidability of the reasoning. The authors extended the abductive reasoning with DL-safe rules in a probabilistic reasoning engine in [25]. Abduction was applied on a robot vision system in order to transform low level sensor data into symbolic representation in [43]. Similarly, the backward reasoning is provided by a set of defined first-order logic rules. The explanation is found by searching the rules when the conditions in the rules are fulfilled in the observation. However, only the concept defined in the rules can be inferred using these formalisms. In [3], the image interpretation was formulated as a concept abduction problem. The DL is expressed in $\mathcal{EL}$. The knowledge base is processed using formal concept analysis and the abductive reasoning is tackled by a recursive erosion on the lattice based representation. In this way, not only defined concepts but also undefined complex concepts can be inferred. To the best of our knowledge, there is not much work that models image interpretation as an abduction problem. Most of it relies on additional rules apart from basic knowledge base which build the causal relations directly between the cause and the effect. The backward reasoning is then seen as a query problem or a searching strategy to find the corresponding defined hypotheses. In this thesis, we attempt to construct a more expressive knowledge base in DLs, which can be reused without specific rules. Beyond restricting the explanation to the defined concepts in the knowledge base, a new generated hypothesis based on the background knowledge is also considered in our formalism.

Apart from the abduction mechanism mentioned above, the tableau method is a powerful approach for solving abduction problems. The tableau method was first adapted in Description Logics formalisms for a market matchmaking problem [13]. Colucci *et al.* modeled this problem as a concept abduction in the DL $\mathcal{ALN}$ [13], where the observations are the demand and the supply is treated as the explanation for the meet of the request. The tableau method has also been studied by Halland *et al.* in [26] for a TBox abduction problem. For a TBox abduction problem, a TBox axiom in the form $\phi = C \sqsubseteq D$ is an explanation enforcing the entailment of the observation, which is also in the form of a TBox subsumption form. Similar to the tableau method for the concept abduction, if the disjunction of two concepts A_1 and $\neg A_2$ can create a clash of the tableau, then $A_2 \sqsubseteq A_1$ is considered as a potential explanation. Their ideas are similar to ours, which are extended from the basic strategy in [2, 13]. However, complex concepts are represented in a restricted form for role qualification. Only $\top$ and $\bot$ can appear in the domain of the role qualification ($\exists r.\top$, $\exists r.\bot$, $\forall r.\top$ and $\forall r.\bot$). In our context, spatial relations are represented in terms of roles, which requires more elaborated and rich complex concepts.

Klarman *et al.* [31] present a tableau method for the ABox abduction in $\mathcal{ALC}$. This method integrates logic reasoning techniques of the first-order logic. First, knowledge and observation are transformed into first-order logic. Then, a tableau in the context of the first-order logic is built and solutions are selected in the open branches. The results are transformed into Description Logic from the first-order logic in the end. Though the transformation from Description Logic formalism to a representation in terms of first-order logic facilitate to reuse the existing tableau method in the developed first-order logic formalism. The computation of a model of first-order logic is undecidable. Infinitely models may be computed. In [15, 16], the authors also focus on ABox abduction problems. In [16], Du *et al.* introduced a tractable approach to ABox abduction, called the query abduction problem. This problem focuses on giving the explanations, which are new facts neither in the observation nor in the assertional knowledge, for an observation. The observation is represented by a Boolean conjunctive query in the form $\exists x \Phi(x, c)$, where $\Phi(x, c)$ is a conjunction of concepts assertions and role assertions, x represents a variable and c represents an individual. However, the potential hypotheses are restricted to atomic concepts and roles in the TBox. As we have described above, the tableau method for an abduction is proposed in a general way. The explanations are formulated without a specific purpose. In image interpretation, a structural formula, which explains the spatial relationships and high-level semantics, is expected to be constructed.

We then move to the other aspect of abduction problem: the selection problem. As a set of syntactical candidates generated using the tableau method, the selection relies on explicit restrictions for choosing the "best" explanation. Restrictions concern filtering out inappropriate hypotheses, for instance inconsistent hypotheses (H_1 such that $\mathcal{K} \cup H_1 \vDash \emptyset$) and independent hypotheses (H_1 entails the observation independently of the background knowledge, such that $H_1 \vDash O$). These types of hypotheses need to be removed. In addition, minimality criteria are required to select the "best" among the filtered candidates. Though the desired candidates are selected, the solutions can be infinite. Therefore, defining minimality criteria is an important manner to find a preference among all the potential hypotheses. Bienvenu discussed a set of basic minimality criteria for abductive reasoning in [8]. These minimality criteria is a general evaluation metric for selecting a preferred explanation. The minimality criteria proposed by Bienvenu are satisfied for a certain type of hypotheses where a hypothesis is a conjunction of atomic concepts in a predefined candidate set. We attempt to find a more reasonable criterion for a specific image interpretation use.

4.2 Abductive reasoning for image interpretation

As introduced above, the tableau method is a powerful way to find an explanation given the observation. In this work, we apply this general strategy to image interpretation expressed as a concept abduction problem. An observed image is represented by an ABox, which is supposed to be consistent with the knowledge base. In the concept abduction problem, the observation concept is constructed on the basis of the individual that is selected to be explained and contextual information in the ABox. In this part, the knowledge base is in the DL $\mathcal{ALC}$ (extension to $\mathcal{ALCHI}_{\mathcal{R}^+}$) are planned for future work).

We then consider extending and automating the tableau method for image interpretation. Following the processing steps presented in Section 3, the given image observation is translated into an ABox. An unknown object is represented by a most specific concept. This concept converts contextual information in the ABox to an appropriate concept to represent the object.

Definition 14 (**Most specific concept** [3])**.** *Given a TBox $\mathcal{T}$ and an associated interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ in a DL $\mathcal{L}$, let $X \subseteq \Delta^{\mathcal{I}}$ be a subset of the interpretation space and E a defined concept in $\mathfrak{C}(\mathcal{L})$. The concept E is defined as the most specific concept of X w.r.t. $\mathcal{I}$ if:*

- $X \subseteq E^{\mathcal{I}}$.
- *for every defined concept $F \in \mathfrak{C}(\mathcal{L})$ with $X \subseteq F^{\mathcal{I}}$, we have $E \sqsubseteq_{\mathcal{T}} F$.*

An example of ABox is given as follows:

$$
\begin{aligned}
\mathcal{A}_{obs} = \{ t_1 &: BrainTumor \\
e_1 &: NonEnhanced \\
l_1 &: LateralVentricle \\
p_1 &: PeripheralCerebralHemisphere \\
(t_1, e_1) &: hasEnhancement \\
(t_1, l_1) &: farFrom \\
(t_1, p_1) &: hasLocation \}.
\end{aligned}
$$

The most specific concept of the $t_1^{\mathcal{I}}$ is :

$$
\begin{aligned}
BrainTumor &\sqcap \exists hasEnhancement.NonEnhanced \\
&\sqcap \exists farFrom.LateralVentricle \\
&\sqcap \exists hasLocation.PeripheralCerebralHemisphere
\end{aligned}
$$

As all observed objects in the ABox can be formulated by the most specific concept, our problem is modeled as a concept abduction. $\mathcal{K} \vDash H \sqsubseteq O$. H is an explanation of the given observation O if H is subsumed by O w.r.t. $\mathcal{K}$. The subsumption problem can be converted into a test of satisfiability which requires to prove that $H \sqcap \neg O$ is unsatisfiable. According to the strategy proposed by Aliseda [2], a potential hypothesis H is the concept which makes the tableau of $H \sqcap \neg O$ closed as a consequence.

In the context of acyclic TBox, the classical tableau method integrates axioms of the TBox using the normalization process. This optimization technique is suitable for forward-chaining inference. For instance, a concept D can be inferred by getting a concept C with the axiom $C \sqsubseteq D$ in a deduction way since a model of the concept C is also a model of D. However, this is not suitable for a backward-chaining inference, which intends to find a concept C as a hypothesis for D. A possible solution is to add internalized concept (see Definition 8) in the tableau.

If $C_i \sqsubseteq D_i$, then $\top \sqsubseteq \neg C_i \sqcup D_i$ and $C_{\mathcal{T}} \equiv \top$. As a consequence, all interpretations of the TBox $\mathcal{T}$ are equivalent to interpretations of the internalized concept $C_{\mathcal{T}}$. Therefore, every interpretation elements belongs to $C_{\mathcal{T}}^{\mathcal{I}}$. Its use has a result in $C \equiv C \sqcap C_{\mathcal{T}}$.

We reformulate the subsumption in terms of satisfiability: the concept $H \sqcap \neg O$ is not satisfiable w.r.t. $\mathcal{T}$, where H is an explanation, O is an observation, $\mathcal{T}$ is a TBox. This problem can be reduced by testing the satisfiability of a concept $H \sqcap \neg O \sqcap C_{\mathcal{T}}$, where $C_{\mathcal{T}}$ is the internalized concept of $\mathcal{T}$. The concept H that causes unsatisfiability of $H \sqcap \neg O \sqcap C_{\mathcal{T}}$ is a potential hypothesis, i.e. the tableau built from this concept is closed. We follow this strategy and propose an extension of the work by Colucci *et al.* in [13].

Each interpretation element in the tableau has now four label functions (instead of $\mathcal{L}(x)$ and $\mathcal{E}(\langle x, y\rangle)$ in Definition 9): $\mathbf{T}(x)$, $\mathbf{F}(x)$, $\mathbf{T}(\langle x, y\rangle)$, $\mathbf{F}(\langle x, y\rangle)$, where x, y are interpretation elements in $\Delta^{\mathcal{I}}$. They are defined as follows:

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A}\rangle$ be a knowledge base, x, y interpretation elements, C, D two concepts and r, s two roles in the given DL, we have:

- $\mathbf{T}(x)$ represents a set of concepts such that x is one of their interpretations: $C \in \mathbf{T}(x)$ iff $x \in C^{\mathcal{I}}$.
- $\mathbf{F}(x)$ represents a set of concepts such that x is not one of their interpretations: $D \in \mathbf{F}(x)$ iff $x \notin D^{\mathcal{I}}$.
- $\mathbf{T}(\langle x, y\rangle)$ represents a set of roles between x and y: $r \in \mathbf{T}(\langle x, y\rangle)$ iff $\langle x, y\rangle \in r^{\mathcal{I}}$.
- $\mathbf{F}(\langle x, y\rangle)$ represents a set of unsatisfiable roles between x and y: $s \in \mathbf{F}(\langle x, y\rangle)$ iff $\langle x, y\rangle \notin s^{\mathcal{I}}$.

In the initialization step, the root node of the tableau is initialized with the concept $C_{\mathcal{T}} \sqcap \neg O$. As $C_{\mathcal{T}} \sqcap \neg O$ belongs to $\mathbf{T}(1)$, we add its negation to $\mathbf{F}(1)$. This technique avoids adding the negation before selected concepts to generate contradictions in the tableau. We can prove the equivalence between $C \in \mathbf{T}(x)$ and $\neg C \in \mathbf{F}(x)$. Suppose that for $x \in \Delta^{\mathcal{I}}$, x is an interpretation element of a concept C, and x is also an interpretation of the concept $\neg C$. As a consequence, x is an interpretation of the concept $C \sqcap \neg C \equiv \bot$. There is no such interpretation. Thus, if $x \in C^{\mathcal{I}}$, then $x \notin (\neg C)^{\mathcal{I}}$.

We assume that the concepts are simplified in the NNF. For a concept $C \in \mathcal{ALC}$, $\neg C$ in the NNF is denoted by $\overline{C}$. The expansion rules used in our work are presented here:

1. Conjunction
   - **T)** if $C \sqcap D \in \mathbf{T}(x)$, we add C and D in $\mathbf{T}(x)$.
   - **F)** if $C \sqcup D \in \mathbf{F}(x)$, we add C and D in $\mathbf{F}(x)$.
2. Disjunction
   - **T)** if $C \sqcup D \in \mathbf{T}(x)$, the branch is divided into two $(\mathbf{T}(x_1), \mathbf{T}(x_2))$. $\mathbf{T}(x_1) = \mathbf{T}(x) \cup \{C\}$ and $\mathbf{T}(x_2) = \mathbf{T}(x) \cup \{D\}$
   - **F)** if $C \sqcap D \in \mathbf{F}(x)$, the branch is divided into two $(\mathbf{F}(x_1), \mathbf{F}(x_2))$. $\mathbf{F}(x_1) = \mathbf{F}(x) \cup \{C\}$ and $\mathbf{F}(x_2) = \mathbf{F}(x) \cup \{D\}$
3. Existential restriction
   - **T)** if $\exists r.C \in \mathbf{T}(x)$ and there does not exist a y such that $r \in \mathbf{T}(\langle x, y\rangle)$ and $C \in \mathbf{T}(y)$, we create a new interpretation element y then add r in $\mathbf{T}(\langle x, y\rangle)$, and C in $\mathbf{T}(y)$.
   - **F)** if $\forall r.C \in \mathbf{F}(x)$ and there does not exist a y such that $r \in \mathbf{T}(\langle x, y\rangle)$ and $C \in \mathbf{T}(y)$, we create a new interpretation element y then add r in $\mathbf{T}(\langle x, y\rangle)$, and C in $\mathbf{F}(y)$.

4. Universal restriction

   T) if $\forall r.C \in \mathbf{T}(x)$ and for all y such that $r \in \mathbf{T}(\langle x, y\rangle)$ and $C \notin \mathbf{T}(y)$, we add C in $\mathbf{T}(y)$.

   F) if $\exists r.C \in \mathbf{F}(x)$ and for all y such that $r \in \mathbf{T}(\langle x, y\rangle)$ and $C \notin \mathbf{T}(y)$, we add C in $\mathbf{F}(y)$.

5. Replacement of axioms in $\mathcal{T}$

   T) if $A \in \mathbf{T}(x)$ and $A \equiv C \in \mathcal{T}$, we add C in $\mathbf{T}(x)$.

   T) if $\neg A \in \mathbf{T}(x)$ and $A \equiv C \in \mathcal{T}$, we add $\overline{C}$ in $\mathbf{T}(x)$.

   F) if $\neg A \in \mathbf{F}(x)$ and $A \equiv C \in \mathcal{T}$, we add $\overline{C}$ in $\mathbf{F}(x)$.

   F) if $A \in \mathbf{F}(x)$ and $A \equiv C \in \mathcal{T}$, we add C in $\mathbf{F}(x)$.

The contradiction in the adapted form is classified in two types: homogeneous clash and heterogeneous clash.

Definition 15 (**Clash** [13])**.** *A clash in a branch can be divided into two categories:*

1. *A branch is defined as a homogeneous clash if:*
   - $\bot \in \mathbf{T}(x)$ *or* $\top \in \mathbf{F}(x)$.
   - $\{A, \neg A\} \in \mathbf{T}(x)$ *or* $\{A, \neg A\} \in \mathbf{F}(x)$.
2. *A branch is defined as a heterogeneous clash if:*
   - $\{A \text{ } or \text{ } \neg A\} \in \mathbf{T}(x) \cap \mathbf{F}(x)$.

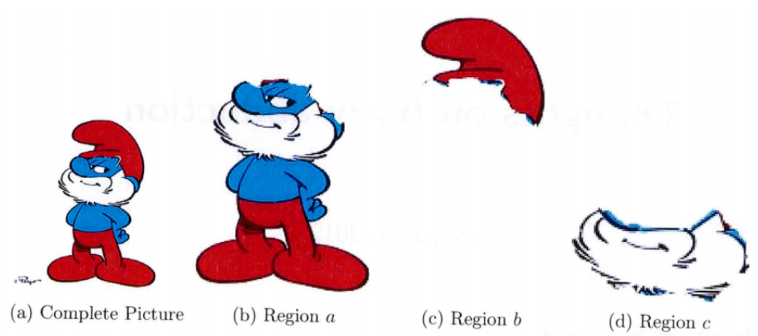

Figure 9: The smurf and its segmentation of different elements.

We illustrate this procedure with the image of the Smurf (Figure 9)[5]. In the TBox, we describe the background knowledge that a leader smurf has a beard and wears a red hat as follows:

$$\begin{aligned}\mathcal{T} = \{&SmurfLeader \sqsubseteq \exists hasPart.Beard \sqcap \exists hasOnTop.RedHat,\\ &RedHat \equiv Hat \sqcap \exists hasColor.Red\}\end{aligned}$$

Suppose that we can recognize three parts a, b, c in an image and the observation is encoded by the following ABox:

$$\begin{aligned}\mathcal{A}_{obs} = \{&(a, b) : hasOnTop,\\ &(a, c) : hasPart,\\ &b : Hat,\\ &b : \exists hasColor.Red,\\ &c : Beard\}.\end{aligned}$$

[5]An example of abduction for the interpretation of a brain tumor image is given in Appendix B.

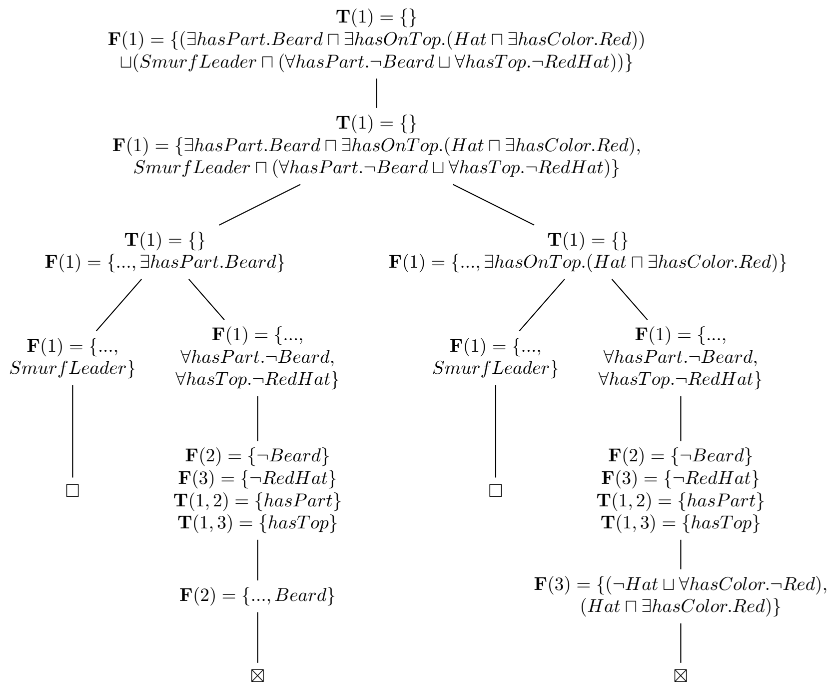

Figure 10: The process of constructing the tableau by applying expansion rules.

Then, the most specific concept of $a^{\mathcal{I}}$ is constructed as:

$$O = \exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$$

In our approach of the construction of the tableau, the initial node consists of two complex concepts. One concept is satisfiable in $\mathbf{T}(1)$. Here, the concept is empty because we do not specify a constraint. The other is non-satisfiable concept in $\mathbf{F}(1)$ (i.e. $\neg(C_{\mathcal{T}} \sqcap O) \equiv \neg C_{\mathcal{T}} \sqcup O$).

By applying expansion rules, the construction process of the tableau is shown in Figure 10. The hypotheses are generated from open branches. In this example, we have two sets of concepts:

$$H_1 = \{SmurfLeader,\ \exists hasPart.Beard\}$$
$$H_2 = \{SmurfLeader,\ \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)\}$$

The concepts in these two sets are basic elements to build a hypothesis H. The hypothesis H is considered as a concept in $\mathbf{T}(1)$. To close the tableau, we can take these concepts in $\mathbf{F}(1)$ to generate a heterogeneous clash. The first branch is closed if one takes the concept $SmurfLeader$, $\exists hasPart.Beard$ or the combination of these two concepts $SmurfLeader \sqcap \exists hasPart.Beard$. The concept $SmurfLeader$ is also a concept for closing the second branch. We can then consider that $H \equiv SmurfLeader$ is a potential hypothesis. Apart from this case, $\exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $SmurfLeader \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $SmurfLeader \sqcap \exists hasPart.Beard$ are also potential hypotheses.

At this stage, we presented a construction process of the tree model using the tableau method. Then we have a set of concepts for each open branch in the tableau. One potential hypothesis is the combination of these concepts. The considered concepts can close the table if at least one concept is selected in each branch. To avoid redundancy, we take the minimum hitting set as the first post-processing on constructing hypotheses from the candidate sets.

Definition 16. *(Hitting set) Let $\{S_1, \ldots, S_n\}$ be a collection of sets. A hitting set T is a subset $T \subseteq \cup_{i=1}^{n} S_i$ such that T contains at least one element of each set in the collection $T \cap S_i \neq \emptyset$ $(1 \leq i \leq n)$. The minimal hitting set is a hitting set T_m if $\nexists$ hitting set T' such that $T' \subset T_m$.*

Such a minimal hitting set of concepts guarantees a syntactical hypothesis in the given DL. The inconsistent hypotheses ($\mathcal{K} \cup H \models \bot$) and irrelevant hypotheses ($H \models O$) are also required to be removed during

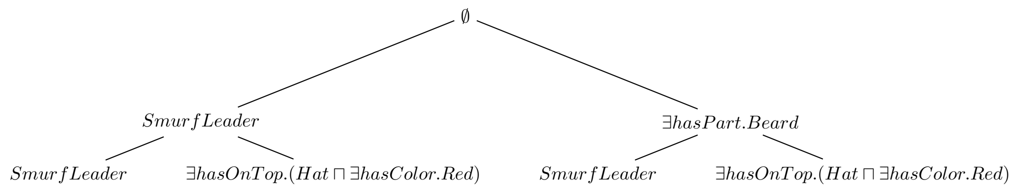

Figure 11: Hitting set construction tree.

the construction process. An exhaustive algorithm is elaborated from the minimal hitting set algorithm [41]:

Input: A collection of sets $\{S_1, \dots, S_n\}$;
Output: A collection of hitting sets $\mathcal{H}$;
Initialization $\mathcal{H} = \{\}$;
Root initialization.;
For (i From 1 to n) **do:**
  Create a new leaf for every S_i in each branch;
  An intermediate hypothesis H_j is the conjunction of all the concepts in the same branch;
  Delete the branch j if H_j is inconsistent w.r.t. the TBox;
End
The conjunction of all concepts in each branch j represents a potential hypothesis H_j;
return: $\mathcal{H}$;

Algorithm 1: Exhaustive search algorithm of selecting hitting sets.

We illustrate the algorithm by the example of Smurf. Note that we got two sets of concepts in the previous step. Applying this algorithm, a tree is initialized with an empty root. Then we construct recursively the tree by adding all concepts in the set h_i as new leaves (Figure 11). In this case, all assumptions are consistent with the TBox: $H_1 = SmurfLeader$, $H_2 = \exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $H_3 = SmurfLeader \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$, $H_4 = SmurfLeader \sqcap \exists hasPart.Beard$. The second hypothesis is removed because $H = \exists hasPart.Beard \sqcap \exists hasOnTop.(Hat \sqcap \exists hasColor.Red)$ is not an independent explainable hypothesis ($H \vDash \mathcal{O}$).

The generated hypotheses from this algorithm are consistent and syntactically minimal. A preference explanation depending on a minimality criterion will be presented in the next subsection.

4.3 Minimality criteria

Abduction can be also defined as inference to the best explanation. A set of potential hypotheses is generated based on the tableau method. However, not all the hypotheses are equally important and only a part of them can be selected according to the defined preference by filtering inconsistent and redundant ones. One potential hypothesis is an explanation if and only if the requirements in the previous section are fulfilled to avoid irrelevant results and observations themselves. The preference relies on different kinds of minimality criteria, which may be different among the application domains. In principle, the minimal hypothesis is the smallest additional information that we add in the knowledge base in order to entail the evidence.

For a concept abduction problem, several minimality criteria are proposed. In [8], Bienvenu provided five basic criteria for abduction problems. An abduction problem is defined as: $\langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$, where $\mathcal{T}$ is a TBox, $\mathcal{H}$ is a set of atomic concepts and $\mathcal{O}$ is the observed concept.

Definition 17. *(Explanation [8])* $\{A_1, \dots, A_n\} \subseteq \mathcal{H}$ *is a subset of* $\mathcal{H}$*. An explanation of abduction problem* $\langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$ *can be formulated from the set if:*

- $A_1 \sqcap \dots \sqcap A_n$ *is satisfiable w.r.t.* $\mathcal{T}$*.*
- $\mathcal{T} \vDash A_1 \sqcap \dots \sqcap A_n \sqsubseteq \mathcal{O}$

In this problem, an hypothesis is formulated with the conjunction of atomic concepts. Five minimality criteria are then defined as follows.

Definition 18. *(Minimality criteria [8]) For an abduction problem* $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$, $\mathcal{A} = \{H_1, \ldots, H_n\} \subseteq \mathcal{H}$ *is an explanation of the problem* $\mathcal{P}$, $\langle H_{(1)}, \ldots, H_{(n)} \rangle$ *is a priority order of* $\mathcal{H}$*, and* $w : \mathcal{H} \rightarrow \mathbb{N}$ *is a weighting function to define the importance of the hypotheses. The criteria can be defined as follows:*

- $\mathcal{A}$ *is* $\subseteq -minimal$ *if there does not exist an explanation* $\mathcal{B}$ *for* $\mathcal{P}$ *such that* $\mathcal{B} \subsetneq \mathcal{A}$,
- $\mathcal{A}$ *is* $\leq -minimal$ *if there does not exist an explanation* $\mathcal{B}$ *for* $\mathcal{P}$ *such that* $|\mathcal{B}| < |\mathcal{A}|$,
- $\mathcal{A}$ *is* $\subseteq_P -minimal$ *if there does not exist an explanation* $\mathcal{B}$ *for* $\mathcal{P}$ *such that* $\mathcal{B} \cap H_i \subsetneq \mathcal{A} \cap H_i$ *and* $\mathcal{B} \cap H_j = \mathcal{A} \cap H_j$ *where* $1 \leq j < i$,
- $\mathcal{A}$ *is* $\leq_P -minimal$ *if there does not exist an explanation* $\mathcal{B}$ *for* $\mathcal{P}$ *such that* $|\mathcal{B} \cap H_i| < |\mathcal{A} \cap H_i|$ *and* $|\mathcal{B} \cap H_j| = |\mathcal{A} \cap H_j|$ *where* $1 \leq j < i$,
- $\mathcal{A}$ *is* $\sqsubseteq_w -minimal$ *if there does not exist an explanation* $\mathcal{B}$ *for* $\mathcal{P}$ *such that* $\sum_{B \in \mathcal{B}} w(B) < \sum_{A \in \mathcal{A}} w(A)$.

The author in [8] defined a particular concept abduction problem where the hypotheses are restricted to the conjunction of a given set of candidate concepts. Therefore, the criteria based on set inclusion relations are suitable for choosing a minimal set as the preferred explanation. In addition, priority order and importance weight of atomic concepts can be used as a metric to evaluate the preference of different hypotheses. In our context, the hypotheses are not limited with a set of atomic concepts, thereby semantic minimality based on concept subsumption is suitable for our goal.

Definition 19. *(Subsumption criterion) For an abduction problem* $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$*, and* $\{\langle P_1, \ldots, P_n \rangle\}$ *potential hypotheses,*

- P_i *is a* $\boldsymbol{\sqsubseteq -minimal}$ *explanation if there does not exist an explanation* P_j *for* $\mathcal{P}$ *such that* $P_i \sqsubseteq P_j$.

In our example of the Smurf, $H_1 = SmurfLeader$ is preferable to $H_4 = SmurfLeader \sqcap \exists hasPart.Beard$ since $H_4 \sqsubseteq H_1$. This example is not illustrative enough because H_1 is also a $\subseteq -minimal$ explanation. We have noted that a $\subseteq -minimal$ explanation in [8] always follows $\sqsubseteq -minimal$ criterion since $A_1 \sqcap \cdots \sqcap A_n \sqsubseteq A_1 \sqcap \cdots \sqcap A_m$ if $\{A_1, \cdots, A_m\} \subseteq \{A_1, \cdots, A_n\}$. Let us consider $Father \sqsubseteq Parent$ and $\exists hasChild.Person \sqsubseteq Parent$. The concept $Father$ can be inferred from $\exists hasChild.Person$. Therefore, $\exists hasChild.Person$ is then the $\sqsubseteq -minimal$ explanation.

Another specific criterion mentioned in [14] is dedicated to the matchmaking application, which is treated as a conditional concept abduction problem. A conditional concept abduction is represented as $\mathcal{T} \models C \sqcap H \sqsubseteq O$. In the formula, C is the condition concept representing the supply in the matchmaking problem and O is the observation representing the request in the matchmaking problem. H is the explanation to be found for the matching process. The authors proposed an irreducible-minimum solution in this formalism. All the concepts used in the problem are transformed into conjunctive normal form where complex concepts are replaced by the conjunction of super concepts considering GCI axioms in the TBox. After this transformation, a hypothesis is represented in terms of corresponding atomic concepts by finding the concepts which appear in the observation concept but not in the condition concept.

In [14], an explanation of the matchmaking problem is the difference between the concept of supply and the concept of the request The solution provides an explanation of why a supply is the compatible and promising one given a request in the market. In contrast with a matchmaking problem, an explanation in terms of high level concepts is favored in our context. An irreducible solution conforms to the criteria $\sqsubseteq -minimal$ and $\leq -minimal$. Therefore, an irreducible criterion is proposed by combining $\sqsubseteq -minimal$ criterion and $\leq -minimal$ criterion. The generated hypotheses and the observation are represented in conjunctive normal form (CNF). As the hypotheses are subsumed by the observation, the conjunct set of concepts of observation is a subset of the conjunct set of concepts of a hypothesis. A metric function $f_{ranking}$ can be selected to assess the importance of the concepts according to different requirements. The cardinality of the different part between the hypothesis and the observation is a possible $f_{ranking}$. Optionally, we can use the order priority or the weighted function proposed in Definition 18.

Definition 20. *(Irreducible criterion) For an abduction problem* $\mathcal{P} = \langle \mathcal{T}, \mathcal{H}, \mathcal{O} \rangle$*, and* $\{\langle P_1, \ldots, P_n \rangle\}$ *are potential hypotheses,*

- *P_i is an* $\boldsymbol{irreductible - minimal}$ *explanation if there does not exist an explanation P_j for $\mathcal{P}$ such that $f_{ranking}(\mathcal{T}, FNC(P_j), FNC(\mathcal{O})) < f_{ranking}(\mathcal{T}, FNC(P_i), FNC(\mathcal{O}))$.*

In the example of interpretation of the Smurf, the explanation $H_1 = SmurfLeader$ is the "best" explanation according to all the criteria presented in this section.

The criteria introduced by Bienvenu are useful when an assumption contains only atomic concepts. A hypothesis is constructed by the conjunction of atomic concepts. The criterion concerns inclusion relations of set theory, possibly with the priority and the weight function. In our problem, a hypothesis can be made by non-atomic concepts. The subsumption test is a good choice in many works, for example those of Colucci *et al.* [13] and those of Atif *et al.* [3]. The hierarchical relationship is considered to seek a more general hypothesis. The irreducibility criterion is a criterion combining a criterion of subsumption and a criterion of cardinality. This criterion has the advantage that the measure $f_{ranking}$ can be adapted to different measures. For example, if we apply the abduction in fuzzy logic, a degree of possibility can be considered as a practical measure.

5 Perspectives

During the first part of the thesis, we have exploited Description Logics and associated tableau method for knowledge representation and reasoning in image interpretation. A first model of background knowledge of brain anatomy including spatial information is proposed. At this stage, we have adapted the tableau method for generating preferred hypotheses w.r.t. the TBox.

Several directions will be considered in the next step. A work in the first place is to generate adaptive hypotheses iteratively. We have demonstrated that the tableau method produces a large amount of hypotheses, however, most of them are irrelevant or unsatisfiable. In order to avoid getting these hypotheses, an iterative method is considered. Instead of adding all internalized concepts into the tableau, we add only the ones corresponding to relative axioms in which observed concepts occur. This part will be explored in detail and be prepared for a submission to an international conference.

Concrete domains are necessary in image interpretation by providing an interface between abstract logical level and concrete image space, because semantic truth models may not have corresponding regions in concrete domains. For example, a concept $CNl \sqcap \exists rightOfCNr$ could be verified to be satisfiable w.r.t. to a defined TBox. However, this concept may not have a model in the image space.

Fuzzy logic is also a useful ingredient in knowledge representation dealing with imprecision and vague information. This aspect has been proved to be expressive for spatial reasoning by combining fuzzy relations in the concrete domains to Description Logics for image interpretation [29]. Another strategy to integrate fuzzy set theory into knowledge representation is to add fuzzy values to terminological and assertional knowledge at abstract logical level. This part of the work will allow dealing directly with satisfaction degrees of spatial relations (without thresholding them).

The minimality criterion is based on semantic subsumption at this stage. Meanwhile, an adapted metric is an additional information to evaluate the preference of the hypotheses. The metric can be associated with truth degrees in terms of fuzzy logic or the distance metric in the taxonomy of the knowledge base.

As we have discussed in Section 3, we have considered spatial relations in a more complex way without approximating an object by a single point, Therefore, qualitative spatial reasoning remains an open problem, especially including the compositions of spatial relations in particular topological ones in abductive reasoning services for the aim of image interpretation. This will be an important part of the thesis.

6 Other Activities

During the first half of my thesis, I also participated in different activities in the TII group and TAO group in LRI[6].

- I presented my work in the meetings of the ANR project LOGIMA[7].

[6]Laboratoire de Recherche en Informatique, Université ParisSud

[7]`http://logima.gforge.inria.fr/doku.php`

- I presented my work in the seminars of the group.
- I have taken doctoral courses in EDITE[8] (Formations in French: Rendre des articles attractifs, Intelligence Economique, Brevets, Pratique de la négociation de projet à l'international).

A An illustrative example using $\mathcal{ALCHI}_{\mathcal{R}^+}$ tableau method

This example can be treated as an abductive reasoning involving qualitative spatial relations. The example is illustrated according to the knowledge base in Section 3. The reasoning concerned testing the subsumption between a potential hypothesis $H \equiv LVl \sqcap \exists isPartOf.Hemisphere$ and an observed concept. The observed concept is represented by the most specific concept (see Definition 14) of $b^{\mathcal{I}}$ ($O \equiv \exists(leftOf^- \sqcap closeTo^-).CNl \sqcap \exists isPartOf.Brain$ and $H \equiv LVl \sqcap \exists isPartOf.Hemisphere$). To check subsumption of the two concepts H and O, $\mathcal{K} \vDash H \sqcap \neg O \sqsubseteq \bot$ is required to prove that $H \sqcap \neg O$ is unsatisfiable. Let x be the interpretation element of the concept $H \sqcap \neg O$.

The tableau is initialized with $\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap \forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain\}$. In the first step, $\sqcap$ and $\sqcup$ rule (Rule 2 and Rule 3) are applied and we obtain:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

To include the terminological knowledge, axioms like $C \sqsubseteq D$ in the TBox can be internalized into single concepts ($\neg C \sqcup D$) and added to $\mathcal{L}(x)$. Here, for the sake of simplicity of demonstration, we only add the internalization of the axiom $LVl \sqsubseteq BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl$ for the first branch.

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

We then apply $\sqcap$ and $\sqcup$ rule (Rule 2 and Rule 3) again on the first branch:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ BrainStructure,\ \exists(rightOf \sqcap closeTo).CNl\}$$

[8] http://edite-de-paris.fr/spip/

A clash (LVl, $\neg LVl$) is detected in the first part of the first branch (closed). We then apply $\exists$ rule (Rule 4) on the second part:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ BrainStructure,\ \exists(rightOf \sqcap closeTo).CNl\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ BrainStructure,\ \exists(rightOf \sqcap closeTo).CNl\}$$
$$\mathcal{L}(y) = \{CNl\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{rightOf, closeTo\}$$

Because of inverse role axiom in the RBox, we can add inverse roles in $\mathcal{E}(\langle x, y\rangle)$ (Rule 7) and apply $\forall$ rule (Rule 5) on the second part:

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ BrainStructure,\ \exists(rightOf \sqcap closeTo).CNl\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ BrainStructure,\ \exists(rightOf \sqcap closeTo).CNl\}$$
$$\mathcal{L}(y) = \{CNl,\ \neg CNl\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{rightOf, closeTo, leftOf^-, closeTo^-\}$$
$$\mathcal{E}(\langle y, x\rangle) = \{rightOf^-, closeTo^-, leftOf, closeTo\}$$

$$\boxtimes$$

The first branch of the tableau is closed because of the clash of CNl and $\neg CNl$ in the second part. We then explore the second branch. At first we apply the $\exists$ rule (Rule 4) and then $\forall$ rule (Rule 5):

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$

The axiom $Hemisphere \sqsubseteq \exists isPartOf.Brain$ is internalized and added into $\mathcal{L}(y)$. Then we continue to extend the second branch with expansion rules (Rules 2, 3, 4, 6, 7):

$$\mathcal{L}(x) = \{LVl \sqcap \exists isPartOf.Hemisphere \sqcap (\forall(leftOf^- \sqcap closeTo^-).\neg CNl \sqcup \forall isPartOf.\neg Brain)\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall(leftOf^- \sqcap closeTo^-).\neg CNl,\ \neg LVl \sqcup (BrainStructure \sqcap \exists(rightOf \sqcap closeTo).CNl)\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\ \neg Hemisphere \sqcup \exists isPartOf.Brain\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain,\ \forall isPartOf.\neg Brain,\ \neg Hemisphere\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$

$$\boxtimes$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\ \exists isPartOf.Brain\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$

$$\mathcal{L}(x) = \{LVl,\ \exists isPartOf.Hemisphere,\ \forall isPartOf.\neg Brain\}$$
$$\mathcal{L}(y) = \{Hemisphere, \neg Brain, \forall isPartOf.\neg Brain,\ \exists isPartOf.Brain\}$$
$$\mathcal{E}(\langle x, y\rangle) = \{isPartOf\}$$
$$\mathcal{L}(z) = \{Brain, \neg Brain, \forall isPartOf.\neg Brain\}$$
$$\mathcal{E}(\langle y, z\rangle) = \{isPartOf\}$$

$$\boxtimes$$

In both parts of the second branch, we get clashes ($Hemisphere$ and $\neg Hemisphere$ in $\mathcal{L}(y)$ for the first part, $Brain$ and $\neg Brain$ in $\mathcal{L}(z)$ for the second part). This implies that we cannot find a model for the concept $H \sqcap \neg O$. Therefore, it is unsatisfiable and we can conclude that $K \vDash H \sqsubseteq O$ and $LVl \sqcap \exists isPartOf.Hemisphere$ is a potential explanation of the observation.

B Abduction example for brain tumor image interpretation

Due to the size of the tableau for this example, we only give the knowledge base and the results for a brain tumor image interpretation problem using the tableau method.

$$
\begin{aligned}
\mathcal{T} = \{\ SmallDeformingTumor \sqsubseteq\ & BrainTumor \\
& \sqcap \exists hasEnhancement.NonEnhanced \\
& \sqcap \exists hasBehavior.Infiltrating \\
PeripheralDeformingTumor \sqsubseteq\ & BrainTumor \\
& \sqcap \exists farFrom.LateralVentricle \\
& \sqcap \exists hasLocation.PeripheralCerebralHemisphere\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O} = \{\ & \exists hasEnhancement.NonEnhanced \\
& \sqcap \exists farFrom.LateralVentricle \\
& \sqcap \exists hasLocation.PeripheralCerebralHemisphere\}
\end{aligned}
$$

In the following part, all the concepts and roles are represented by an abbreviation including the first letter and capital letters. In this case the initial node is composed by $\mathbf{T}(1) = \{\}$ and $\mathbf{F}(1) = \{\neg C_{\mathcal{T}} \sqcap \mathcal{O}\}$.

$$
\begin{aligned}
\neg C_{\mathcal{T}} \sqcap \mathcal{O} = \{ & (SDT \sqcap \neg BT \sqcup \forall hE.\neg NE \sqcup \forall hB.\neg In) \sqcup \\
& (PDT \sqcap \neg BT \sqcup \forall fF.\neg LV \sqcup \forall hL.\neg PCH) \sqcup \\
& (\exists hE.NE \sqcap \exists fF.LV \sqcup \exists hL.PCH)\}
\end{aligned}
$$

Six branches are open in the tableau at the end. In each one, the set of concepts that cannot be decomposed are:

$$
\begin{aligned}
H_1 &= \{SDT, PDT, \exists hE.NE\} \\
H_2 &= \{SDT, PDT, \exists fF.LV\} \\
H_3 &= \{SDT, PDT, \exists hL.PCH\} \\
H_4 &= \{SDT, \neg BT, \forall fF.\neg LV, \forall hL.\neg PCH, \exists hE.NE\} \\
H_5 &= \{PDT, \neg BT, \forall hE.\neg NE, \forall hB.\neg In, \exists fF.LV\} \\
H_6 &= \{PDT, \neg BT, \forall hE.\neg NE, \forall hB.\neg In, \exists hL.PCH\}
\end{aligned}
$$

One potential hypothesis is the combination of these concepts using Algorithm 1. Inconsistent hypotheses are eliminated, e.g. $SDT \sqcap \neg BT$, $PDT \sqcap \forall fF \neg LV$, etc. Similarly, $\exists hE.NE \sqcap \exists fF.LV \sqcup \exists hL.PCH$ are removed since the hypothesis is irrelevant.

According to the subsumption criterion, $SDT \sqcap \exists fF.LV \sqcap \exists hL.PCH$ and $PDT \sqcap \exists hE.NE$ are considered as preferred explanations. For example, $SDT \sqcap PDT$ is subsumed by these two hypotheses. These two results fulfill the consistency, relevance and the minimality criteria:

- $PeripheralDeformingTumor \sqcap \exists hasEnhancement.NonEnhanced$
- $SmallDeformingTumor \sqcap \exists farFrom.LateralVentricle$
  $\sqcap \exists hasLocation.PeripheralCerebralHemisphere$

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