ECE108 Assignment 1

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1 Set Operation

a)

$R \subseteq S \iff R \subseteq ((S - T) \cup (R \cap T))$
This statement is false because the implication is only unidirectional.

Proving $R \subseteq S \rightarrow R \subseteq ((S - T) \cup (R \cap T))$
$R \subseteq ((S - T) \cup (R \cap T))$ can be simplified to using distributivity

$R \subseteq (((S - T) \cup R) \cap ((S - T) \cup T))$ where
$(S - T) \cup R$ gives you a set X such that $R \subseteq X$
$(S - T) \cup T$ gives you S ...

So... we get $R \subseteq (X \cap S)$ since $R \subseteq X$ and from our assumption $R \subseteq S$
The intersection gives at least R as an answer
Therefore $R \subseteq R$ is true

The opposite way cannot be proved because R does'nt have to be $\subseteq$ S
Counter-example $S = \{4\}$ $R = \{1, 3, 5\}$ $T = R = \{1, 3, 5\}$
$S - T = \{4\}$ $R \cap T = \{1, 3, 5\}$
$R = (S - T) \cup (R \cap T) = \{1, 3, 4, 5\}$ in this case R is definitely not a subset of S

a) $R \subseteq T$ proves it wrong then you are done
2.) Given sets A and B under what condition does $A - B = B - A$
need to prove that $A = B \iff A - B = B - A$

starting with $A = B \rightarrow A - B = B - A$
if $A = B$, then $A - B = B - A = \emptyset$

continuing with $A = B \leftarrow A - B = B - A$

PBC if $A \neq B$ then $A \not\subseteq B \vee B \not\subseteq A$ then $\exists x \in A \mid x \notin B$ OR $\exists x \in B \mid x \notin A$

but by definition of difference of sets:

$A - B = \{x \mid x \in A \wedge x \notin B\}$

$B - A = \{x \mid x \in B \wedge x \notin A\}$

if $A - B = B - A$

then it means that $\exists x \in A \mid x \notin B$

because of the equality but the same x must exists in B not in A

we can see that no element satisfies this condition

We can then conclude that

$A - B = \emptyset$ $B - A = \emptyset$ proving also that $\forall x \in A \mid x \in B$ $\forall x \in B \mid x \in A$ therefor proving the equality

i. Define $T \subseteq A2$ st $XTy \iff (xRyANDxSY)$ show T is refl sym and transitive to prove it

4. Given poset (x, smallerEq) prove or disprove (a) x ¿= y iff y ¡=-1 x

R-1 = (b,a) — (a,b) ∈ R) (y,x) ∈ ¡= so it mean (x,y) ∈ ¡=-1

(b) x ¿= y $\iff$ y ¡='x (2,2) will prove it false;