ECE108 Assignment 1

Yi Fan Yu (yf3yu@edu.uwaterloo.ca)

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1 Set Operation

a) $R \subseteq S \iff R \subseteq ((S - T) \cup (R \cap T))$
This statement is false because the implication is only unidirectional.

Proving $R \subseteq S \rightarrow R \subseteq ((S - T) \cup (R \cap T))$
$R \subseteq ((S - T) \cup (R \cap T))$ can be simplified to using distributivity

$R \subseteq (((S - T) \cup R) \cap ((S - T) \cup T))$ where
$(S - T) \cup R$ gives you a set X such that $R \subseteq X$
$(S - T) \cup T$ gives you S ...

So... we get $R \subseteq (X \cap S)$ since $R \subseteq X$ and from our assumption $R \subseteq S$
The intersection gives at least R as an answer
Therefore $R \subseteq R$ is true

The opposite way cannot be proved because R does'nt have to be $\subseteq$ S $R \subseteq T$ will suffice the counter-example

Counter-example $S = \{4\}$ $R = \{1, 3, 5\}$ $T = R = \{1, 3, 5\}$
$S - T = \{4\}$ $R \cap T = \{1, 3, 5\}$
$R = (S - T) \cup (R \cap T) = \{1, 3, 4, 5\}$ in this case R is definitely not a subset of S

(b) $(A \cap C) \subseteq (B \cap C) \to A \subseteq B$ proof: let $x \in A \cap B$ we can conclude that...

by the definition of intersection, $\forall x \in A, x \in C$
we deduce that x must be in both A and C

by the definition of subset, $\forall x \in (A \cap C), x \in (B \cap C)$
we can say $\forall x \in A, x \in (B \cap C)$

which means by the definition of intersection that
$\forall x \in A, x \in B \wedge x \in C$
So... $\forall x \in A, x$ must be $\in B$
which is the definition of $A \subseteq B$

(c) $A \in B \wedge B \in C \to A \in C$
this statement is obviously false since
let $A = \{3\}$ let $B = \{\{3\}, 4\}$ let $C = \{\{\{3\}, 4\}, 5\}$

we can see that $A \in B \wedge B \in C$
but $A \notin C$

(d) $A \in B \wedge B \in C \to A \subseteq C$
this statement is obviously false since
let $A = \{3\}$ let $B = \{\{3\}, 4\}$ let $C = \{\{\{3\}, 4\}, 5\}$

we can see that $A \in B \wedge B \in C$
but $A \not\subseteq C$

(e) $A \in B \wedge B \subseteq C \to A \in C$
proof:
if $B \subseteq C$
that means $\forall x \in B, x \in C$
now $A \in B$ means that A is an element of B represented by $\forall x$
replacing $\forall x$ by A
we can then conclude $A \in B$ means $A \in C$

(f) $A \in B \wedge B \subseteq C \to A \subseteq C$
this is obviously false since we proved that $A \in C$ is true let $A = \{3\}$ let $B = \{\{3\}, 4\}$ let $C = \{\{3\}, 4, 5\}$

we can see that $A \in B \wedge B \subseteq C$
but $A \not\subseteq C$

2 Set operations

Given sets A and B under what condition does $A - B = B - A$
need to prove that $A = B \iff A - B = B - A$

Proof:
starting with $A = B \rightarrow A - B = B - A$
if $A = B$, then $A - B = B - A = \emptyset$

continuing with $A = B \leftarrow A - B = B - A$ Proof:

the definitions for differences are:
$A - B = \{x \mid x \in A \wedge x \notin B\}$
$B - A = \{x \mid x \in B \wedge x \notin A\}$
if $A - B = B - A$
then it means that $\exists x \in A \mid x \notin B$
because of the equality,
but the same x must exists in B not in A
we can see that no element
We can then conclude that
$A - B = \emptyset \wedge B - A = \emptyset$
using the definition of difference we can see that
in order for $A - B = \emptyset$, that means that $A \subseteq B$
in order for $B - A = \emptyset$, that means that $B \subseteq A$

therefore $A \subseteq B \wedge B \subseteq A$
this is the definition of equality $A = B$.

3 Functions

(a)
(i) if f is injective, we don't know anything about the relationship between co-domain and image we only know about that image $\subseteq$ co-domain
(ii) image = co-domain when it is surjective
(iii) image = co-domain when it is bijective

(b)
(i) if f is injective, it means that the function is invertible and we can map the image ($\neq$ co-domain) back to it's domain
so f^{-1} has image the full codomain of f
image (f^{-1}) = dom (f)
(ii) turns out that if f is not injective, it cannot be inverted
since f^{-1} does not exist,
(iii) image (f^{-1}) = dom (f) when it is bijective

4 Functions

I assume that both X and Y are not empty sets....
(a) there exists an injection $f : X \to Y$

Proof:
if $X \subseteq Y$, this means
$\forall a(a \in X \to a \in Y)$
a "same" function can be applied that maps all the values in X to its same value, but in Y

$f : X \to Y$
$x \mapsto x$

Since all values in X is present in Y,
and every single value in X maps to one value inside of Y
and sets don't have duplicates
we've got an injective "equivalence" function

(b) there exists a surjection $g : Y \to X$
well we know that $X \subseteq Y$
so cardinality$(X) \leq$ cardinality(Y)
I can conclude that my domain will be either equal or bigger than my codomain
Therefore I can guarantee that my function will not be injective if I need my function to be surjective

so a function that

$$x \mapsto \begin{cases} x, & \text{if } x \in (X \cap Y) \\ \text{any Value in Y}, & \text{if } x \in (Y - X) \end{cases} \tag{1}$$

the mapping to any value will make sure that our function definition maps all the domain to satisfy the definition of a function

5 Functions.. Even more

(a) $f : \mathbb{N} \to \mathbb{N}$ where $f : x \mapsto x$

Since the dom = codom, and the function maps the value to itself
we can conclude that:

function is injective because all values of the domain is mapped to a unique value in the codomain

function is surjective because all values of the codomain is being mapped to (range = codomain)
therefore, the function is bijective

(b) $g : \mathbb{N} \to \mathbb{N}$ where $g : x \mapsto x^2$
in this case, the function maps all the values in the domain to the square of itself.
since all values in the domain has a unique square value in the codomain, the function is injective
since range $\neq$ codomain because $3 \in \mathbb{N}$ but 3 is not mapped by any value in the domain, the function is **NOT** surjective
therefore not bijective

(c) $h : \mathbb{Q}^+ \to \mathbb{Q}^+$ where $h : x \mapsto 1/x$
I assume that the function maps x to the inverse of its most simplified version else, it is not even a function because $4 \mapsto \frac{1}{4}$ but also $\mapsto \frac{2}{8}$
right off the bat, we can see that the same number could be represented by 2 different values of the domain i.e $\frac{1}{2}$ $\frac{2}{4}$
since the inverse operation of these 2 fractions all map to 2, the function does not satisfy the injective definition.
similiarly, the surjective definition is not satisfied since $\frac{2}{8}$ will never be mapped to. refer to the assumption
obviously not bijective, so **None of the Above**

(d) possible to compose fog foh goh

6 Closure

i. Define $T \subseteq A2$ st $XTy \iff (xRyANDxSY)$ show T is refl sym and transitive to prove it

4. Given poset (x, smallerEq) prove or disprove (a) x ¿= y iff y ¡=-1 x

R-1 = (b,a) — (a,b) $\in$ R) (y,x) $\in$ ¡= so it mean (x,y) $\in$ ¡=-1

(b) x ¿= y $\iff$ y ¡='x (2,2) will prove it false;