# ECE 108 WRITTEN 3: COMBINATORICS AND GRAPHS

DUE: SECTION 001:     WEDNESDAY, APRIL 4<sup>th</sup> AT 10:00 PM

This assignment consists of 4 questions worth 4 marks.

If you are in Section 001 you submit your work to the Assignment 3 DropBox on Learn.

*Quidquid Latine dictum sit, altum viditur*

**1. Rolling Dice [1 marks]** If you roll a fair dice $n$ times:

**(a)** In how many ways can the sequence 1,2,3,4,5,6 occur at least once, assuming $n \geq 6$? What is the probability of this occurring?

**(b)** In how many ways will the sum of the number of 1's and 2's equal the sum of the number of 3's, 4's, 5's and 6's, assuming $n = 2i$ for some positive integer $i$? What is the probability of this occurring?

**(c)** In how many ways will the sum of the number of 1's and 2's equal the sum of the number of 3's and 4's and equal the sum of the number of 5's and 6's, assuming $n = 3i$ for some positive integer $i$? What is the probability of this occurring?

**(d)** In how many ways will you roll a "1" twice in succession at least once, assuming $n \geq 2$? What is the probability of this occurring?

**(e)** In how many ways will you roll a "1" $k$-times in succession at least once, assuming $n \geq k$? What is the probability of this occurring?

**(f)** You have a communications system that transmits bundles of data from a source to a destination. Most bundles are received correctly. However, there is a random chance of the data in a bundle being in error. Roughly every $E^{th}$ bundle is in error. If you transmit $n$ bundles, what is the probability of at least two bundles in sequence being in error? what is the probability of at least $k$ bundles in sequence being in error?

You may wish to modify your Lab3 program `dice` to give you a sense of whether or not your answers look correct.

**2. Binomial Theorem [1 marks]** Prove (using induction) the binomial theorem:

$$(x + a)^n = \sum_{r=0}^{n} C(nr)x^{n-r}a^r$$

**3. Graph Questions [1 marks]** Is there

**(a)** a graph with 5 vertices and 3 edges?

**(b)** a graph with 5 odd-degree vertices with 3 edges?

**(c)** a connected graph with 5 vertices and 3 edges?

**(d)** a directed graph with 5 vertices and 3 edges?

**(e)** a disconnected graph (a graph that is not connected) with 5 vertices and 7 edges?

**(f)** a disconnected, directed graph with 5 vertices and 7 edges?

**(g)** a graph with 3 vertices and 5 edges?

**(h)** a directed graph with 3 vertices and 5 edges?

In each case, if there exists such a graph, illustrate it; if not, explain (precisely) why not.

**4. Given a Graph [1 marks]** Given the following graph:
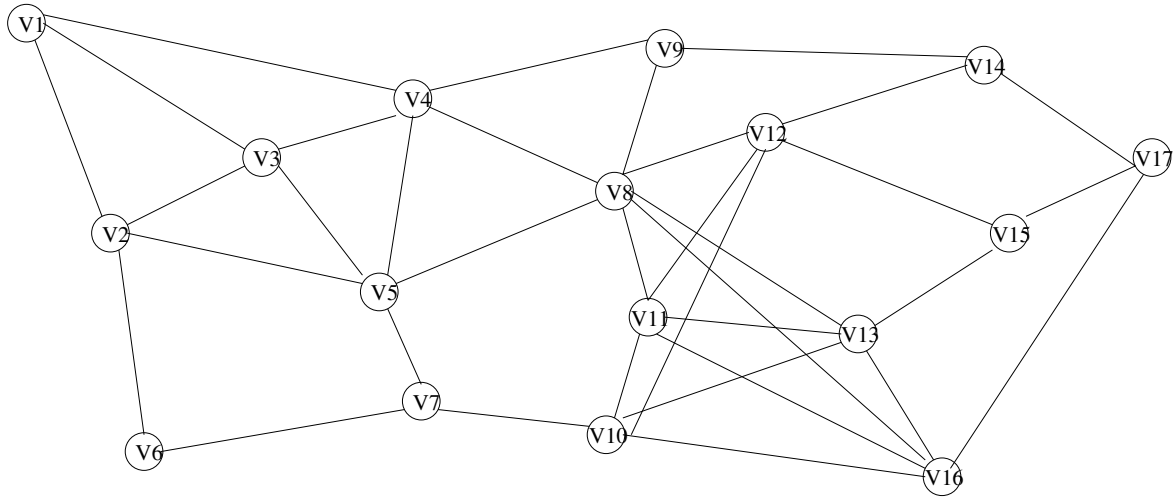


**(a)** Is this graph connected?

**(b)** Is this graph complete? Why/Why not?

**(c)** What is the minimum degree of the graph?

**(d)** What is the maximum degree of the graph?

**(e)** What is the largest clique in the graph?

**(f)** Is the graph planar? Why/Why not?

**(g)** What is the shortest path from V1 to V17? How long is it?

**(h)** What is the longest path from V1 to V17? How long is it?