ECE108 Assignment3

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1 Rolling Dice

a Dicerolling1,2,3,4,5,6

How many times will the sequence 1,2,3,4,5,6 occur? What is the probability of this occurring?

well if $n \leq 6$, there is 0 chance of any sequence occuring
if $n = 6$, there is 1 occurence which means $\frac{1}{6^6}$ chance
if $n \geq 6$,there are

$$(n-5)! * 6^{n-6} \text{occurences}$$

which leads to a probability of occurence to

$$\frac{(n-5)! * 6^{n-6}}{6^n}$$

b DicerollingA,B,C

How many times will the sum of the number of 1's and 2's equal the sum of the number of 3's, 4's, 5's and 6's? What is the probability of this occurring?
let's say

$$1\&2 \mapsto A$$

$$3\&4 \mapsto B$$

$$5\&6 \mapsto C$$

$A\&B\&C$ with all equal possibilities of showing up (1/3)
Then we can say we are only throwing a 3 sided dice, therefore the total amount of possibilities while throwing is

$$3^n$$

using the analogy of undistinguishable As and $B\&C$s our equation for $\|A\| = \|B\| + \|C\|$ is

$$\frac{n!}{\frac{n}{3}! * \frac{2n}{3}!}$$

$$\frac{\frac{n!}{\frac{n}{3}! * \frac{2n}{3}!}}{3^n}$$

c DicerollingA,B,C Part2

How many times will the sum of the number of 1's and 2's equal the sum of the number of 3's and 4's and equal the sum of the number of 5's and 6's? What is the probability of this occurring?
Same thing as part I, but now As are indisguishable from Bs and from Cs. Therefore,

$$\frac{n!}{\frac{n}{3}*!\frac{n}{3}! * \frac{n}{3}!}$$

with the total probability at :

$$\frac{\frac{n!}{\frac{n}{3}*!\frac{n}{3}!*\frac{n}{3}!}}{3^n}$$

d Rolling 1s

How many times will you roll a 1 twice in succession? For the purpose of this calculation, the sequence ...11.... counts as once, but so does ...111.... The sequence ...1111... counts as twice. What is the probability of this occurring?

e Rolling 1s

How many times will you roll a 1 twice in succession? For the How many times will you roll a 1 k-times in succession? As with the previous case, when a 1 is counted as part of a k-sequence, it is not counted as part of a subsequent k-sequence. What is the probability of this occurring?

f Communication

How many times will you roll a 1 twice in succession? For the You have a communications system that transmits bundles of data from a source to a destination. Most bundles are received correctly. However, there is a random chance of the data in a bundle being in error. Roughly every E^{th} bundle is in error. If you transmit n bundles, what is the probability of two bundles in sequence being in error? what is the probability of k bundles in sequence being in error?

2 Binomial Theorem

Prove (using induction) the binomial theorem:

$$(x+a)^n = \sum_{r=0}^{n} C(nr)x^{n-r}a^r$$

a Base Case

n = 1 case:

$$(x+a) = \sum_{r=0}^{1} C(nr)x^{n-r}a^r$$

$$(x+a) = \binom{1}{0}x^1 + \binom{1}{1}a^1$$

$$(x+a) = x + a\blacksquare$$

b N+1 Case

Assuming:

$$(x+a)^n = \sum_{r=0}^{n} C(nr)x^{n-r}a^r$$

Show:

$$(x+a)^{n+1} = \sum_{r=0}^{n+1} C((n+1)r)x^{(n+1)-r}a^r$$

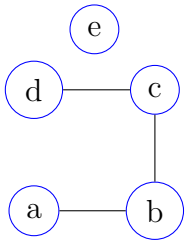
$$(x+a)^{n+1} = (\sum_{r=0}^{N} C(nr)x^{n-r}a^r) + \binom{N+1}{1}$$

3 Graph Questions

a a graph with 5 vertices and 3 edges

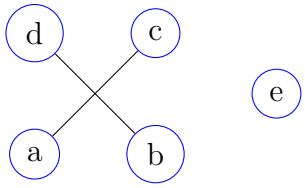
yes, any graph can exist with any amount of vertices and edges provided the number of edges are not bigger than the maximum possible amount.
for 5 vertices, the maximum amount of edges

$$\binom{5}{2} = 10$$



b a graph with 5 odd-degree vertices with 3 edges

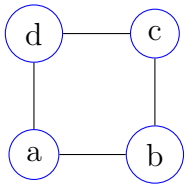
it is quite clear that with 3 edges, we are thinking of vertices with degree = 1.



it is clear that if we try to connect node e, with our remaining 1 edge, we will create a vertex of degree 2, therefore it is impossible.

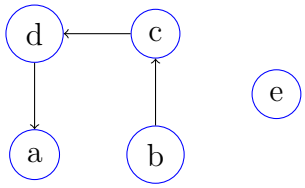
c a connected graph with 5 vertices and 3 edges?

a connected graph with 4 vertices requires at least 4 edges. therefore it is impossible to create a connected graph of 5 vertices with only 3 edges...



d a directed graph with 5 vertices and 3 edges?

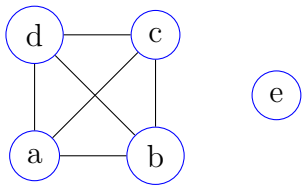
assuming you mean arcs by edges since in a digraph, there are no edges.



yes it it possible

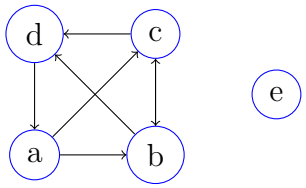
e a disconnected graph (a graph that is not connected) with 5 ices and 7 edges

no it is not possible, since K_4 has 6 edges (the maximum amount of edges we can connect with 4 vertices) so we have to connect the 5th node e using our last remaining edge.



f a disconnected, directed graph with 5 vertices and 7 edges

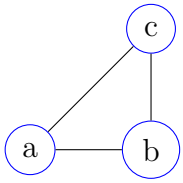
I assume edges mean arcs (directed edges). Then yes it is possible since the maximum amount of arcs for K_4 is 12.
So we can completely ignore the 5th node



g a graph with 3 vertices and 5 edges

No, since K_3 has 3 edges and the maximum amount of edges for 3 vertices is represented by

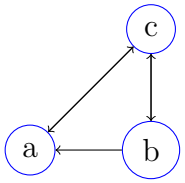
$$E(K_3) = 3$$



we can't fit 5 edges between 3 vertices.

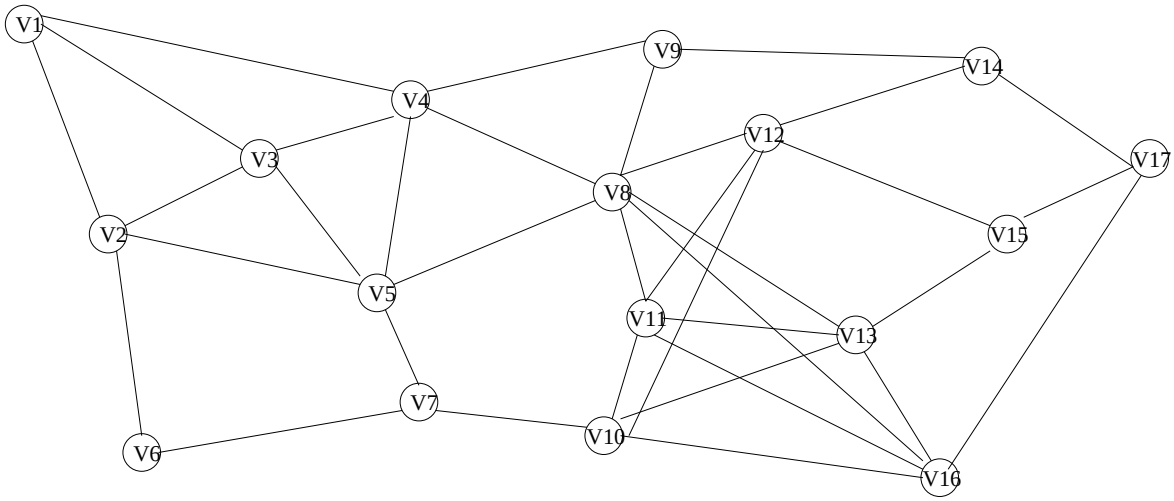
h a directed graph with 3 vertices and 5 edges

I assume that by edges, we are meaning arcs. then yes it is possible since the maximum amount of arcs for 3 vertices is 6



4 Given a Graph

Given the following graph:



a Is this graph connected?

Yes it is complete, since I can go from any node to any other node.

b Is this graph complete? Why/Why not?

No, there is no edge between V1 and V6.
However, there exists a subgraph K_4

c What is the minimum degree of the graph?

minimum degree is 2, on node V6

d What is the maximum degree of the graph?

maximum degree is 7, on node V8

e What is the largest clique in the graph?

$$K_4$$

$$V8, V11, V16, V13$$

f Is the graph planar? Why/Why not?

No, there exists a subgraph K_4 $V8, V11, V16, V13$ inside.
We know that K_4 is non-planar.

g What is the shortest path from V1 to V17? How long is it?

$$V1 \to V4 \to V8 \to V16 \to V17$$

it is length 4

h What is the longest path from V1 to V17? How long is it?

$$V1 \to V4 \to V9 \to V14 \to V12 \to$$

$$V15 \to V13 \to V10 \to V7 \to$$

$$V6 \to V2 \to V3 \to V5 \to$$

$$V8 \to V11 \to V16 \to V17$$

length of 16. All the vertices are visited exactly once