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上海交通大学在线考试诚信承诺书

SJTU Online Examination Honor Code Letter

考试不仅是对学习成效的检查，更是对道德品质的检验。自觉维护学校的考风考纪，营造公平、公正的考试环境是全体同学的共同责任和义务。特别在疫情防控的特殊时期，更应强化自律意识，恪守诚信，拒绝舞弊，做一名诚实守信的新时代大学生，用诚信的考试构筑诚信的人生。

Examination is the evaluation of both learning effect and morality. It is the responsibility and obligation of all students to consciously maintain the school's common examination practice, abide by the discipline and create a fair and just examination environment. Especially in the special period of epidemic prevention and control, we should strengthen the consciousness of self-discipline, abide by the integrity, refuse to cheat, be an honest and trustworthy college student in the new era, and build an honest life from the integrity test.

我郑重承诺 I solemnly promise：

（1）本人将履约践诺，知行统一；遵从诚信规范，恪守学术道德；自尊自爱，自省自律。I will fulfill my promise, unify between knowledge and action, abide by the rules of integrity, academic ethics, be self-respected and self-disciplined.

（2）在线考试过程中，自觉遵守学校和老师宣布的考试纪律（详见《上海交通大学本科生学生手册》中的《学生考试纪律规定》，沪交教【2019】28号），不剽窃，不违纪，不作弊。In the process of online examination, I will consciously abide by the examination discipline announced by the school and the teachers (see the regulations on student examination discipline in the undergraduate student handbook of Shanghai Jiao Tong University, HJJ [2019] No. 28), and do not plagiarize, violate discipline or cheat.

（3）若违反相关考试规定和纪律要求，自愿接受学校的严肃处理或处分。In case of violation of relevant examination regulations and discipline, students shall bear the serious treatment or punishment from the school.

承诺人 Committed by：杨一凡

（学号 Student No：520021911080）

日期 Date (Y/M/D)：2022 年 6 月 6 日

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上 海 交 通 大 学 答 题 纸

(20__ 至 20____ 学年 第___学期)

班级号 F2003602　　学号 520021911080　　姓名 杨一凡

课程名称 量子力学　　成绩 ______

题号										
得分										
批阅人(流水阅卷教师签名处)										

我承诺，我将严格遵守考试纪律。

一、填空 (答案下标有下划线)

1. $\frac{h}{\sqrt{2mUe}}$

$\frac{1}{2}mv^2 = Ue \Rightarrow p = \sqrt{2mE} = \sqrt{2mUe}$

$\Rightarrow \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mUe}}$

2. $\sqrt{6}\hbar$　$2\hbar$　$\frac{\sqrt{3}}{2}\hbar$

$n=3$　$l=2$　$L = \sqrt{l(l+1)}\,\hbar = \sqrt{2\times3}\,\hbar = \sqrt{6}\,\hbar$

$L_z = m\hbar = 2\hbar$

$S = \frac{\sqrt{3}}{2}\hbar$

3. 玻色 ; 玻色

4. $a_n = \int_{-\infty}^{+\infty} \varphi_n^*(x) \cdot \psi(x)\, dx$

5. $\frac{b}{\lambda}$　$\sigma\frac{b^4}{\lambda^4}$　$\sigma\frac{b^4R^2}{\lambda^4 d^2}$

(1) $\lambda \cdot T = b \Rightarrow T = \frac{b}{\lambda}$

(2) $M_0(T) = \sigma T^4 = \sigma\frac{b^4}{\lambda^4}$

(3) $E = \frac{\sigma T^4 \cdot 4\pi R^2}{4\pi d^2} = \frac{\sigma b^4 R^2}{\lambda^4 d^2}$

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6. 本征值；本征波函数

7. $[x^2, \hat{p_x}]\varphi$

$= x^2(-i\hbar\frac{\partial}{\partial x})\varphi - \hat{p_x}x^2\varphi$

$= -i\hbar x^2\frac{\partial}{\partial x}\varphi + i\hbar\frac{\partial}{\partial x}(x^2\varphi)$

$= -i\hbar x^2\frac{\partial}{\partial x}\varphi + i\hbar\cdot 2x\cdot\varphi + i\hbar x^2\frac{\partial}{\partial x}\varphi$

$= i\hbar\cdot 2x\cdot\varphi$

$\Rightarrow [x^2, \hat{p_x}] = i\hbar\cdot 2x$

8. 不能　　$\frac{h\nu}{c} = \vec{p}\cdot\cos\theta + \frac{h\nu'}{c}\cos\phi$

$\frac{h}{\lambda} = \frac{h}{\frac{c}{\nu}} = \frac{h\nu}{c}$　　　$\frac{h\nu}{c} = p\cdot\cos\theta + \frac{h\nu'}{c}\cos\phi$

9. $\frac{(U_1-U_2)\lambda_1\lambda_2\cdot e}{c(\lambda_2-\lambda_1)}$

$h\nu = h\nu_0 + \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = h\nu - h\nu_0 = Ue$

$\Rightarrow U = \frac{h}{e}(\nu-\nu_0)$

$U_1 = \frac{h}{e}(\frac{c}{\lambda_1} - \frac{c}{\lambda_0})$

$U_2 = \frac{h}{e}(\frac{c}{\lambda_2} - \frac{c}{\lambda_0})$

$\Rightarrow U_1 - U_2 = \frac{h}{e}\frac{c}{\lambda_1} - \frac{h}{e}\frac{c}{\lambda_2} = \frac{h}{e}\frac{c(\lambda_2-\lambda_1)}{\lambda_1\lambda_2}$

$\Rightarrow h = \frac{(U_1-U_2)\lambda_1\lambda_2\cdot e}{c(\lambda_2-\lambda_1)}$

10. $\left(-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + mgz\right)\varphi(x) = E\varphi(x)$

/ $(-\frac{\hbar^2}{2m}\nabla^2 + mgz)\varphi(x) = E\varphi(x)$

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11. $\psi(t) = 0.6 e^{-i\frac{E_0}{\hbar}t} \cdot \varphi_1 + 0.8 e^{-i\frac{2E_0}{\hbar}t} \cdot \varphi_2$

$H = \begin{pmatrix} E_0 & 0 \\ 0 & 2E_0 \end{pmatrix}$

$\begin{pmatrix} E_0 & 0 \\ 0 & 2E_0 \end{pmatrix} \psi(t) = E\psi(t)$

$\begin{pmatrix} E_0 - E & 0 \\ 0 & 2E_0 - E \end{pmatrix} \psi(t) = 0 \quad (E_0 - E)(2E_0 - E) = 0$

$\Rightarrow E_0 = E \quad E_1 = 2E_0$

$\Rightarrow i\hbar \frac{\partial}{\partial t} C_1(t) = E_0 C_1(t) \quad \Rightarrow C_1(t) = C_1 e^{-i\frac{E_0}{\hbar}t}$

$i\hbar \frac{\partial}{\partial t} C_2(t) = 2E_0 C_2(t) \quad \Rightarrow C_2(t) = C_2 e^{-i\frac{2E_0}{\hbar}t}$

$t=0$ 时 $C_1 = 0.6 \quad C_2 = 0.8$

$\Rightarrow C_1(t) C_1(t)^* = C_1^2 = 0.36$

$C_2(t) C_2(t)^* = C_2^2 = 0.64$

$\Rightarrow \psi(t) = C_1(t)\varphi_1 + C_2(t)\varphi_2(t)$

$= 0.6 e^{-i\frac{E_0}{\hbar}t} + 0.8 e^{-i\frac{2E_0}{\hbar}t}\varphi_2(t)$

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二. 计算题:

1. $\psi(r,\theta,\varphi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}}$

由方程可知.

(1) 电子的径向概率密度 $\rho(r) = 4\pi r^2 \cdot |\psi(r,\theta,\varphi)|^2$

$= 4\pi r^2 \cdot \frac{1}{\pi a_0^3} e^{-\frac{2r}{a_0}}$

$= \frac{4r^2}{a_0^3} e^{-\frac{2r}{a_0}}$

$\int_0^{\infty} 4\pi r^2 |\psi(r,\theta,\varphi)|^2 dr = 1$ 成立.

(2) $\frac{d\rho(r)}{dr} = \frac{8r}{a_0^3} e^{-\frac{2r}{a_0}} - \frac{2}{a_0}\frac{4r^2}{a_0^3} e^{-\frac{2r}{a_0}}$

$= \frac{8r}{a_0^3} e^{-\frac{2r}{a_0}} - \frac{8r^2}{a_0^4} e^{-\frac{2r}{a}}$

$= \frac{8r}{a_0^3} e^{-\frac{2r}{a_0}} (1 - \frac{r}{a_0}) = 0 \implies r = a_0$

$\implies$ 最可能半径为 $r = a_0$

(3) $\overline{\frac{1}{r^2}} = \int \psi(r,\theta,\varphi)^* \frac{1}{r^2} \psi(r,\theta,\varphi) d\tau$

$= \int_0^{+\infty} \psi(r,\theta,\varphi)^* \frac{1}{r^2} \psi(r,\theta,\varphi) \cdot 4\pi r^2 dr$

$= \int_0^{+\infty} 4\pi \cdot \frac{1}{\pi a_0^3} e^{-\frac{2r}{a_0}} dr$

$= \frac{4}{a_0^3} \left(-\frac{a_0}{2} e^{-\frac{2r}{a_0}}\right)\Big|_0^{+\infty}$

$= \frac{4}{a_0^3} \frac{a_0}{2} = \frac{2}{a_0^2}$

$\implies \frac{1}{r^2}$ 的平均值为 $\frac{2}{a_0^2}$

2. $[\hat{F}, \hat{G}] = [\hat{L}_x + i\hat{L}_y, \hat{L}_x - i\hat{L}_y]$

$= [\hat{L}_x + i\hat{L}_y, \hat{L}_x] - [\hat{L}_x + i\hat{L}_y, i\hat{L}_y]$

$= [\hat{L}_x, \hat{L}_x] + [i\hat{L}_y, \hat{L}_x] - [\hat{L}_x, i\hat{L}_y] - [i\hat{L}_y, i\hat{L}_y]$

$= i[\hat{L}_y, \hat{L}_x] - i[\hat{L}_x, \hat{L}_y]$

$= -i\hbar \cdot i\hat{L}_z - i \cdot i\hbar \hat{L}_z = 2\hbar \hat{L}_z \implies [\hat{F}, \hat{G}] = 2\hbar \hat{L}_z$

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$$[\hat{L}^2, \hat{F}] = [\hat{L}_x^2+\hat{L}_y^2+\hat{L}_z^2, \hat{L}_x + i\hat{L}_y]$$

$$= [\hat{L}^2, \hat{L}_x] + i[\hat{L}^2, \hat{L}_y] = 0$$

$$[\hat{L}^2, \hat{G}] = [\hat{L}^2, \hat{L}_x - i\hat{L}_y]$$

$$= [\hat{L}^2, \hat{L}_x] - i[\hat{L}^2, \hat{L}_y] = 0$$

综上所述 $[\hat{F}, \hat{G}] = 2\hbar\hat{L}_z$

$$[\hat{L}^2, \hat{F}] = [\hat{L}^2, \hat{G}] = 0$$

3. 由归一化可知

$$\int_0^a \psi^*(x)\psi(x)\,dx = 1$$

$$\Rightarrow \int_0^a A^2x^2(a-x)^2\,dx = 1 \quad \Rightarrow A = \sqrt{\frac{30}{a^5}}$$

由分析可知

$$\bar{E} = \int_{-\infty}^{+\infty} \psi(x)^* \hat{H}\psi(x)\,dx$$

在一维无限深势阱中 $0 \le x \le a$ 时 $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$

$$\therefore \bar{E} = \int_0^a Ax(a-x)\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)(Aax - Ax^2)\,dx$$

$$= -\frac{\hbar^2}{2m}\int_0^a Ax(a-x)\cdot(-2A)\,dx$$

$$= \frac{\hbar^2A^2}{m}\int_0^a ax - x^2\,dx$$

$$= \frac{\hbar^2A^2}{m}\left(\frac{1}{2}ax^2 - \frac{1}{3}x^3 \Big|_0^a\right)$$

$$= \frac{\hbar^2A^2}{m}\left(\frac{1}{2}a^3 - \frac{1}{3}a^3\right) = \frac{\hbar^2A^2}{m}\frac{1}{6}a^3 = \frac{\hbar^2A^2a^3}{6m}$$

$$A^2 = \frac{30}{a^5}$$

$$\therefore \bar{E} = \frac{\hbar^2a^3}{6m}\frac{30}{a^5} = \frac{5\hbar^2}{ma^2}$$

$$\Rightarrow \text{电子能量的平均值为 } \bar{E} = \frac{5\hbar^2}{ma^2}$$

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4. $\psi = \frac{2}{3} Y_{31}(\theta,\varphi) + \frac{2}{3} Y_{21}(\theta,\varphi) - \frac{1}{3} Y_{1-1}(\theta,\varphi)$

由函数可知.

L^2的可能取值为 $\sqrt{3(3+1)}^2 \hbar^2 = 2\sqrt{3}^2 \hbar^2 = 12\hbar^2$

$\sqrt{2(2+1)}^2 \hbar^2 = \sqrt{6}^2 \hbar^2 = 6\hbar^2$

$\sqrt{1(1+1)}^2 \hbar^2 = \sqrt{2}^2 \hbar^2 = 2\hbar^2$

$\Rightarrow P(2^2\sqrt{3}^2\hbar^2) = \frac{4}{9} \quad P(\sqrt{6}^2\hbar^2) = \frac{4}{9} \quad P(\sqrt{2}^2\hbar^2) = \frac{1}{9}$

L_z的可能取值 $m\hbar = \hbar \quad 2\hbar$ 以及. $-\hbar$

$\Rightarrow P(\hbar) = \frac{4}{9} \quad P(2\hbar) = \frac{4}{9} \quad P(-\hbar) = \frac{1}{9}$

L^2的平均值 $\overline{L^2} = 12\hbar^2 \times \frac{4}{9} + 6\hbar^2 \frac{4}{9} + 2\hbar^2 \frac{1}{9}$

$= 8\hbar^2 + \frac{2}{9}\hbar^2 = \frac{74}{9}\hbar^2$

L_z的平均值 $\overline{L_z} = \hbar \frac{4}{9} + 2\hbar \frac{4}{9} - \hbar \frac{1}{9}$

$= \frac{11}{9}\hbar$

5. 对于厄米算符 $\hat{A}$ 设 $\hat{A}\psi = \lambda\psi$ $d\tau$为空间微分

(1) $\int_{-\infty}^{+\infty} (\psi)^* \hat{A}\psi\, d\tau = \int \psi^* \lambda \psi\, d\tau = \lambda \int \psi^* \psi\, d\tau$

$\int_{-\infty}^{+\infty} (\hat{A}\psi)^* \psi\, d\tau = \int (\lambda\psi)^* \psi\, d\tau = \lambda^* \int \psi^* \psi\, d\tau$

$\Rightarrow \lambda \int \psi^* \psi\, d\tau = \lambda^* \int \psi^* \psi\, d\tau$

$\Rightarrow \lambda = \lambda^*$ 因此其本征值为实数

(2) 设 $\hat{A}\varphi = \lambda_1 \varphi$ $\hat{A}\psi = \lambda_2 \psi$ $\lambda_1 \neq \lambda_2$

$\int_{-\infty}^{+\infty} \psi^* \hat{A}\varphi\, d\tau = \int \psi^* \lambda_1 \varphi\, d\tau = \lambda_1 \int \psi^* \varphi\, d\tau$

$\int_{-\infty}^{+\infty} (\hat{A}\psi)^* \varphi\, d\tau = \int \lambda_2^* \psi^* \varphi\, d\tau = \lambda_2 \int \psi^* \varphi\, d\tau$

$\Rightarrow \int_{-\infty}^{+\infty} \psi^* \hat{A}\varphi\, d\tau = \int_{-\infty}^{+\infty} (\hat{A}\psi)^* \varphi\, d\tau \Rightarrow \lambda_1 \int \psi^* \varphi\, d\tau = \lambda_2 \int \psi^* \varphi\, d\tau$

$\Rightarrow (\lambda_1 - \lambda_2) \int \psi^* \varphi\, d\tau = 0 \quad \lambda_1 \neq \lambda_2 \quad \therefore \int \psi^* \varphi\, d\tau = 0$ 因此相互正交

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6. (1) $\psi(x,0) = A[1+\cos(\frac{\pi x}{a})]\sin\frac{\pi x}{a}$

$= A\sin\frac{\pi x}{a} + \frac{A}{2}\sin\frac{2\pi x}{a}$

归一化条件：$\int_0^a \psi(x,0)^* \psi(x,0)\,dx = 1$

$\Rightarrow \int_0^a (A\sin\frac{\pi x}{a} + \frac{A}{2}\sin\frac{2\pi x}{a})^2\,dx = 1$

其为相应的叠加态 $\Rightarrow A^2 + \frac{A^2}{4} = 1$ $A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\Rightarrow A = \frac{2\sqrt{5}}{5}$

(2) E的可能取值为 $E_1 = \frac{\pi^2\hbar^2}{2ma^2}$ $E_2 = \frac{4\pi^2\hbar^2}{2ma^2}$

$\Rightarrow P(E_1) = A^2 = \frac{4}{5}$

$P(E_2) = \frac{A^2}{4} = \frac{1}{5}$

$\bar{E} = P(E_1)E_1 + P(E_2)E_2$

$= \frac{4}{5}\frac{\pi^2\hbar^2}{2ma^2} + \frac{1}{5}\frac{4\pi^2\hbar^2}{2ma^2} = \frac{8\pi^2\hbar^2}{10ma^2} = \frac{4\pi^2\hbar^2}{5ma^2}$

(3) 由分离变量 $\psi(x,t) = \psi(x)\varphi(t)$

$\Rightarrow i\hbar\frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,t)$

$\therefore i\hbar\,\psi(x)\frac{\partial}{\partial t}\varphi(t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x)\cdot\varphi(t)$

$\int\frac{\partial\varphi(t)}{\varphi(t)} = \int\frac{i\hbar}{2m}\frac{1}{\varphi(x)}\frac{\partial^2}{\partial x^2}\psi(x)\,dt$

$\Rightarrow \varphi(t) = C e^{\frac{i\hbar}{2m}\frac{1}{\varphi(x)}\frac{\partial^2}{\partial x^2}\psi(x)\cdot t}$ 由 $\varphi(0)=1$

$\Rightarrow \varphi(t) = e^{\frac{i\hbar}{2m}\frac{1}{\varphi(x)}\frac{\partial^2}{\partial x^2}\psi(x)\cdot t}$

$\varphi(x) = A\sin\frac{\pi x}{a} + \frac{A}{2}\sin\frac{2\pi x}{a}$

$\frac{d^2}{dx^2}\varphi(x) = -(\frac{A\pi^2}{a^2}\sin\frac{\pi x}{a} + \frac{2A\pi^2}{a^2}\sin\frac{2\pi x}{a})$

$= -\frac{A\pi^2}{a^2}(\sin\frac{\pi x}{a} + 2\sin\frac{2\pi x}{a})$

$\Rightarrow \varphi(t) = e^{\frac{i\hbar}{2m}\frac{-\pi^2}{a^2}\frac{\sin\frac{\pi x}{a}+2\sin\frac{2\pi x}{a}}{\sin\frac{\pi x}{a}+\frac{1}{2}\sin\frac{2\pi x}{a}}\cdot t}$

$\Rightarrow \varphi(t)\psi(x) = \psi(x,t)$

其中 $\psi(x) = A[1+\cos(\frac{\pi x}{a})]\sin(\frac{\pi x}{a}) = \frac{2\sqrt{5}}{5}[1+\cos(\frac{\pi x}{a})]\sin\frac{\pi x}{a}$

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$\rho(x,t) = \psi_1(x,t)^* \psi_1(x,t)$

$= |\psi_1(x,t)|^2$

$= A^2 [1+\cos(\frac{\pi x}{a})]^2 (\sin\frac{\pi x}{a})^2 = \frac{4}{5}[1+\cos(\frac{\pi x}{a})]^2 \sin^2(\frac{\pi x}{a})$

与$\varphi(t)$并不存在关系

(4) $\psi_1(x,t) = A\varphi(t)\sin\frac{\pi x}{a} + \frac{A}{2}\varphi(t)\sin\frac{2\pi x}{a}$

$\varphi^*(t)\varphi(t)=1$

因此 (2)中的结果与时间t没有关系

1. $H = \begin{pmatrix} E_0 & -A \\ -A & E_0 \end{pmatrix}$

$H C(t) = E C(t)$

$\Rightarrow \begin{pmatrix} E_0-E & -A \\ -A & E_0-E \end{pmatrix} C(t) = 0 \Rightarrow (E_0-E)^2 = A^2$

$\Rightarrow E_1 = E_0 - A$

$E_2 = E_0 + A$

(1) 当$E = E_0 - A$时

$\begin{pmatrix} A & -A \\ -A & A \end{pmatrix}\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = 0 \Rightarrow \begin{cases} Ac_1 - Ac_2 = 0 \\ -Ac_1 + Ac_2 = 0 \end{cases}$

且 $c_1^2 + c_2^2 = 1 \Rightarrow c_1 = c_2 = \frac{\sqrt{2}}{2}$ 或 $c_1 = c_2 = -\frac{\sqrt{2}}{2}$

$\Rightarrow$ 其本征矢为 $\pm\frac{\sqrt{2}}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(2) 当$E = E_0 + A$时

$\begin{pmatrix} -A & -A \\ -A & -A \end{pmatrix} C(t) = 0 \quad C(t) = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$

$c_1 + c_2 = 0$

$c_1^2 + c_2^2 = 1$

$\Rightarrow c_1 = -c_2 = \frac{\sqrt{2}}{2}$

若 $c_1 = -c_2 = -\frac{\sqrt{2}}{2}$ 其本征矢为 $\pm\frac{\sqrt{2}}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

本征值为 $E_0 - A$

对应本征矢为 $\pm\frac{\sqrt{2}}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

本征值为 $E_0 + A$

对应本征矢为 $\pm\frac{\sqrt{2}}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$