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Issues and Opportunities in Exotic Hadrons*

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The last few years have been witness to a proliferation of new results concerning heavy exotic hadrons. Experimentally, many new signals have been discovered that could be pointing towards the existence of tetraquarks, pentaquarks, and other exotic configurations of quarks and gluons. Theoretically, advances in lattice field theory techniques place us at the cusp of understanding complex coupled-channel phenomena, modelling grows more sophisticated, and effective field theories are being applied to an ever greater range of situations. It is thus an opportune time to evaluate the status of the field. In the following, a series of high priority experimental and theoretical issues concerning heavy exotic hadrons is presented.

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I. INTRODUCTION

In 2007 the Belle Collaboration claimed the discovery of the Z(4430). This state attracted considerable attention because it is charged and couples to charmonium, implying that the most economical interpretation of its quark content is $c\bar{c}\bar{u}d$. The recent high statistics confirmation of the Z by the LHCb collaboration, and the startling demonstration of phase motion, has brought sharp focus on exotic hadronic matter. Indeed, the $Z(4430)$ joins a long list of other putative exotic states, several of which have been reported within the past year:

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$c\bar{c}$ **multiquarks**

X(3872), $Z_c(3900)$, Y(3940), $Z_c(4020)$, $Z_1(4050)$, $Z_2(4250)$, Y(4140)

$b\bar{b}$ **multiquarks**

$Z_b(10610)$, $Z_b(10650)$

other unusual states

$D_s(2317)$, H dibaryon, Y(2175), Y(4260), Y(4660), $Y_b(10888)$, $\pi_1(1600)$, $\pi(1800)$, $f_0(1500)$.

Although hadronic exotics such as glueballs, hybrids, and multiquark states have been long expected, the understanding of these states is primarily at the level of conjecture. Certainly, if the confirmation of the $Z(4430)$ marks the beginning of the exploration of a new sector of matter, the current phenomenology concerning quark interactions will need to be radically overhauled. A compelling and unified understanding of the new states has not yet emerged, and the gap between theory and experiment remains a major deficiency in our current level of understanding of elementary particle physics.

This gap has its roots in the famously difficult problem of solving QCD in its many-body, strongly interacting, relativistic regime. Current effective field theories are inoperable in the excited spectrum, lattice field theory has difficulties with weakly bound diffuse coupled-channel systems, and extant phenomenological models are insufficiently well constrained to be confidently applied to exotic states. Even the lack of knowledge of relatively simple dynamics, such as interactions in the $K\pi$ system, can affect the analysis of data concerning the new states.

The flood of information initiated by B factories (CLEO, BaBar, Belle), τ-charm facilities (CLEO-c, BESIII), and hadron machines (CDF, D0, LHCb, ATLAS, CMS) is not expected to abate soon. LHCb will continue to deliver new results in heavy quark spectroscopy for at least a decade. At the same time, BESIII at the Beijing Electron Positron Collider will continue its program to collect and analyze e^+e^- data in the energy region of the putative exotic states of charmonium. Furthermore, the GlueX experiment is due to start taking data in 2015. This experiment, situated at Hall D at JLab, is designed to discover and explore the properties of light hybrid mesons. The COMPASS experiment at CERN has been, and will continue to be, very active in hadron spectroscopy. The PANDA experiment at FAIR is expected to start taking data in 2019; amongst its goals is the exploration of charmonium hybrids and other exotic states.

In view of this situation, a workshop was convened at the Institute of Nuclear Theory, Seattle, with the aim of assessing the status of the field and drawing up a short list of questions that have the potential to move the field forward. This document is the outcome. We stress that this is not meant as a review, for which the reader is directed to Refs. [1–5]. Furthermore, the topics contained herein are not meant to comprehensive, but are offered in the hope that progress will be spurred in various directions.

The next three sections provide specific queries in the areas of lattice field theory, experiment, and theory. The lattice method has been singled out because it has advanced to the stage where modelling issues are minimal, but where results are sufficiently complex that experimental methods must sometimes be invoked to interpret them. Finally, the interface between theory and experiment is addressed in section V. Here the emphasis is on smoothing the interaction between theorists and experimental collaborations with the hope of drawing on the strengths of both communities.

II. LATTICE QCD CALCULATIONS

1.1 **Compute quantities as a function of light quark mass.** [Estia]

A better determination of the contribution of (virtual) two heavy-light meson loops in $Q\bar{Q}$ states below threshold is needed. Coupled channel phenomenological models suggest that for $Q\bar{Q}$ states near threshold these contributions are significant. After renormalization of the bare model parameters, one finds modest shifts in the leading nonrelativistic mass spectrum. Spin splittings between ground state heavy-light mesons induce spin-dependent effects in the spectrum and allow hadronic transitions that violate the Heavy Quark Spin Symmetry expectations. Furthermore, the mass splittings between the $Q\bar{u}$, $Q\bar{d}$ and $Q\bar{s}$ ground state heavy-light mesons allows small isospin breaking and considerable SU(3) breaking effects. In particular, this may be evident in the large $X(3872) \to \rho J/\psi$ and $\psi'(2S) \to \eta\psi$ transition rates.

In lattice QCD calculations, as the light quark masses are varied down from infinite (quenched approximation) to the scale of the momentum in the $Q\bar{Q}$ system the dominate effect of light quark loops is to modify the running of the QCD coupling (α_s). But as we vary the masses below this scale and below Λ_{QCD}, the effects of the light quark loops is essentially in probing the effects of large quark loops, i.e. the effects of coupled channels in the hadronic basis of states.

We suggest that detailed lattice studies of the $Q\bar{Q}$ mass spectrum as a function of light quark masses (m_u, m_d, m_s) for masses in a range between their physical values and $\approx 2 \times \Lambda_{QCD}$ will give much insight into the effects of coupling to decay channels in a model independent way. Furthermore, a calculation of hadronic transition rates as a function of light quark masses would be very illuminating.

1.2 **Develop and implement coupled-channel scattering formalism.** [Dudek]

The recent publication [6–8] of the first determinations of *coupled-channel* scattering amplitudes from lattice QCD offers promise that this first-principles approach to QCD might shed light on the exotic behavior being observed in charmonium. For the case of coupled two-body scattering, resonant singularities in the amplitudes can be explored using parameterizations of the t-matrix, where the parameters are tuned to describe the finite-volume spectra calculated in lattice QCD. From the pole positions and residues, masses, widths, and branching fractions of resonances can be determined – the distribution of poles across unphysical Riemann sheets may offer a discriminator for the internal structure of the resonances.

There has been no application of these coupled-channel techniques to meson systems featuring charm quarks, and only limited studies of elastic scattering, which is a situation in need of remedy. Early targets will be charmed systems near threshold like $DK, D_s\eta$ and $D\pi, D\eta$ as well as exotic isospin and strangeness channels. Double charmed channels like DD are also relatively simple. Hidden charm channels are challenging, because while all the tools are in place to deal with the coupled $D\bar{D}, D\bar{D}^*, D^*\bar{D}^*, D_s\bar{D}_s, \ldots$ system, the opening of three-body channels like $\eta_c\pi\pi$ and $J/\psi\pi\pi$ occurs at rather low energies. No complete formalism to relate finite-volume spectra to three-body scattering amplitudes yet exists – such a formalism will be required to study such systems in detail.

Calculations of meson-baryon scattering are at a less advanced stage than for meson-meson. In principle a calculation of the $J/\psi p$ scattering amplitude should be relatively straightforward, but some progress in correlator construction techniques is required to get the computational cost down to a reasonable level.

The decay constants of the vector resonances determine their rate of production in e^+e^- and radiative transitions to lighter states offer a way to produce states of other J^{PC}. The rigorously correct way to determine these in lattice QCD is to first determine the scattering amplitudes and their resonant content as described above, and to then introduce an external vector current. By extrapolating the calculated vector-current matrix elements to the resonance poles, off in the complex energy plane, the decay constants and radiative transition rates for resonances can be obtained. A slightly less rigorous approach, which may be acceptable for narrow resonances, is to ignore the hadronic decay of the states by excluding meson-meson-like operators from the basis used to determine the spectrum of states – a first round of calculations of this simplified type may be justified to aid our phenomenological intuition of the vector spectrum, extending the limited calculations presented in [9–14], using the excited state technology presented in [15].

1.3 **Investigate static quark interactions.** [Peardon]

Recent improvements to the set of techniques available for computing light quark propagation on the lattice should encourage practitioners to revisit the problem of computing potentials between static color sources and their excitation spectra [16, 17]. These calculations have a long history and the static potential in the $SU(3)$ Yang-Mills theory was amongst the first lattice Monte Carlo computations. Revisiting the potentials in the presence of light dynamical quarks [18–21] will give useful insight into the nature of the XYZ and pentaquark experimental signals. In particular, the bottom quark sector could be modelled very effectively with this data while exotic mesons in the charm sector are more sensitive to finite mass corrections. Phenomenological models of the exotic hadrons based on the Born-Oppenheimer picture would use these potentials as input.

With a static color and anti-color source, separated along an axis at distance R, the eigenstates of the Hamiltonian are irreducible representations of the little group of symmetries that preserve this axis. The energy of these states as a function of distance defines the potential, $V(R)$. The residual symmetry means these potentials are labelled by $\Sigma = 0^{\pm}, 1/2, 1, 3/2, \ldots$, where there are two spin-zero potentials since a mirror symmetry is also a good symmetry for this case. The half-integer spin potentials do not appear in a theory of gluons alone but would be present in QCD. With two flavors of light quarks, QCD energy eigenstates are classified with an extra quantum number, light isospin, and this property would be inherited by states built from static sources. There would thus be a new multiplicity of spectra with isospin $I = 0, 1/2, 1, \ldots$. The isospin 0 and 1 spectra would be the relevant ones for studies of hidden charm or bottom tetraquarks and in particular, since the Z^+ states are charged, the isospin 1 spectrum is of particular interest. This spectrum has not been computed in lattice QCD to date.

For pentaquarks containing hidden charm $c\bar{c}$ or bottom $b\bar{b}$ the isospin 1/2 and 3/2 potentials are relevant for modelling these states. Again, there is no counterpart for this potential in the theory of strongly-coupled gluons alone. Another possible potential that might usefully be investigated and which has no counterpart in the pure gauge theory are those associated with two color sources, $Q(x)Q(y)$ [19, 20]. In order to neutralize this color charge, at least one light quark field must be included in the creation operator. These potentials would help model doubly-charmed or doubly-bottom baryons.

1.4 **Compute form factors relevant to exotic states.** [Raul]

The determination of the elastic and inelastic form factors of the XZY resonances directly from lattice QCD would have three major impacts. First, it would lead to the theoretical reproduction of experimentally observed production or decay rates in a model-independent way. Second, it will give access to poorly constrained quantities that would elucidate the nature and structure of these exotic states. Examples of such quantities include the radii and electromagnetic moments of tentative molecular states. Third, it will guide future experimental searches of exotics. Although the studies of resonant electromagnetic processes are presently at their early stages, there have been a great deal of theoretical [22–24] and numerical [11, 15] developments that demonstrate that they are in fact accessible from lattice QCD. This progress resulted in the first calculation of a radiative transition of a hadronic resonance [25]. This calculation was performed in the light sector for $\pi\gamma^\star \to \rho \to \pi\pi$. The same technology will be applicable for future calculations in the heavy quark sector.

1.5 **Compute decay constants for exotic states.** [Dudek]

The decay constants of the vector resonances determine their rate of production in e^+e^- and radiative transitions to lighter states offer a way to produce states of other J^{PC}. The rigorously correct way to determine these in lattice QCD is to first determine the scattering amplitudes and their resonant content as described above, and to then introduce an external vector current. By extrapolating the calculated vector-current matrix elements to the resonance poles, off in the complex energy plane, the decay constants and radiative transition rates for resonances can be obtained. A slightly less rigorous approach, which may be acceptable for narrow resonances, is to ignore the hadronic decay of the states by excluding meson-meson-like operators from the basis used to determine the spectrum of states – a first round of calculations of this simplified type may be justified to aid our phenomenological intuition of the vector spectrum.

1.6 **In-medium hadron properties.** [Gastao]

Several model calculations predict that charmonium-nucleus exotic bound states should exist [references]. Two independent, equally important binding mechanisms have been identified: multigluon exchanges in the form of color van der Waals forces [references], and D, D^* meson loop contributions to charmonium self-energy with medium-modified masses [references]. A first, recent lattice calculation [reference] confirms model calculations expectations, finding relatively deeply bound states of J/Ψ and η_c to several light nuclei. Another interesting class of charmed-hadrons nuclear bound states are D-mesic nuclei [references], which are an important source of information on chiral symmetry restoration in medium [reference]. A lattice calculation of the $D-$meson interaction with nucleons and of $D-$nuclei binding energies would be of great importance for constraining models, given the present lack of experimental information on the D-nucleon interaction.

III. EXPERIMENT

2.1 **Publish upper limits for negative searches.** [Ryan]

Candidates for exotic hadrons have been observed in many channels. While it is clearly important to find new decay channels for these states, it is also important to limit their decays to other channels when these searches are negative. This is a reminder to experimentalists to publish upper limits on cross sections and branching fractions in a wide range of channels.

2.2 **Confirm marginal states.** [Swanson]

A variety of signals have been observed that require confirmation. There is some urgency in achieving this because attempts to understand the data can be seriously misled by the acceptance of spurious signals as hadronic states. Alternatively, many signals are statistically significant but contain unknown systematic errors due to assumptions in modelling (for example using interfering Breit-Wigner amplitudes to obtain asymmetric line shapes). Additional and more varied amplitude analysis is required in these cases. Amongst states requiring confirmation are $X(3940)$, $Y(4008)$, $Z_1(4050)$, $X(4160)$, $Z_2(4250)$, and $Y(4350)$.

2.3 **Unravel the excited χ_{cJ} spectrum.** [Changzheng]

The masses of the charmonium $2P$ states are expected to be around 3.8–4.0 GeV/c^2 [26, 27], while $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$ are well above the $D\bar{D}$ threshold but below $D^*\bar{D}^*$ threshold; they are expected to be wide. If the mass of $\chi_{c1}(2P)$ is high enough, $\chi_{c1}(2P) \to D^*\bar{D} + c.c.$ will be its dominant decay mode. The $\chi_{c2}(2P)$ may decay into $D^*\bar{D} + c.c.$ as well.

So far the $Z(3930)$ observed in $\gamma\gamma \to D\bar{D}$ [28] is regarded as the $\chi_{c2}(2P)$ state, and the $X(3915)$ observed in $\gamma\gamma \to \omega J/\psi$ [29] is supposed to be a $\chi_{c0}(2P)$ candidate, although its mass is a bit too close to $\chi_{c2}(2P)$, and it was not observed in $\gamma\gamma \to D\bar{D}$.

Further study on $\chi_{cJ}(2P)$ decaying to $D\bar{D}$ and $D^*\bar{D} + c.c.$ should be performed to identify $\chi_{c0}(2P)$ and $\chi_{c1}(2P)$ and to confirm the $\chi_{c2}(2P)$.

With more data collected in e^+e^- annihilation at the $\psi(4040)$ and $\psi(4160)$ peaks, the E1 transitions $\psi(3S)$ and $\psi(2D) \to \gamma\chi_{cJ}(2P)$ should be searched for; E1 transitions of $\chi_{cJ}(2P) \to \gamma\psi(2S)$ are also expected to be large compared with $\chi_{cJ}(2P) \to \gamma J/\psi$ and $\gamma\psi(1^3D_J)$.

Hadronic transitions $\chi_{cJ}(2P) \to \pi\pi\chi_{cJ}(1P)$ should be searched for, and the reaction $\chi_{cJ}(2P) \to \omega J/\psi$ may occur if the mass difference between $\chi_{cJ}(2P)$ and J/ψ is large enough. The spin-parity of the $\chi_{c0}(2P)$ candidate, $X(3915)$, needs to be measured and its production and decay patterns should be examined carefully to see if it is the $\chi_{c0}(2P)$.

2.4 **Measure e^+e^- cross sections.** [Changzheng]

The region at center-of-mass energies above the open charm threshold is of great interest due the plethora of vector charmonium states: the $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, and $\psi(4415)$ observed in the inclusive hadronic cross section, and the vector charmonium-like states, the $Y(4008)$, $Y(4260)$, $Y(4360)$, $Y(4630)$, and $Y(4660)$ observed in exclusive hadronic modes. These states were discovered in one specific mode and are not observed in other modes. Searches for these states in all possible final states are desired. This suggests high precision measurements of as many as possible exclusive e^+e^- annihilation modes, including multi-body open charm modes, hadronic transitions, radiative transitions, and even exclusive light hadron final states.

Fig. 1 shows an example of measured cross sections of two-body open charm final states and two- or three-body hadronic transition modes. Common features of the distributions are a richness of structures and a lack of precision. With more data from open charm threshold up to about 5 GeV and improved precision, better theoretical models will likely be needed to describe the line shapes of all the final states simultaneously. In this way better knowledge on the excited ψ and the Y states can be extracted. This may result in an understanding of the nature of these states and reveal if any are charmonium hybrids.

With the existing data samples, BESIII can already improve precision of the open charm cross sections significantly [30], considering the BESIII experiment will continue run for another 6-8 years, better measurements at more energy points are expected. Belle-II [31] will start data taking in 2018 with a data sample 50 times more than at Belle experiment, the precisions of all the measurements with initial state radiation will be improved.

The cross sections of e^+e^- annihilation into open bottom and bottomonium final states should also be measured to understand the excited bottomonium and bottomoniumlike states. This can only be done at the Belle-II experiment [31].

2.5 **Search for flavor analog exotic states.** [Swanson]

The majority of recently discovered exotic states are placed firmly in the charmonium spectrum. Flavor-independence of gluon exchange implies that flavor analog states should exist. For example, the $Z_b(10610)$ and $Z_b(10650)$ are evidently hidden bottom partners to the $Z_c(3900)$ and $Z_c(4020)$ multiquark candidate states. It is possible that the $Y(2175)$ is the hidden strange partner of the $Y(4260)$. Finding flavor-analog states will yield valuable information on the dynamics underlying the new states and will probe the robustness of putative models.

The case of a bottomonium analog of the $X(3872)$ is interesting, both because of the novelty of the X and because of differences that may arise. For example, if the X is a weakly bound $D\bar{D}^*$ system then the X_b would be expected at mass of 10604 MeV. However, some models [32] rely on the proximity of the hidden charm $\rho - J/\psi$ and $\omega - J/\psi$ channels to stabilize the X. This coincidence is *not* repeated in the case of the X_b, where the $\omega - \Upsilon$ threshold lies 370 MeV away. This also implies that the novel isospin-breaking features of the X will not be repeated in the X_b (isospin symmetry breaking is related to the hidden flavor mixing and to the splitting between charged and neutral $D\bar{D}^*$ channels – neither of which is repeated in the case of the X_b). Finally the proximity of the χ'_{c1} to the $X(3872)$ is likely to be important. Again, this numerical coincidence is not repeated in the case of the X_b, where nearby χ_{b1} states are at 10255 MeV (1P), 10512 MeV (2P), or 10788 MeV (3P[33]).

2.6 **Search for flavor analogs of the P_c.** [Richard]

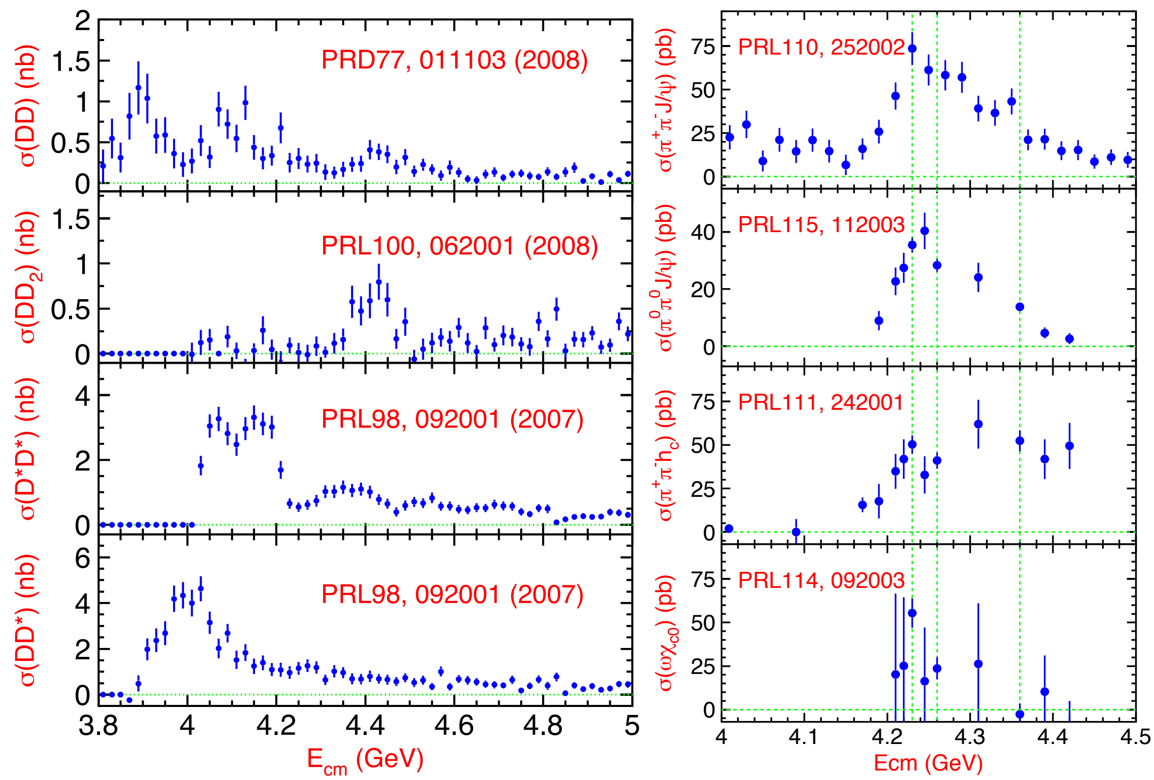

FIG. 1: The cross sections of e^+e^- annihilation into open charm final states (left panel, from Belle experiment) and charmonium final states (right panel, the top is from Belle and the others from BESIII experiments.) The vertical lines are at 4.23, 4.26, and 4.36 GeV.

Most established and candidate exotic hadron states contain hidden heavy flavor, $Q\bar{Q}$. This is mainly due to experimental constraints for production and detection. However, other sectors of flavor deserve to be investigated. Let us give two examples.

The isospin partner ($\bar{c}cudd$) and the strangeness partners such as ($\bar{c}cuds$) should be searched for, along with their $\bar{b}b$ analogs. One should not restrict to hidden heavy flavor. Pentaquark states ($\bar{Q}q^4$), where q^4 denotes $uuds$, $ddsu$ or $ssdu$ were predicted in 1987 on the basis of a chromomagnetic mechanism very similar to the one leading to speculations about the H dibaryon. This flavored pentaquark has been searched for at Fermilab and HERA. Searches with higher statistics are desirable, especially if more hidden-flavor states such as ($\bar{Q}q'q^3$) are found [34, 35].

Exotic mesons with double heavy flavor, ($QQ'\bar{q}\bar{q}$) have been predicted for many years, and their existence is confirmed by different groups working with different approaches: potential models, QCD sum rules, lattice QCD and meson-meson molecules. It has also been stressed that more effort should be put on double-charm and other doubly-heavy baryons. We thus suggest a search of doubly-heavy hadrons besides B_c: double-charm baryons, double-charm mesons and double-charm dibaryons, and in the future, their analogs with charm and beauty or double beauty[36, 37].

2.7 **Search for quantum number partners of the $Y(4260)$.** [Swanson]

If the $Y(4260)$ is a hybrid state it represents the first example of – what is expected to be – a large array of novel hadrons. Specifically, a spin multiplet analogous to those in the conventional spectrum is expected. An example of this multiplet in the light quark sector is shown in Fig. 2. A clear structure with quantum numbers 1^{--} and $(0,1,2)^{-+}$ is seen. This multiplet can be conveniently interpreted as arising due to an effective constituent gluon with quantum numbers $(J^{PC})_g = 1^{+-}$ mixing with conventional quark-antiquark degrees of freedom[38].

Given this information one can expect the spectrum shown in Table I. The increasing luminosity expected at the colliders raises interesting possibilities for detecting these states. For example, the 0^{-+} and 1^{-+} lie below the $Y(4260)$ and therefore should be accessible in radiative decays in P-wave. The hybrids can then be detected in decay modes such as $\eta\,\chi_{cJ}$ or $f_J\,\eta_c$ (see Table II).

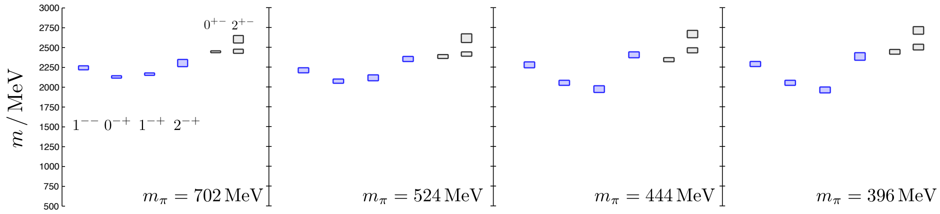

FIG. 2: The lightest hybrid multiplet as a function of quark mass. Figure reproduced with permission from Ref. [39].

TABLE I: Expected Hybrid Multiplet [11].

J^{PC}	mass (MeV)
2^{-+}	~ 4320
1^{--}	4260
1^{-+}	~ 4200
0^{-+}	~ 4190

2.8 **Pursue properties of the** $X(3872)$**.** [Swanson]

Although properties of the $X(3872)$ are reasonably well known, additional experimental effort can greatly assist in improving the understanding of this state. For example, the rate for decays to light hadrons, such as $X \to \eta\pi\pi$, can be compared to those for χ_{c1} states in an effort to determine the expected mixing of the X with the bare $\chi_{c1}(2P)$.

Analog hidden charm states are predicted in some models and can be searched for. For example a 0^{++} $D^*\bar{D}^*$ state is expected at 4019 MeV in pion-exchange models[1, 40].

Intriguing analog flavor-exotic states are also expected in QCD. In particular, it has been argued that $QQ\bar{q}\bar{q}$ states must exist in the limit where the heavy quark mass goes to infinity[37]. The phenomenology of such states is discussed by Tornqvist[40] and was anticipated long ago[41]. Specific possibilities include isoscalar KK^*, DD^*, BB^* states with $J^P = 1^-$ and K^*K^*, D^*D^*, B^*B^*, etc. Nevertheless, flavor exotic vector-vector bound states are unlikely, except possibly in the doubly charged bottom sector[40].

2.9 **Measure additional channels to investigate the** P_c**.** [Tomasz]

The recent evidence for the resonant P_c^+ structures in $J/\psi p$ in the $\Lambda_b \to J/\psi p K^-$ decays by the LHCb experiment, has renewed interest of the experimental and theoretical communities in pentaquark states. Further experimental work is critical for clarification of the nature of these structures. Tightly bound pentaquarks, molecular states and rescattering effects have been proposed. More accurate determination of the quantum numbers of the pentaquark candidates would greatly help their interpretation. Even before more data is accumulated by the LHCb, improving parameterizations in the amplitude fits to the existing data may help this end. For example, models of Λ excitations, which dominate the data via decays to pK^-, need to be checked for completeness since the previous experiments may have not discerned all the relevant states, especially at high masses. Non-resonant terms with slowly varying magnitude and phase can also be significant. Alternative approaches to the isobar model may be helpful, like, for example, the recently published coupled-channel model by Fernandez-Ramirez et al.[42]. See also Refs. [43–45].

It is important to confirm the P_c via other channels. There are already suggestions[43] such as $\Lambda_b \to J/\psi p\pi^-$, or $\Xi_b \to J/\psi K^- p$, which are Cabibbo suppressed, or $B \to J/\psi p\bar{p}$. The predictions of rescattering models for the P_c^+ amplitudes can be tested by fitting them directly to the data. The presence (or lack thereof) of the same structures in the other channels, like $\Lambda_b \to J/\psi p\pi^-$ or $\Lambda_b \to J/\psi pK^0\pi^-$ is of great importance.

Rescattering models predict presence of structures related to the P_c^+ peaks induced by the analyticity in the coupled channels, like $\Lambda_b \to \chi_{cJ} pK^-$, $\Lambda_b \to \Sigma_c^{(*)+}\bar{D}^{(*)0}$ and $\Lambda_b \to \Lambda_c^{(*)+}\bar{D}^{(*)0}$. Ideally, simultaneous coupled channel analysis of the related final states should be performed. The investigation of possible structures, which may include depressions rather than peaks, is a good start. Even total relative rates between the different channels would be interesting. Negative searches have theoretical implications and should be published.

TABLE II: Possible production and decay modes of hybrid charmonium.

$Y(4260) \to \gamma\, 1^{-+} \to \gamma\, \eta\, \chi_{c1},\, \gamma\, f_1\, \eta_c$
$Y(4260) \to \gamma\, 0^{-+} \to \gamma\, \eta\, \chi_{c0},\, \gamma\, f_0\, \eta_c$
$Y(4360) \to \gamma\, 2^{-+} \to \gamma\, \eta\, \chi_{c2},\, \gamma\, f_2\, \eta_c$

Bound-state models for the observed P_c^+ states predict other pentaquark states built by the same binding mechanisms. The same P_c^+ states may be observable in the other decay modes too. Thus, every accessible decay mode of Λ_b with c and $\bar{c}$ quarks among the final states hadrons should be examined, e.g. $\eta_c p K^-$. Other final states can be accessible from Ξ_b to charmonium decays.

Different productions mechanisms for the P_c^+ states, or their siblings, should be investigated. Examples are prompt production at LHC or photo-production at JLab.

2.10 **Test ideas for meson-nuclear interactions.** [Gastao]

Presently there is a complete lack of experimental information on the low-energy interactions of charmed mesons and charmonium with nucleons and nuclei. We look forward to several forthcoming experimental programs in this area: the near-threshold experiments by the ATHENA collaboration [reference] as part of the 12 GeV program at Jefferson Lab, the proton-antiproton experiments by the PANDA collaboration at FAIR [references], and the experiments with 50 GeV high-intensity proton beams at the J-PARC complex [reference]. We also envisage opportunities for finding exotic charmonium-nucleon and charmonium-nucleus bound states with the ongoing heavy-ion experiments at RHIC and LHC. In particular, we suggest studies on the formation of such exotic bound states by coalescence in the late-stage evolution of heavy-ion collisions, as their production yields should be of comparable magnitude to those of anti-nuclei and anti-hypernuclei recently observed at RHIC [reference] and LHC [reference].

2.11 **Improve meson classification scheme.** [Swanson]

There is a wide range in signal robustness in the spectrum of new states. Because this can lead to confusion amongst theorists who are attempting phenomenological descriptions of these new states, we recommend that a star system for mesons be implemented by the Particle Data Group for use in the *Review of Particle Properties*. Furthermore, the current exotic particle naming scheme is somewhat confused and is applied inconsistently; we therefore recommend that a consistent and flexible nomenclature be implemented.

2.12 **Search for $p\overline{p}$ in decays at LHC for PANDA.** [Richard]

Heavy-flavor physics will benefit from experiments with medium-energy antiproton beams. In the past, a precursor signal of the h_c was seen at the CERN ISR, and many properties of the $\chi_{c,J}$ and other charmonium states were obtained from the $\bar{p}p$ experiment at the Fermilab accumulator.

To assist future experiments, it is desirable to get information on the coupling of heavy hadronic systems to proton-antiproton pairs by detecting $\bar{p}$ production at B-factories and at the LHC. This is already under way, and this should be accompanied by more theoretical studies. For instance, it remains rather mysterious that $\eta_c(1s)$ decays more often to $p\bar{p}$ than suggested by simple perturbative QCD, while $\eta_c(2s)$ is much less coupled to that channel.

IV. THEORY AND PHENOMENOLOGY

3.1 **Study exclusive e^+e^- cross sections using better coupled-channel formalism.** [Adam]

The identification of possible new resonances implied by the X,Y,Z phenomena requires studies of analytical amplitudes that describe the relevant production and decay characteristics. For example, in the case of the $Z_c(3900)$ that is observed in the $\pi^\pm J/\psi$ spectrum in decays of the $Y(4260)$ to $\pi^+\pi^- J/\psi$, the relevant direct channels involve the nearby open-charm, $D\bar{D}^* + c.c$ states and need to be included in a coupled channel formalism. The open charm resonances in the production channel, e.g., the $D_1(2420)$ can produce rapid variations of the direct channel partial waves near the Z_c signal and need to be taken into account in production. The singularity structure of partial wave amplitudes is constrained by unitarity, therefore a comprehensive analysis requires implementation of unitarity constraints in all relevant channels. This requires simultaneous studies of quasi two-to-two scattering amplitudes of open flavor and heavy quarkonia, e.g. $D\bar{D}^* \to J/\psi\pi$, and eventually a study of three-to-three scattering, i.e. $D\bar{D}\pi \to D\bar{D}\pi$ amplitudes.

3.2 **Develop tests for the dynamical diquark picture.** [Lebed]

In an alternate proposal for the structure of the heavy quarkonium-like exotics, both for the tetraquarks [46] and pentaquarks [47], the states are composed of compact diquark-antidiquark (-antitriquark) pairs rapidly separating (and hence ultimately achieving large ($\approx$ 1 fm or greater) separation before decay. This picture has features in common with the diquark models previously mentioned [48, 49], but differs in that the states are extended, dynamical rather than compact, static objects, and therefore does not necessarily admit a Hamiltonian description. Nevertheless, in the limit of small separation, the two pictures should coincide. The first priority in this case is therefore the development of a formalism in which the spectrum can reliably be predicted. A first attempt in the pentaquark sector [50], still using a Hamiltonian formalism, gives a natural explanation for a broad $\frac{3}{2}^-$ lying just below a narrow $\frac{5}{2}^+$, consistent with the LHCb findings [51], but also predicts a large number of undiscovered states. A lattice calculation of the potential corresponding to a well-separated static diquark-antidiquark pair may provide valuable information on the possible spectrum. Since the exotics are so prominent in the $c\bar{c}$ and $b\bar{b}$ channels, some hints of the same mechanism with $s\bar{s}$ (hidden-strangeness pentaquarks) should appear in processes such as $\Lambda_c \to \phi\pi^0 p$ [52] or ϕ photoproduction [53]. A primary benefit of the dynamical picture is its natural explanation of strong overlaps with spatially larger states, so a precision measurement of the ratio $Z(4475) \to \psi(2S)\pi$ vs. J/ψ and to other states will be illuminating. The dynamical and compact diquark models share an expected enhancement of $Z_c \to \eta_c\rho$ compared to the corresponding rate in molecular models [54]. The extended structure of the state may also offer interesting opportunities for the production of unusual final-state particle correlations. The multiparticle nature of states produced via, say, electroproduction or $p\bar{p}$ annihilation (at JLab or PANDA, respectively) can be probed by means of constituent counting rules [55], and can help to distinguish whether compact multiquark components are produced.

3.3 **Develop experimental tests for tetraquarks.** [Lebed]

Compact tetraquark configurations, in which all four quarks participate in strong mutual interactions, can be distinguished from the hadron molecular picture or threshold effects through a variety of experiments. The most well developed tetraquark models are of the diquark-antidiquark class [48, 49], and rely on Hamiltonians with spin-spin interactions between the quark pairs. A comparison of the expected spectra in this tetraquark model versus hadronic molecular models (and also hadrocharmonium) [56] indicates that many more states should arise if tetraquarks are the dominant exotic component; for example, the $X(3872)$ should have isotriplet charged partners of the same G parity [and opposite that of the $Z(3900)$ and $Z(4020)$]. Due to the proximity of thresholds, such states might exist only as very broad yet-undiscovered resonances. Large prompt production cross sections at colliders [57] argue against $X(3872)$ being a $D\bar{D}^*$ molecule forming through coalescence; indeed, an extrapolation [58] of data from ALICE shows that production of loosely bound hadronic molecules such as d and ^{3}He at high $p_\perp$ will be quite suppressed, unlike current indications for $X(3872)$, an effect that can be decisively checked in future ALICE and LHCb experiments. The molecular and diquark pictures also differ radically in the ratios of their branching fractions of $Z_c \to \eta_c\rho$ vs. $J/\psi\pi$ or $h_c\pi$ [54, 59], the former being dozens of times less frequent in molecular models. Loosely bound molecules also must obey well-known universal relations (independent of the potential) between binding energy and width, and precision measurements of the resonance widths and constituent masses can help determine whether these constraints are satisfied [60].

3.4 **Develop techniques for $5q$ and $6q$ systems** [Richard]

Potential models provide some guidance for QCD calculations. Two-body calculations are obvious once an explicit potential is given. Three-body and four-body computational methods new yield accurate spectra, although they require more delicate tools. The case of five-body and six-body systems are still debated. For instance, with similar Ansatze for the interaction, the $H = (uuddss)$ is can be found to be either stable or unbound. We suggest to publish a set of benchmark calculations to remove the ambiguities.

3.5 **Pursue the Born-Oppenheimer method (adiabatic surface mixing).** [Morningstar]

The presence of heavy charm or bottom quarks in the new tetraquark mesons suggests that they may be successfully studied using the *Born-Oppenheimer expansion*. This approach was introduced by Born and Oppenheimer in 1920 [61] to understand the binding of atoms into molecules by exploiting the large ratio between the mass of an atomic nucleus and an electron, which implies that the time scale for the motion of electrons is orders of magnitude faster than that for the motion of the nuclei. The energies of stationary states of the electrons in the presence of fixed nuclei can be calculated as functions of the separation of the nuclei. The resulting functions are called *Born-Oppenheimer potentials*. In the Born-Oppenheimer approximation, these functions are used as potential energies in the Schrödinger equation for the nuclei, under the assumption that the electrons respond very rapidly to the motion of the nuclei. The Born-Oppenheimer *expansion* involves taking the large mass ratio into account more systematically by incorporating non-adiabatic couplings between different stationary states of the electrons. This results in coupled-channel Schrödinger equations that systematically improves the description of a molecule.

The Born-Oppenheimer expansion was applied to mesons containing a heavy quark (Q) and antiquark ($\bar{Q}$) in 1999[62], exploiting the fact that, since the mass of the heavy quark is much larger than the typical energies of the gluons and light quarks, the time scale for the evolution of the gluon and light-quark fields is much faster than that for the motion of the Q and $\bar{Q}$. In Ref. [62], lattice QCD was used to calculate the Born-Oppenheimer potentials defined by the energies of the gluons in the presence of fixed Q and $\bar{Q}$ as functions of the $Q\bar{Q}$ separation. These energies were then used as the potential energies in the Schrödinger equation for the Q and $\bar{Q}$. The bound states in the Born-Oppenheimer potentials were interpreted as meson resonances. These bound-state energies were compared with corresponding meson masses computed directly using lattice QCD and agreement in the level spacings to within 10% was found, strongly supporting the validity of the Born-Oppenheimer expansion for such systems.

The approach used in Ref. [62] should be extended to apply to the XYZ mesons and to include nonadiabatic effects that can be incorporated through coupled-channel Schrödinger equations. The Born-Oppenheimer potentials for heavy tetraquark mesons and the nonadiabatic couplings between the potentials could be calculated using lattice QCD. The heavy quark and antiquark would be treated as static, and the energies of the gluons and light quarks could then be computed as a function of the separation between the quark and the antiquark. The resulting coupled-channel Schrödinger equations could then be solved to determine the energies and widths of resonances, which can be compared with the observed XYZ mesons, and possibly to predict new tetraquark mesons.

3.6 **Revisit conventional meson models.** [Susana/Swanson]

While the successes of the constituent quark model are well-known in the heavy quark sector, the efficacy of the model is not expected to survive higher in the spectrum, where gluonic and coupled channel effects become important. Of course, it should be possible to extend constituent models to include these additional degrees of freedom, but experimental and theoretical guidance will be required.

Even the simple problem of assessing the accuracy of the constituent quark model above threshold has difficulties. For example, there are eight charmonium states below $D\bar{D}$ threshold that are all well-described by models. Alternatively, the situation above threshold is considerably more confused; of the approximately twenty claimed states, most of them are not understood, and even well-known states such as the $\psi(3770)$ lie 50 MeV below the prediction of the Godfrey-Isgur model. In the bottom sector the 22 states that lie below $B\bar{B}$ are well-described. In this case there are only six states above threshold, but, again, the experimental and theoretical situation is confused.

Since the new experimental data lie firmly in the continuum region, it is very likely that more sophisticated versions of the quark model must be developed. Of course, this has been known in the community for many decades, and much work has been done[63–84]. There are daunting issues to be overcome, including determining the form of the nonperturbative gluonic transition operator and evaluating the (divergent) sum over infinitely many virtual channels[85]. Nevertheless it is difficult to imagine progress being made without a successful outcome to this effort. Alternatives approaches exist of course: lattice gauge theory is rapidly making progress in working in the coupled channel regime, and one hopes that effective field theory approaches will be developed that can accommodate the extra scales present.

3.7 **Develop the Schwinger-Dyson Formalism.** [Christian]

The Dyson-Schwinger equations (DSEs) of QCD, together with various many-body equations for bound states (Bethe-Salpeter equations for the two-body problem, Faddeev and Faddeev-Yakubovsky equations for the three- and four-body problem) have the potential to reveal the connections between the physics in different sectors of QCD. They encode the running of QCD Green's functions as, e.g. the quark mass function, and therefore connect the perturbative current quark region with the non-perturbative constituent quark domain. Furthermore, they connect the heavy quark regime, where NRQCD or potential models are applicable, with the light quark sector, where the concept of a potential is not vey well defined.

The explanatory power of the DSE framework with respect to exotic hadrons is hardly explored so far. For a start, light scalar mesons have been treated as tetraquarks in an approach which takes into account all possible two-body correlations within the bound state equation for two quarks and two antiquarks [86, 87]. The resulting Bethe-Salpeter amplitude for scalar tetraquarks is dominated by pseudoscalar meson-meson correlations. For the lightest state, the $f_0(500)$, this explains its large decay width into two pions, whereas the a_0/f_0 are dominated by their $K\bar{K}$ components. In general, it turns out that the wave functions can be controlled by either pair of two-body correlations inside the tetraquark (i.e. (anti-)diquarks or mesons) and it is a question of the internal dynamics which is the dominant cluster. For the light scalar mesons this is the 'meson molecule' configuration, but other results are in principle possible for other quantum numbers and different quark flavors and masses.

Whether this mechanism has the potential to shed some light on the question of the internal structure of the tetraquarks among the XYZ-states, in particular their (anti-)diquark, molecular or hadrocharmonium nature, needs to be explored. To this end, quantum numbers other than scalar need to be studied and the framework needs to be extended towards heavy-light systems. Furthermore, more quantitative precision is needed to confirm the prediction of an all-charm tetraquark in the 5.0 - 6.5 GeV mass region [86, 87].

Complementary ongoing projects within the DSE framework concern the glueball spectrum [88, 89] and the question whether states with exotic quantum numbers can be accounted for with relativistic quark-antiquark systems (in contrast to the non-relativistic quark model) [90].

3.8 **The status of Large N_c Considerations.** [Cohen]

One striking thing about modern exotics— the X,Y, Z states—is that they all involve the physics of heavy quarks. This raises an interesting issue: are heavy quarks necessary for the formation of exotics or do exotics exist for light quarks systems. The experimental data on this is murky. The large N_c limit may provide a bit of insight. The subtle point is that the large N_c and heavy quark limits may not commute so that generic large N_c arguments based on scaling arguments really apply for light quark systems. The standard version of the large N_c limit with quarks in the fundamental representation of SU(N_c) can be shown not to have narrow tetraquarks at large N_c[91], apparently supporting the proposition that the heavy quarks are necessary for the existence of tetraquark states. However, there is a variant of the large N_c limit where quarks are in the two-index anti-symmetric representation of SU(N_c) in which it can be shown that states with exotic tetraquark quantum numbers must exist as narrow resonances (i.e. states whose widths go to zero as N_c goes to infinity) regardless of the mass of the quarks[92]. Minimally this shows that QCD-like gauge theories are not excluded from having tetraquark states even if the quarks are light.

V. THEORY-EXPERIMENT COLLABORATION

The following are a few suggestions that could help facilitate collaboration between theory and experiment.

4.1 **Improve parameterizations of the data.**

One of the challenges in many of the experimental studies of the XYZ states is to develop correct parameterizations of the data. For example, amplitude analyses often find a need to introduce non-resonant terms. At present, very little theoretical guidance is provided except for the amplitude formulations based on the K-matrix approach. However, the latter is not always practical.

To improve this situation, we have two recommendations.

First, we encourage that, when appropriate and beneficial, experimentalists and theorists directly work together on the analysis of data. This could be accommodated by theorists becoming co-authors on specific experimental papers they substantially contributed to, or joint submission of experimental and theoretical papers cross-referencing each other. The experiments are encouraged to formalize procedures making such collaboration possible, and theorists are encouraged to approach the experiments when they think they might directly aid specific data analysis topics. Further progress could be made by more persistent forms of collaboration, including direct involvement of theorists in the data analysis process within the established procedures of the experimental collaborations.

Second, we encourage theorists, when possible, to publish complete functional forms (amplitudes, etc.) that could be used in the fitting of data. One example of this is in the parameterization of rescattering amplitudes. The current theoretical calculations are dependent only on the center of mass energy [93–95], whereas amplitudes used in fitting require a flexible parameterization involving the angular information of the decay. If theorists develop more complete rescattering amplitudes, experimentalists could use them in analyses. This would most likely involve some collaborative effort in understanding both how the experimental analysis is performed and what are the theoretical requirements for such amplitudes. Another example is in resonance parameterizations: it would be useful for experimentalists if a number of alternate resonance parameterizations were available that could be used in systematic studies.

4.2 **Make experimental results more accessible for subsequent interpretation.**

The analysis of data from many modern experiments often necessitates complying with internal rules designed to provide collaborative controls over the quality of statistical methods used and the proper evaluation of systematic

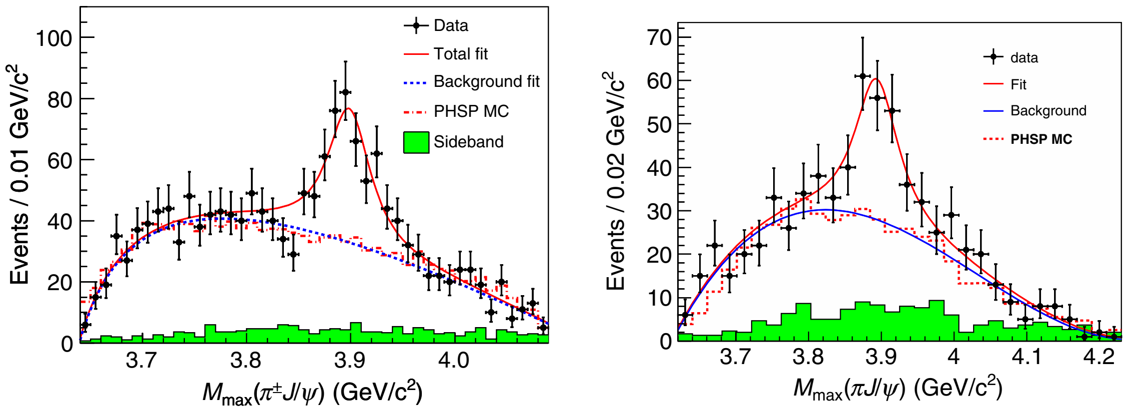

FIG. 3: The observation of the $Z_c(3900)$ from BESIII [96] (left) and from Belle [97] (right). The different shapes at low $M(\pi^{\pm}J/\psi)$ mass are due to differences in experimental detection efficiencies.

uncertainties. Therefore it is unrealistic to expect that all data will be made available for analyses outside of this collaborative setting. A correct analysis of data would benefit from the types of closer interaction between experiment and theory discussed in the previous bullet.

A different issue is how published data (for example, Dalitz plots) should be subsequently interpreted. It often occurs that experimental results are made public in a manner that does not easily allow for subsequent interpretation.

One example is the discovery of the $Z_c(3900)$ decaying to $\pi^{\pm}J/\psi$ in the process $e^+e^- \to \pi^+\pi^-J/\psi$ [96, 97]. The data presented in the discovery papers include effects due to experimental efficiencies that must be taken into account when performing theoretical fits. The importance of these effects can be seen when comparing the $M(\pi^{\pm}J/\psi)$ plots from BESIII and Belle, which differ substantially due to different efficiencies (Fig. 3). It is therefore not clear how one could correctly analyze the published data with various new parameterizations to test, for example, differences between cusp and resonant models of the $Z_c(3900)$.

When deemed appropriate, experiments are therefore encouraged to make efficiency-corrected data available for external analyses. Or, when possible or desired, experiments could make published plots publicly available along with efficiency curves and instructions for how to use the plots for subsequent analysis. This could be provided as supplemental information to a publication. This may be easy for simple three-body final states (like $\pi\pi J/\psi$, where the Dalitz plot could be provided), but impractical for more complicated final states.

Another suggestion, especially when amplitude analyses have been performed, is to publish a complete parameterization of the data, including both the formulas and the numerical values for each fit parameter. Making data public in this way could help facilitate ongoing efforts to test and build models. It could also permit combined fits of data from different experiments or different channels, thus helping a more global picture to emerge.

4.3 Preview upcoming analysis results.

It may be useful, in some circumstances, for experimental collaborations to provide a list of upcoming experimental results. This might be a fruitful way to elicit new theoretical predictions or ideas. For example, if the community knows a certain measurement is being performed, there is then a chance to make predictions prior to the publication of experimental results. Hosting such a list on a common platform may prove useful and should be explored.

4.4 Create a list of publications in XYZ physics.

It would be helpful to have a centralized running bibliography of publications relevant for XYZ physics.

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