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# DICTIONARY OF ANGULAR DISTRIBUTIONS OF PARTICLES FROM $e^+e^-$ ANNIHILATION

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Abstract

This is the summary of works done by Y.S. Tsai and the author about the polar angle distributions. They are investigated in both all pair of combinations for scalar-, pseudoscalar-, vector-, axialvector-, and tensor particles, and one aprticle productions with a photon.

Recent experimental developments in $e^+e^-$ colliding beams have opened up various exciting and fascinating facts: New hadron family $\psi$ series,[1] jets in inclusive hadron production,[2] a narrow state at 1865 MeV/$c^2$,[3] anomalous $e\mu$ events,[4] and so on.

We shall investigate polar angle distributions for all pair of combinations of S(scalar), P(pseudoscalar), V(vector), A(axialvector), T(tensor), and ~~that~~ those for one particle productions with a $\gamma$-emission, which are produced by $e^+e^-$ annihilation. The calculation is carried out by adopting the effective covariant vertices.

The importance of the angular distribution is as follows: First in $\psi$ series, the $J^{PC}$-determination of $P_c$ and $\chi$ states will play a major role for the selection of models[5] and gaining an insight into the dynamics for constituents of new hadrons. Several works on angular distributions of $\psi' \to \gamma\chi$ have already been done[6] using the non-relativistic approximation. But in some cases, e.g. $\phi'$*), the recoil effect cannot be neglected. We take it into account by making the vertices Lorentz invariant. Second, hadron production by $e^+e^-$ annihilation has shown that the distribtuion of jets is $( 1 + \cos^2\theta )$ where $\theta$ is the polar angle. It exists various possibilities for the interpretation of this distribution besides a pair of spin 1/2 particles. Y.S. Tsai[7] discussed them, but some cases were omitted. We include the cases done by Tsai for completeness of our study. And last, a new narrow state at 1865 MeV/$c^2$ must show $( 1 + \cos^2\theta )$ distribution if it is $D^0$ accompanied by $D^{*0}$. This is the easiest method for the elimination of other possibilities, e.g., SS', PP', VV', etc.

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*) The prime denotes the second radial excited state of $\phi$.



It should be noted that the azimuthal angle disribution for polarized beams can be derived from our results, because the general form can be written as[8]

$$\frac{d\sigma}{d\Omega} = ( \sigma_T + \sigma_L )( 1 + \frac{\sigma_T-\sigma_L}{\sigma_T+\sigma_L} \cos^2\theta + \xi^2 \frac{\sigma_T-\sigma_L}{\sigma_T+\sigma_L} \sin^2\theta \cos 2\phi ),$$

where $\theta$ ( $\phi$ ) is the polar (azimuthal) angle and $\xi$ is the polarization parameter.

The cross section is proportional to $L^{\mu\nu}M_{\mu\nu}$ where $L^{\mu\nu}(M_{\mu\nu})$ is the tensor formed by $e^+e^-$(final particles). The general form of $M_{\mu\nu}$ is

$$M_{\mu\nu} = ( - g_{\mu\nu} + \frac{P_\mu P_\nu}{P^2} ) W_1 - ( k_\mu - \frac{(kP)}{P^2} P_\mu )( k_\nu - \frac{(kP)}{P^2} P_\nu ) \frac{W_2}{M^2} .$$

We denote the total momentum by $P_\mu$ and $k_\mu$ is the four momentum carried by final one particle. The angular distribution takes the form of

$$( 2 W_1 - \vec{k}^2 \frac{W_2}{M^2} ) + \vec{k}^2 \frac{W_2}{M^2} \cos^2\theta ,$$

where $\vec{k}$ is the momentum of a final particle.

Now we are ready to open the dictionary.

Table I. Angular Distributions for $e^+e^- \longrightarrow XY$

| Mode | Vertex | Structure Functions | Angular Distribution[a)] | |
|---|---|---|---|---|
| | | | | $\lvert\vec{k}\rvert >> M_X, M_Y$ |
| $e^+e^- \longrightarrow N\bar{N}$ | $F_1 A^\mu \bar{\psi}\gamma_\mu\psi + F_2 F_{\mu\sigma}\bar{\psi}\sigma^{\mu\nu}\psi$ | $W_1 = 2G_M^2 P^2$<br>$\vec{k}^2 \frac{W_2}{M^2} = 2(G_M^2 P^2 - 4m^2 G_E^2)$ | $(sG_M^2 + 4m^2 G_E^2) + (sG_M^2 - 4m^2 G_E^2)\cos^2\theta$ | $1 + \cos^2\theta$ |
| $e^+e^- \longrightarrow S\bar{S}$, $SS'$ | (minimal) $+ F_{\mu\nu}\partial^\mu S \partial^\nu S'$ | $W_1 = 0$ | $1 - \cos^2\theta$ | $1 - \cos^2\theta$ |
| $e^+e^- \longrightarrow SP$ | This is forbidden. | | | |
| $e^+e^- \longrightarrow SV$ | $F_{\mu\nu}V^{\mu\nu}S$ | $W_1 = (kP)^2$, $\frac{W_2}{M^2} = P^2$ | $(1 + \frac{\vec{k}^2}{2M_V^2}) + \frac{\vec{k}^2}{2M_V^2}\cos^2\theta$ | $1 + \cos^2\theta$ |
| $e^+e^- \longrightarrow SA$ | $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}F^{\mu\nu}\partial^\lambda S A^\sigma$ | $W_1 = (kP)^2 - k^2P^2$, $\frac{W_2}{M^2} = P^2$ | $1 + \cos^2\theta$ | $1 + \cos^2\theta$ |
| $e^+e^- \longrightarrow ST$ | $F_{\mu\nu}T^{\mu\lambda}\partial_\lambda\partial^\nu S$ | $W_1 = \frac{1}{2}(kP)^2(PI_qP)$<br>$\frac{W_2}{M^2} = \frac{P^2}{2q^2}(kP)^2 - \frac{2}{3}\frac{(k\cdot q)^2}{q^4}P^4$ | $\{7 + (3 + 4\frac{s}{M_t^2})\frac{\vec{k}^2}{M_s^2}\} - \{1 + (-3 + 4\frac{s}{M_t^2})\frac{\vec{k}^2}{M_s^2}\} \times \cos^2\theta$ | $1 - \cos^2\theta$ |
| $e^+e^- \longrightarrow \bar{P}P$, $PP'$ | the same as that of $e^+e^- \longrightarrow S\bar{S}$, $SS'$ | | $1 - \cos^2\theta$ | $1 - \cos^2\theta$ |
| $e^+e^- \longrightarrow PV$ | $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}\partial^\mu V^\lambda F^{\lambda\sigma}P$ | $W_1 = (kP)^2 - k^2P^2$, $\frac{W_2}{M^2} = P^2$ | $1 + \cos^2\theta$ | $1 + \cos^2\theta$ |
| $e^+e^- \longrightarrow PA$ | $F_{\mu\nu}A^\mu\partial^\nu P$ | $W_1 = (kP)^2$, $\frac{W_2}{M^2} = P^2(1 - \frac{P^2}{q^2})$ | $\{1 + (1 + \frac{s}{M_A^2})\frac{\vec{k}^2}{2M_p^2}\} + (1 - \frac{s}{M_A^2})\frac{\vec{k}^2}{2M_p^2}\cos^2\theta$ | $1 - \cos^2\theta$ |
| $e^+e^- \longrightarrow PT$ | $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}T^{\mu\delta}F^{\nu\lambda}\partial_\delta\partial^\sigma P$ | $W_1 = (kP)^2 - k^2P^2$, $\frac{W_2}{M^2} = P^2$ | $1 + \cos^2\theta$ | $1 + \cos^2\theta$ |

Table I — Continued

| | | | | |
|---|---|---|---|---|
| $e^+e^- \longrightarrow V\bar{V}$ | $F_1$ (minimal) $+ F_2 F_{\mu\nu} V^{\mu} \bar{V}^{\nu}$ | $W_1 = F_1 \frac{k^2+q^2}{k^2q^2}((k\cdot q)^2 - k^2q^2)$ <br> $+ F_2((PI_qP) + (PI_kP))$ <br> $\frac{W_2}{M^2} = F_1(2 \frac{k^2+q^2}{k^2q^2} - 8)$ <br> $+ F_2 P^2 \frac{1}{k^2q^2}[k^2 + q^2 - P^2]$ | $\{F_2 + (F_1 + F_2)\frac{2M_V^2}{s} - 12F_1 \frac{M_V^4}{s^2}\}$ <br> $- \{F_2 - (F_1 + F_2)\frac{2M_V^2}{s} + 12F_1 \frac{M_V^4}{s^2}\} \cos^2\theta$ | $1 - \cos^2\theta$ <br> $\begin{pmatrix} \text{If } F_2 = 0 \\ 1 + \cos^2\theta \end{pmatrix}$ |
| $e^+e^- \longrightarrow VV'$ | $F_{\mu\nu} V^{\mu} V'^{\nu}$ | | | $1 - \cos^2\theta$ |
| $e^+e^- \longrightarrow VA$ | $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma} V^{\mu} A^{\nu} F^{\lambda\sigma}$ | $W_1 = \frac{(kP)^2}{k^2} + \frac{(qP)^2}{q^2}$ <br> $\frac{W_2}{M^2} = P^2 \frac{k^2+q^2}{k^2q^2}$ | $(1 + \frac{\vec{k}^2}{4} \frac{M_V^2+M_A^2}{M_V^2M_A^2}) + \frac{\vec{k}^2}{4} \frac{M_t^2+M_A^2}{M_V^2M_A^2} \cos^2\theta$ | $1 + \cos^2\theta$ |
| $e^+e^- \longrightarrow VT$[b)] | $f^{(1)} \partial_{\mu} V^{\nu} F^{\mu}{}_{\lambda} T^{\lambda}{}_{\nu}$ <br> $+ f^{(2)} \partial^{\mu} V_{\nu} T^{\nu}{}_{\lambda} T^{\lambda}{}_{\mu}$ | * * * | $[(\frac{5}{3} + \frac{s}{2M_V^2M_t^2} \vec{k}^2) + \frac{\vec{k}^2}{2M_V^2}(\frac{5}{3} + \frac{11}{6} \frac{s}{M_t^2})]$ <br> $+ \frac{\vec{k}^2}{2M_V^2}(\frac{5}{3} - \frac{11}{6} \frac{s}{M_t^2}) \cos^2\theta$ | $1 - \cos^2\theta$ |
| $e^+e^- \longrightarrow A\bar{A}$ <br> $AA'$ | the same as that of <br> $e^+e^- \longrightarrow V\bar{V}$ | | | $1 - \cos^2\theta$ |
| $e^+e^- \longrightarrow AT$ | $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu} T^{\lambda\delta} \partial_{\delta} A^{\sigma}$ | $W_1 = (PI_qP)[\frac{7}{6}P^2 + \frac{2}{3}(PI_qP)]$ <br> $\frac{W_2}{M^2} = \frac{P^2}{q^2}[-\frac{1}{2}P^2 + \frac{2}{3}\frac{(qP)^2}{q^2}]$ | $(13 + 4\frac{\vec{k}^2}{M_t^2}) + (1 + 4\frac{\vec{k}^2}{M_t^2}) \cos^2\theta$ | $1 + \cos^2\theta$ |

a) $s = P^2$

b) Reference 9.

Table II. Angular Distributions for $e^+e^- \longrightarrow V \longrightarrow \gamma X$

| Mode | Vertex | Angular Distribution |
|---|---|---|
| $e^+e^- \longrightarrow V \longrightarrow S\gamma$ | $F_{\mu\nu}V^{\mu\nu}S$ | $1 + \cos^2\theta$ |
| $e^+e^- \longrightarrow V \longrightarrow P\gamma$ | $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}\partial^{\mu}V^{\nu}F^{\lambda\sigma}P$ | $1 + \cos^2\theta$ |
| $e^+e^- \longrightarrow V \longrightarrow V'\gamma$ | $F_{\mu\nu}V^{\mu}V'^{\nu}$ | $(3 + 2\frac{\sqrt{s}}{M_{V'}^2}\lvert\vec{k}\rvert) - (1 - 2\frac{\sqrt{s}}{M_{V'}^2}\lvert\vec{k}\rvert)\cos^2\theta$ |
| $e^+e^- \longrightarrow V \longrightarrow A\gamma$ | $\frac{1}{2}\epsilon_{\mu\nu\lambda\sigma}F^{\mu\nu}V^{\lambda}A^{\sigma}$ | $(3 + 2\frac{\sqrt{s}}{M_A^2}\lvert\vec{k}\rvert) - (1 - 2\frac{\sqrt{s}}{M_A^2}\lvert\vec{k}\rvert)\cos^2\theta$ |
| $e^+e^- \longrightarrow V \longrightarrow T\gamma$[a] | $f^{(1)}\partial_{\mu}V^{\nu}F^{\mu}{}_{\sigma}T^{\sigma}{}_{\nu}$ <br> $+ f^{(2)}\partial_{\mu}V^{\nu}F_{\nu}{}^{\sigma}T_{\sigma}{}^{\mu}$ | $[(13 + 6\frac{\vec{k}^2}{M_t^2}) - 4g\frac{\lvert\vec{k}\rvert}{\sqrt{s}}(4 + 10\frac{\sqrt{s}}{M_t^2}\lvert\vec{k}\rvert) + 2g^2\frac{\vec{k}^2}{M_t^2}(5 + 4\frac{\sqrt{s}}{M_t^2}\lvert\vec{k}\rvert)]$ <br> $+ [(1 - 6\frac{\vec{k}^2}{M_t^2}) + 4g\frac{\lvert\vec{k}\rvert}{\sqrt{s}}(2 + \frac{\sqrt{s}}{M_t^2}\lvert\vec{k}\rvert) - 2g^2\frac{\vec{k}^2}{M_t^2}(1 - 4\frac{\sqrt{s}}{M_t^2}\lvert\vec{k}\rvert)]\cos^2\theta$ <br> where <br> $g = f^{(2)}/f^{(1)}$ |

a) Reference 9.

In conclusion, we have found that the angular distributions for all pair of combinations of S, V, T, P, A produced by $e^+e^-$ annihilation can be classified in two groups according to their behaviors in the relativistic limit. One group, which shows $( 1 + \cos^2\theta )$ distribution, consists of SV, SA, PV, PT, VA, AT, while the other is composed of SS', ST, PP', PA, VV', VT, AA', which are produced with $( 1 - \cos^2\theta )$. The former may also be relevant to the observed jet structure.



References.


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5) Some models require that $P_c$ and $\chi$ states are all natural parity particles, in contrast to charmoniums. See Y.Tosa, O.Shito, T. Kaji, and J. Iizuka, DPNU-28 (November, 1975), and N.P. Chang and C.A. Nelson, Phys. Rev. D13, 1345 (1976).
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