Bayesian Personalized Feature Interaction Selection for Factorization Machines

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Factorization Machines

What is Factorization Machine?

- generic supervised learning method
- account for feature interactions with factored parameters
  - the combination of features

#Hashtag

"comics"

"marvel"

"avengers"

Feature combinations

("comics" , "marvel")

("comics" , "avengers")

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Factorization Machines

- Linear regression: $O(d)$

$$\hat{r}(\boldsymbol{x}) = b_0 + \sum_{i=1}^{d} w_i x_i$$

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- Factorization machine: $O(dk)$

$$\hat{r}(\boldsymbol{x}) = b_0 + \sum_{i=1}^{d} w_i x_i + \sum_{i=1}^{d} \sum_{j=i+1}^{d} \langle \boldsymbol{v}_i, \boldsymbol{v}_j \rangle \cdot x_i x_j$$

Factorization Machines

Example

$$\hat{r}(\text{spider-man}) = b_0 + w_{\text{comics}} + w_{\text{marvel}} + w_{\text{avengers}} + \langle \boldsymbol{v}_{\text{comics}}, \boldsymbol{v}_{\text{marvel}} \rangle + \langle \boldsymbol{v}_{\text{comics}}, \boldsymbol{v}_{\text{avengers}} \rangle + \langle \boldsymbol{v}_{\text{marvel}}, \boldsymbol{v}_{\text{avengers}} \rangle$$

#Hashtag	Feature combinations
"comics"	("comics" , "marvel")
"marvel"	("comics" , "avengers")
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Factorization Machines for Recommendation

- Effective use of **historical interactions** between users and items
- Incorporate **additional information** associated with users or items
- High-dimensional feature space
  - #feature = #user + #item + #additional
  - not all features or feature interactions are helpful

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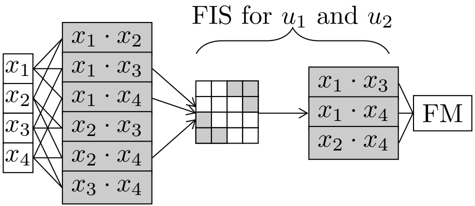
Feature Interaction Selection (FIS)

Filter out useless feature interactions

- P-FIS: Select feature interactions for users personally
- FIS: select a common set of interactions



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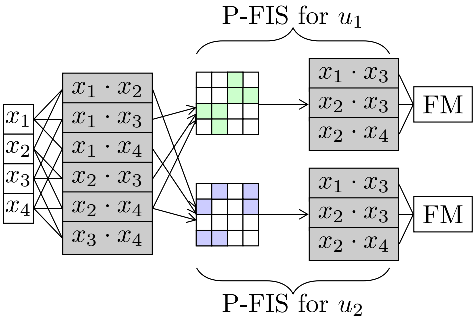


Personalized Factorization Machines (PFM)

FM

$$\hat{r}(\boldsymbol{x}) = b_0 + \sum_{i=1}^{d} w_i x_i + \sum_{i=1}^{d} \sum_{j=i+1}^{d} w_{ij} \cdot x_i x_j$$

PFM

$$\hat{r}(\boldsymbol{x}) = b_u + \sum_{i=1}^{d} w_{ui} x_i + \sum_{i=1}^{d} \sum_{j=i+1}^{d} w_{uij} \cdot x_i x_j$$

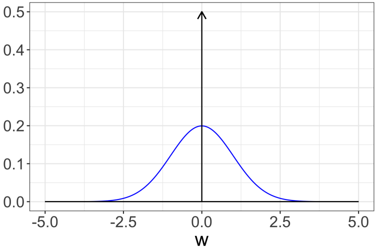
Select 1$^{\text{st}}$-order interactions $\{x_i\}$ and 2$^{\text{nd}}$-order interactions $\{x_i x_j\}$ by $\{w_{ui}\}$ and $\{w_{uij}\}$

Bayesian Variable Selection (BVS)

- Apply BVS to select feature interactions
  - avoid expensive cross-validation
- Priors for BVS
  - sparsity priors
  - spike-and-slab

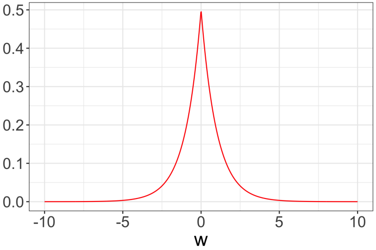
Bayesian Variable Selection

Spike-and-slab



- Spike (black arrow): $p(w=0)=0.5$
- Slab (blue line)

Sparsity priors



- $f(w)=\frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$
- $p(w=0)=0$

Hereditary Spike-and-Slab Priors

- Spike-and-slab

$$s \sim Bernoulli(\pi), \quad \tilde{w} \sim \mathcal{N}(0,1), \quad w = \tilde{w} \cdot s.$$

- Hereditary spike-and-slab
  - capture the relations between 1st-order and 2nd-order feature interactions

$$\begin{aligned} s_{ui}, s_{uj} &\sim Bernoulli(\pi_1) \\ p(s_{uij} = 1 \mid s_{ui} s_{uj} = 1) &= 1 && (\textit{Strong heredity}) \\ p(s_{uij} = 1 \mid s_{ui} + s_{uj} = 1) &= \pi_2 && (\textit{Weak heredity}) \\ p(s_{uij} = 1 \mid s_{ui} + s_{uj} = 0) &= 0 \end{aligned}$$

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Generative Procedure of BP-FIS

Algorithm Generation procedure

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```
for each user $u \in \mathcal{U}$ do
    for each feature $i \in \mathcal{F}$ do
        draw first-order interaction selection variable $s_{ui} \sim Bernoulli(\pi_1)$
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2:     **for** each feature $i \in \mathcal{F}$ **do**
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    for each feature pair $i,j \in \mathcal{F}$ do
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for each user u ∈ 𝒰 do
    for each feature i ∈ ℱ do
        draw first-order interaction selection variable s_ui ∼ Bernoulli(π_1)
        draw first-order interaction weight w̃_i ∼ 𝒩(0, 1)
        w_ui = s_ui · w̃_i
    for each feature pair i, j ∈ ℱ do
        draw second-order interaction selection variable s_uij ∼ p(s_uij | s_ui, s_uj)
        draw second-order interaction weight w̃_ij ∼ 𝒩(0, 1)
        w_uij = s_uij · w̃_ij
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  calculate the rating prediction $\hat{r}(\boldsymbol{x})$ by PFM

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for each feature vector $\boldsymbol{x} \in \mathcal{X}$ **do**
    calculate the rating prediction $\hat{r}(\boldsymbol{x})$ by PFM
    draw $r(\boldsymbol{x}) \sim p(r \mid \hat{r}(\boldsymbol{x}))$

Optimization

Maximum A Posteriori: $\arg\max_{\tilde{W},S} p(\tilde{W}, S \mid \mathcal{R}, \mathcal{X})$

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Infeasible exact inference

- space complexity: $O(md^2)$
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Variational inference

- approximate $p(\tilde{W}, S \mid \mathcal{R}, \mathcal{X})$ by $q(\tilde{W}, S)$
  - space complexity: $O(md)$
- Stochastic Gradient Variational Bayes (SGVB)
  - time complexity: $O(dk)$, same as FMs

Experimental Setup

Datasets

HetRec: Information Heterogeneity and Fusion in Recommender Systems

- **MovieLens**: rating and tagging
- **LastFM**: rating, tagging, social networking
- **Delicious**: rating, tagging, social networking

Baselines

- Factorization Machine (FM)
- Sparse Factorization Machine (SFM)
- Attentional Factorization Machine (AFM)
- Neural Factorization Machine (NFM)

Experimental Setup

Our methods

apply BP-FIS to a linear FM and a non-linear FM

- BP-FM
- BP-NFM

Evaluation

Top-N recommendation

- Leave-One-Out-Cross-Validation (LOOCV)
- ranking among 100 items
- metrics: HR@N and ARHR@N

Overall Performance

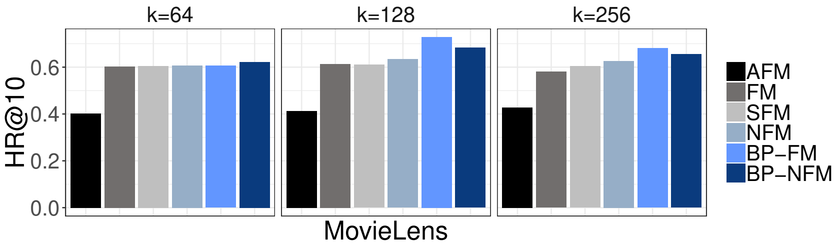
Table: Delicious

Method	HR@1	HR@10	ARHR@10
FM	0.0202	0.1147	0.0440
SFM	0.0229	0.1212	0.0465
AFM	0.0274	0.1169	0.0494
BP-FM	**0.0278**	**0.1240****	**0.0509***
NFM	0.0229	0.1065	0.0426
BP-NFM	**0.0268**	**0.1289****	**0.0504****

* and ** indicate that the best score is significantly better than the second best score with $p < 0.1$ and $p < 0.05$, respectively.

- SFM outperforms FM and AFM on HR@10: need for FIS
- BP-FM and BP-NFM significantly outperforms FMs and NFM, respectively: effect of P-FIS

Impact of Embedding Size



- $k = 64$: P-FIS has insignificant effect of FMs
- $k = 128, 256$:
  - BP-FM and BP-NFM significantly outperform FMs and NFM
  - BP-NFM does not outperform BP-FM

Case study

#hashtag

"action"
"buddy"
"comedy"
"sequel"

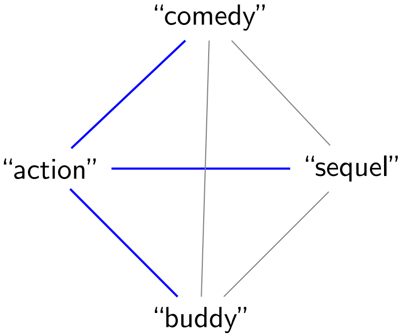


Figure: User 1, top-1 recommendation

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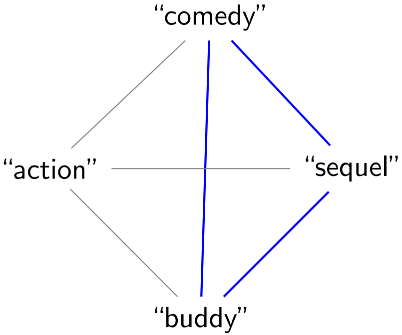

Figure: User 2, top-5 recommendation

Case study

#hashtag

"action"
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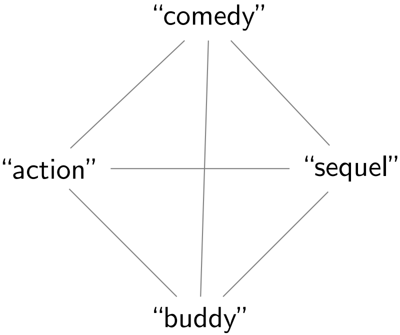

Figure: User 3, not recommended

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   - We propose hereditary spike-and-slab as priors to achieve P-FIS.
   - BP-FIS is a plug-and-play framework for FMs
3. We design an efficient optimization algorithm based on Stochastic Gradient Variational Bayes (SGVB).

Future Work

1. Extend BP-FIS to select higher-order feature interactions
2. Consider group-level personalization via clustering to speed up training

Thank you

Source code

https://github.com/yifanclifford/BP-FIS