FE5213 Group Project

Spring 2025

1 Project 1: An Optimal Monetary Policy Problem

1.1 Overview

Central banks aim to stabilize inflation and output fluctuations using monetary policy instruments such as the interest rate. This problem can be formulated as a Linear-Quadratic Regulator (LQR), where the policymaker minimizes deviations of inflation and output from their targets.

This project focuses on applying Linear-Quadratic Dynamic Programming (LQDP) to study optimal monetary policy in a simple New Keynesian framework. Students will implement and solve a central bank's optimal policy problem using Python. The project involves formulating the central bank's objective function, setting up the state-space representation, solving the optimal policy problem using numerical methods, analyzing the optimal policy response to economic shocks, and doing comparative statics when the model parameters change.

1.2 Problem Setup

Model Dynamics. We consider a simple New Keynesian model, where the economy is described by a Phillips curve and IS curve. In particular,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{2}$$

where (1) is the New Keynesian Phillips curve and (2) is the IS curve. In these equations, π_t denotes inflation, x_t output gap, i_t nominal interest rate (policy instrument), u_t cost-push shock, r_t^n demand shock, β discount factor, κ slope of the Phillips curve, σ sensitivity to the real interest rate.

Assume that u_t and r_t^n follow the following AR(1) process:

$$u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_{ut} \tag{3}$$

$$r_t^n = \rho_r r_{t-1}^n + \sigma_r \varepsilon_{rt} \tag{4}$$

where $\varepsilon_{ut} \sim N(0,1)$ and $\varepsilon_{rt} \sim N(0,1)$

Central Bank's Loss Function. The central bank seeks to minimize a quadratic loss function:

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_x (x_t - x^*)^2 + \lambda_i (i_t - i_t^*)^2\right) \tag{5}$$

where weights $\lambda_x, \lambda_i > 0$ reflect relative weight on output stabilization and penalty on interest rate volatility; x^* and i_t^* represent some optimal levels of output gap and nominal interest rate.

Let's focus on a class of monetary policy rules, the Taylor rule, which takes the following form:

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \phi_0 \tag{6}$$

Parameters. Use the following calibrated parameters that are taken from the literature.

Parameter	**Value**
β	0.99
σ	6
κ	0.024
ρ_r	0.35
ρ_u	0.35
σ_r	13.8
σ_u	0.17
λ_x	0.048
λ_i	0.236

Table 1: Parameter Values

1.3 Problems to Address

- Solve the optimal Taylor rule (6).
- Plot and discuss central bank's optimal response to shocks ε_{rt} and ε_{ut}.
- Investigate how the results would change if we change $\sigma, \rho_u, \rho_r, \lambda_x, \lambda_i$. Show the results using plots.