FE5213 Group Project

Spring 2025

1 Project 1: Linear Dynamics in a Monetary Model

1.1 Overview

Central banks aim to stabilize inflation and output fluctuations using monetary policy instruments such as the interest rate, typically via a Taylor rule. The analysis of monetary policy is an important issue in the New Keynesian monetary economics. In this project, students will analyze the linear dynamics in a monetary model using Python. It involves formulating and setting up the state-space representation, solving impulse responses to economic and monetary shocks, and conducting comparative statics as the model parameters change.

1.2 Problem Setup

Economic Dynamics. We consider a simple New Keynesian model, where the economy is described by a Phillips curve and IS curve. In particular,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

$$x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) \tag{2}$$

where (1) is the New Keynesian Phillips curve and (2) is the IS curve. In these equations, π_t denotes inflation, x_t output gap, i_t nominal interest rate (policy instrument), u_t cost-push shock, r_t^n demand shock, β discount factor, κ slope of the Phillips curve, σ sensitivity to the real interest rate.

Assume that u_t and r_t^n follow the following AR(1) process:

$$u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_{ut} \tag{3}$$

$$r_t^n = \rho_r r_{t-1}^n + \sigma_r \varepsilon_{rt} \tag{4}$$

where $\varepsilon_{ut} \sim N(0,1)$ and $\varepsilon_{rt} \sim N(0,1)$

Central Bank's Policy Rule. The central bank sets interest rate via the Taylor rule, which takes the following form:

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \nu_t \tag{5}$$

where

$$\nu_t = \rho_\nu \nu_{t-1} + \sigma_\nu \varepsilon_{\nu t} \tag{6}$$

$\varepsilon_{\nu t} \sim N(0,1)$ is the monetary policy surprise/shock.

1.3 Model Solution

Solve the Model. The Rational Expectations equilibrium for this model can be solved as follows. Conjecture a model solution where the output is linear in the state comprised of three shocks (u_t, r_t^n, ν_t):

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \underbrace{\begin{bmatrix} \gamma_\pi^u & \gamma_\pi^r & \gamma_\pi^\nu \\ \gamma_x^u & \gamma_x^r & \gamma_x^\nu \\ \gamma_i^u & \gamma_i^r & \gamma_i^\nu \end{bmatrix}}_{\text{denoted by } P} \begin{bmatrix} u_t \\ r_t^n \\ \nu_t \end{bmatrix}. \tag{7}$$

With (7), we can write the one-period-ahead expectations accordingly:

$$\mathbb{E}_t \pi_{t+1} = \gamma_\pi^u \rho_u u_t + \gamma_\pi^r \rho_r r_t^n + \gamma_\pi^\nu \rho_\nu \nu_t, \tag{8}$$

$$\mathbb{E}_t x_{t+1} = \gamma_x^u \rho_u u_t + \gamma_x^r \rho_r r_t^n + \gamma_x^\nu \rho_\nu \nu_t, \tag{9}$$

$$\mathbb{E}_t i_{t+1} = \gamma_i^u \rho_u u_t + \gamma_i^r \rho_r r_t^n + \gamma_i^\nu \rho_\nu \nu_t, \tag{10}$$

To solve for P, we can substitute items in (1), (2) and (5) with (7) and (8)-(10) and solve P using the method of undetermined coefficients. In particular, for (1):

$$\gamma_\pi^u u_t + \gamma_\pi^r r_t^n + \gamma_\pi^\nu \nu_t = \beta(\gamma_\pi^u \rho_u u_t + \gamma_\pi^r \rho_r r_t^n + \gamma_\pi^\nu \rho_\nu \nu_t) + \kappa(\gamma_x^u u_t + \gamma_x^r r_t^n + \gamma_x^\nu \nu_t) + u_t,$$

collecting items and re-arranging:

$$(\gamma_\pi^u - \beta\gamma_\pi^u \rho_u - \kappa\gamma_x^u - 1)u_t + (\gamma_\pi^r - \beta\gamma_\pi^r \rho_r - \kappa\gamma_x^r) r_t^n + (\gamma_\pi^\nu - \beta\gamma_\pi^\nu \rho_\nu - \kappa\gamma_x^\nu)\nu_t = 0.$$

This implies that

$$\gamma_\pi^u = \beta\gamma_\pi^u\rho_u + \kappa\gamma_x^u + 1, \tag{11}$$

$$\gamma_\pi^r = \beta\gamma_\pi^r\rho_r + \kappa\gamma_x^r, \tag{12}$$

$$\gamma_\pi^\nu = \beta\gamma_\pi^\nu\rho_\nu + \kappa\gamma_x^\nu. \tag{13}$$

Similarly, for (2)

$$\begin{aligned}\gamma_x^u u_t + \gamma_x^r r_t^n + \gamma_x^\nu \nu_t = {} & (\gamma_x^u\rho_u u_t + \gamma_x^r\rho_r r_t^n + \gamma_x^\nu\rho_\nu\nu_t) \\ & - \sigma[(\gamma_i^u u_t + \gamma_i^r r_t^n + \gamma_i^\nu \nu_t) - (\gamma_\pi^u\rho_u u_t + \gamma_\pi^r\rho_r r_t^n + \gamma_\pi^\nu\rho_\nu\nu_t) - r_t^n],\end{aligned}$$

collecting items, we have

$$\gamma_x^u = \gamma_x^u\rho_u - \sigma(\gamma_i^u - \gamma_\pi^u\rho_u), \tag{14}$$

$$\gamma_x^r = \gamma_x^r\rho_r - \sigma(\gamma_i^r - \gamma_\pi^r\rho_r - 1), \tag{15}$$

$$\gamma_x^\nu = \gamma_x^\nu\rho_\nu - \sigma(\gamma_i^\nu - \gamma_\pi^\nu\rho_\nu). \tag{16}$$

For (5):

$$\gamma_i^u u_t + \gamma_i^r r_t^n + \gamma_i^\nu \nu_t = \phi_\pi(\gamma_\pi^u u_t + \gamma_\pi^r r_t^n + \gamma_\pi^\nu\nu_t) + \phi_x(\gamma_x^u u_t + \gamma_x^r r_t^n + \gamma_x^\nu\nu_t) + \nu_t,$$

So we have

$$\gamma_i^u = \phi_\pi\gamma_\pi^u + \phi_x\gamma_x^u, \tag{17}$$

$$\gamma_i^r = \phi_\pi\gamma_\pi^r + \phi_x\gamma_x^r, \tag{18}$$

$$\gamma_i^\nu = \phi_\pi\gamma_\pi^\nu + \phi_x\gamma_x^\nu + 1. \tag{19}$$

Note that in the system of equations (11) through (19), there are 9 equations with 9 unknowns in the matrix P:

$$P = \begin{bmatrix} \gamma_\pi^u & \gamma_\pi^r & \gamma_\pi^\nu \\ \gamma_x^u & \gamma_x^r & \gamma_x^\nu \\ \gamma_i^u & \gamma_i^r & \gamma_i^\nu \end{bmatrix}$$

which can be easily solved using matrix inversion.

State-Space Form. With P, the model is characterized by a linear state-space system with the measurement equation

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = P \begin{bmatrix} u_t \\ r_t^n \\ \nu_t \end{bmatrix}, \tag{20}$$

and the transition equation

$$\begin{bmatrix} u_t \\ r_t^n \\ \nu_t \end{bmatrix} = \begin{bmatrix} \rho_u & 0 & 0 \\ 0 & \rho_r & 0 \\ 0 & 0 & \rho_\nu \end{bmatrix} \begin{bmatrix} u_{t-1} \\ r_{t-1}^n \\ \nu_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_u & 0 & 0 \\ 0 & \sigma_r & 0 \\ 0 & 0 & \sigma_\nu \end{bmatrix} \begin{bmatrix} \varepsilon_{ut} \\ \varepsilon_{rt} \\ \varepsilon_{\nu t} \end{bmatrix}. \tag{21}$$

1.4 Parameters

In the benchmark analysis, use the following calibrated parameters that are taken from the literature.

Parameter	**Value**
β	0.99
σ	1/6
κ	0.024
ρ_r	0.35
ρ_u	0.35
ρ_ν	0.35
σ_r	3.7
σ_u	0.4
σ_ν	1
ϕ_π	1.5
ϕ_x	0.5

1.5 Main Problems to Address

- Solve P and formulate the model solutions as a linear state space system.
- Plot and discuss the dynamic responses of $\pi_{t+i}, x_{t+i}, i_{t+i}$ to a one standard deviation change in economic shocks $\varepsilon_{rt}, \varepsilon_{ut}$ and the monetary policy shock $\varepsilon_{\nu t}$.
- Investigate how the results would change if we change $\kappa, \rho_u, \rho_r, \phi_\pi, \phi_x$. Discuss your results.